Entrepreneurial Optimism and Venture Capital: A Quantitative Analysis

Richard Fu* Ajay Subramanian†

February 24, 2022

Abstract

We develop a dynamic structural model to quantitatively explore the impact of asymmetric beliefs and agency conflicts on venture capital relationships. In our multi-period principal-agent framework, a representative venture capitalist (VC) and an entrepreneur (EN) have imperfect information and differing beliefs about the intrinsic quality of a project in addition to having differing attitudes towards its risk. We derive the equilibrium of the stochastic dynamic game in which the VC's investments, the EN's compensation and effort choices, and the project's duration are derived endogenously. Consistent with observed contractual structures, the equilibrium dynamic contract features staged investments by the VC, progressive vesting of the EN's stake, and the presence of inter-temporal performance targets that must be realized for the project to continue. We use the simulated method of moments to estimate the model parameters by matching a broad set of moments on the round-by-round returns, risks and termination probabilities of VC projects. In particular, we indirectly infer the average degree of entrepreneurial optimism implied by the data and show that it is significant enough to reconcile the discrepancy between the discount rates used by VCs (35-50%), which adjust for optimistic payoff projections by ENs, and the average expected return of VC projects (15%). EN optimism enhances VC project values by over 80% on average. Relative to the benchmark scenario with symmetric beliefs, VCs are able to exploit EN optimism to increase EN pay-performance sensitivities by almost 100% and garner a major portion of the rents generated by EN optimism. We derive several testable implications for how the degree of EN optimism, permanent and transitory components of projects' risks, and projects' physical and human capital intensities influence VC project values, implied discount rates that adjust for EN optimism, EN compensation contracts, and project durations.

*University of Alabama at Birmingham, RichardFu@uab.edu

†Georgia State University, asubramanian@gsu.edu

1 Introduction

A growing literature in behavioral economics argues that individual behavior often departs from the neoclassical paradigm of complete rationality. A number of studies demonstrate that agents' decisions are significantly influenced by dispositional characteristics such as optimism (see Baker et al, 2005). We develop a dynamic, structural model of venture capital investment to derive quantitative assessments of the impact of entrepreneurial optimism on the characteristics of venture capital relationships: |the economic value they generate, the structures of dynamic contracts between venture capitalists (VCs) and entrepreneurs (ENs), the durations of VC projects, the manner in which VC investment is staged over time, and the extent to which EN optimism could mitigate agency costs of risk-sharing between VCs and ENs. Although the model we develop is potentially applicable in other economic settings, we focus on VC-EN relationships for two principal reasons. First, venture capital is one of the most important mediums through which technological innovation, a key driver of economic growth, is financed. Second, because innovation is often characterized by high levels of uncertainty and differing beliefs about project quality, it is anecdotally suggested that optimism significantly affects VC investments. If optimism, indeed, influences VC projects and thus the financing of innovation, it clearly has important aggregate welfare consequences.

In our dynamic principal-agent model, VCs and ENs could have asymmetric beliefs about the intrinsic quality of projects in addition to having asymmetric attitudes towards their risk. We derive the equilibrium in which the VC's investments, the EN's compensation and effort choices, and the project's duration are derived endogenously. Consistent with observed contractual structures, the equilibrium dynamic contract features staged investments by the VC, progressive vesting of the EN's stake, and the presence of inter-temporal performance targets that must be realized for the project to continue. We structurally estimate the model parameters by matching disparate statistics on the round-by-round returns, risks and termination probabilities of VC projects predicted by the model to their observed values in the data. Our estimates shows that the data are consistent with a significant level of EN optimism that is high enough to explain the discrepancy between the discount rates used by VCs to value projects (˜ 40%), which adjust for optimistic payoff projections by ENs, and the average expected return of VC projects (˜ 15%) (e.g. Fuerst and Geiger (2005)). EN optimism offsets the agency costs of risk-sharing between VCs and EN's, thereby leading to substantially more powerful EN incentives, and increases VC project values by over 80%. Optimism is, therefore, a major determinant of the durations and economic values of VC relationships. We derive novel testable implications for the effects of the degree of EN optimism, the true intrinsic qualities of projects, and the permanent and transitory components of their risks on project values and durations, the values of VC stakes, the implied discount rates of VCs, and the characteristics of contracts between VCs and ENs. The tractability of the model and its ability to match disparate statistics associated with VC projects suggest that it could be a useful tool to value risky ventures.

We develop a discrete-time, multi-period framework with a finite horizon. A cash-constrained, risk-averse EN with a risky project approaches a VC (who could represent a group of investors) for funding at date zero. Each period represents a *round* of financing. The project generates earnings through physical capital investments by the VC and human capital (effort) investments by the EN. The project requires a "startup" capital investment that represents the initial capital stock. If the VC decides to invest in the project, she provides the initial capital and offers the EN a long-term contract that stipulates her subsequent capital investments in the project in each round (conditional on the project's continuation) and the evolution of the EN's stake in the project's payoffs. Depending on the project's interim performance, termination could correspond to a "negative" outcome such as liquidation or a "positive" outcome such as an IPO or sale to outside investors.

We model the project's *termination payoff* in each round which represents the payoff from the project if the VC-EN relationship is terminated. The termination payoff is the net present value of future earnings from the project if there are no further physical and human capital investments by the VC and EN, respectively. It can, therefore, be interpreted as the *outside value* of the project. As in Neher (1999), the VC and the EN possess project-specific skills that are not transferrable. Thus, the outside value of the project diifers, in general, from its *inside value*, which includes the value generated by future physical and human capital investments by the VC and EN, respectively. As in Holmstrom and Milgrom (1987), we assume for simplicity that there are no interim cash flows from the project or, alternatively, any interim cash flows are reinvested and, thereby, reflected in the project's termination payoff. The change in the project's termination payoff in each round is proportional to the capital stock, which evolves stochastically as a lognormal process, but increases in expectation with the VC's physical capital investment during the round. Consistent with observed VC project payoffs, the lognormal evolution of the capital stock process generates skewness in the project's payoffs (Cochrane, 2005).

The *productivity* of capital has three components: a fixed, non-discretionary component that represents the project's *intrinsic quality*; a discretionary component that is determined by the VC's investment and the EN's effort during the round; and a normally distributed productivity shock that is independent across rounds. The VC and EN both have imperfect information about the project's intrinsic quality and possibly differing beliefs. The VC's and EN's initial priors are normally distributed with potentially different means, but a common variance, that is, they *agree to disagree* about their respective initial assessments. The VC and EN rationally update their assessments of the project's intrinsic quality from observations of the project's termination payoff and capital stock processes. The difference between the EN's and VC's mean assessments of the project quality is the *degree of entrepreneurial (EN) optimism.* The common variance of the VC's and EN's assessments of the project's quality is the project's *transient risk*, which declines over time as the VC and EN update their assessments. The second discretionary component of the project's productivity is determined by the VC's capital investment and the EN's effort via a Cobb-Douglas production function. As in the literature on incomplete contracting (see, for example, Hart and Moore, 1998, Aghion and Bolton, 1992), the discretionary output is *observable* but *non-verifiable*, hence *non-contractible*. Because the discretionary output is non-contractible, the EN must be provided with appropriate incentives to exert effort through an explicit contract with the VC that can only be contingent upon the observable and verifiable termination payoff and capital stock processes. The variance of the productivity shock—the third component of the project's productivity—is the project's *intrinsic risk* that is constant through time.

The VC is risk neutral whereas the EN is risk-averse with linear inter-temporal *recursive* preferences described by a subjective *stochastic discount factor* that reflects his costs of bearing risk (see Duffie and Epstein, 1992; Alvarez and Jermann, 2000). We derive the equilibrium of the stochastic dynamic game in which the VC's investments over time, the EN's path-dependent payoff upon termination, and the inter-temporal performance targets that must be met for the project to continue are endogenously determined. The change in the EN's promised payoff or stake in the project over any round has two components: a *performance-sensitive* component contingent on the change in the termination payoff, and a *performance-invariant* component that depends on the project's performance history through its effect on the updated assessments of the project's intrinsic quality.

As the change in the termination payoff in any round is proportional to the project's capital stock, the EN's pay-performance sensitivity and effort as well as the VC's investments as a proportion of the capital stock are deterministic functions of time conditional on the project's continuation. The performance-invariant component of the change in the EN's stake over a round is, however, stochastic and depends on the project's termination payoff history through its effect on the VC's and EN's updated assessments of the project's intrinsic quality. The structure of the equilibrium dynamic contract between the VC and EN is broadly consistent with observed VC contractual structures: (i) the VC's payoff has "debt" and "equity" components; (ii) the VC optimally stages her investments; (iii) the EN's stakes in the project are progressively vested over time; and (iv) the project is continued if and only if periodic milestones or performance targets are met (see Gompers, 1995, and Kaplan and Stromberg, 2003).

We analytically derive testable implications for how project characteristics influence the dynamics of contractual variables. The optimal contract balances the costs of risk-sharing between the VC and the EN with the positive rents that the VC is able to extract due to the EN's optimism. Indeed, as an optimistic EN has a higher expectation of future project payoffs, he overvalues the performance-sensitive component of his compensation relative to the performance invariant component. The optimal contract, therefore, exploits the EN's optimism by making his pay-performance sensitivity increase with his optimism. Because the degree of EN optimism declines over time as the VC and EN learn about the project's intrinsic quality, the rents from the EN's optimism decrease relative to the costs of risk-sharing. The VC's proportional investment rates, the EN's pay-performance sensitivities, and the EN's effort, therefore, decrease monotonically over time to their respective steady state values. The paths of the VC's proportional investment rate, the EN's pay-performance sensitivity, and the EN's effort (conditional on continuation of the relationship) all decline with the project's intrinsic and transient risks and increase with the degree of EN optimism. The duration of the project increases with the degree of EN optimism and decreases with the EN's cost of risk. These results illustrate the negative effects of risk and the positive effects of EN optimism on risk sharing between the VC and EN and, therefore, the power of incentives for the EN. The negative relation between duration and the degree of EN optimism is consistent with the evidence in Kaplan and Stromberg (2003) that experienced entrepreneurs, who are likely to have more realistic beliefs, receive fewer rounds of financing.

We take our structural model to the data to derive quantitative implications for how entrepreneural optimism impacts VC projects. We use the simulated method of moments to estimate the model parameters by matching statistics on the round-by-round returns, risks and termination rates of VC projects reported in Cochrane (2005). The observed statistics all lie within the 95% confidence intervals of their model-predicted values. Our estimation allows us to indirectly infer values of deep structural parameters that are difficult to directly measure in the data: the average intrinsic quality of VC projects, the degree of EN optimism, the EN's cost of risk and disutility of effort, the capital and labor elasticities of VC projects, and projects' intrinsic and transient risks. Consistent with anecdotal evidence, our estimation shows that the EN is, indeed, significantly optimistic relative to the VC. The baseline estimate of the EN's mean assessment of the project's intrinsic quality is approximately five times the VC's mean assessment. Relative to the benchmark scenario with symmetric beliefs, the EN's pay-performance sensitivities are approximately twice as high, which reflects the VC's exploitation of EN optimism. The higher pay-performance sensitivities, in turn, generate significantly greater effort by the EN and higher investments by the VC. We find that, by mitigating the detrimental effects of the agency costs of risk sharing, EN optimism enhances VC project values by over 80% in the baseline model. Due to the substantial rents the VC extracts by exploiting EN optimism, we also find that the benefits of EN optimism more than completely offset the loss in the value of the VC's stake in the project due to the agency costs of risk-sharing. The positive effects of EN optimism on project value are consistent with the empirical evidence reported in Gelderen et al (2005).

We examine whether the estimated degree of EN optimism can reconcile the discrepancy between the high discount rates (˜ 35-50%) used by VCs (see Sahlman (1990), Gladstone and Gladstone (2003), Fuerst and Geiger (2005)) and the average expected return of VC projects (˜ 15%) (Cochrane (2005)). Although it has been anecdotally suggested that the higher discount rates adjust for optimistic projections by ENs, there is no prior research that rigorously analyzes whether EN optimism is, indeed, high enough to reconcile the discrepancy. We define the *implied discount rate* (IDR) as the rate at which the VC would discount the EN's projections of the project's payoffs to equal her own valuation. We find that the IDR in our baseline model is 42%, which implies that the EN "optimism premium" is, indeed, high enough to explain the discount rates used by VCs in reality.

We obtain additional testable implications of the model by examining how the project's true intrinsic quality, the degree of EN optimism, the project's physical and human capital intensities, and the project's intrinsic and transient risks affect fundamental characteristics of VC-EN relationships: project value and duration, the value of the VC's stake, and the implied discount rate used by the VC to pare down the EN's optimistic payoff projections. Consistent with our earlier analytical results, EN optimism significantly increases these characteristics. Unsurprisingly, an increase in the project's true mean intrinsic quality causes increases the project value, VC value and expected duration to increase. The VC's implied discount rate, however, declines because the increase in the VC's valuation of the project necessitates a lower discount rate to adjust for the EN's optimistic payoff projections.

The project value, VC value and the expected project duration all decline with the project's intrinsic and transient risks. An increase in the transient risk lowers the degree of EN optimism at each date because the "signal to noise ratio" is increased so that the EN "learns faster". Hence, the economic rents to the VC in each round from the EN's optimism are lowered relative to the costs of risk-sharing so that the EN's pay-performance sensitivity, effort, and the VC's investment declines. In contrast, an increase in the intrinsic risk increases the degree of EN optimism at each date because the EN "learns more slowly" but also increases the costs of risk-sharing. The costs of risk-sharing, however, outweigh the benefits of the EN's optimism so that the EN's pay-performance sensitivity, effort, and the VC's investment also decrease with the intrinsic risk. The opposing impacts of the transient risk and intrinsic risk on EN optimism, however, cause the VC's implied discount rate to vary in opposing directions. In all our results, we find that project value is positively related to project duration, which is consistent with the evidence in Gompers (1995).

We contribute to the literature by developing a dynamic structural model to quantitatively analyze the impact of asymmetric beliefs and agency conflicts on venture capital relationships. Our theoretical framework belongs to the class of dynamic principal-agent models (see Laffont and Martimort (2002) and Bolton and Dewatripont (2005) for surveys). Starting with Gibbons and Murphy (1992), several studies in this literature incorporate imperfect public information, asymmetric beliefs and learning in dynamic principal-agent frameworks (Adrian and Westerfield (2009), Giat et al. (2010), Giat and Subramanian (2013), Prat and Jovanovic (2014), He et al. (2017), DeMarzo and Sannikov (2017)). From a theoretical standpoint, the themes we explore—the impact of heterogeneous beliefs and learning on incentives—have been highlighted in this literature. From a technical standpoint, however, we differ from these studies in that the project's earnings process is lognormally distributed due to the capital stock process being lognormal, thereby resulting in a skewed project payoff distribution that better captures the observed characteristics of VC projects (Cochrane (2005)). Consequently, the capital stock process is an additional state variable, which increases the dimensionality of the state space in the recursive formulation of the contracting problem. Our modeling of the entrepreneur's preferences via a stochastic discount factor that belongs to the class of recursive stochastic differential utilities (Duffie and Epstein (1992)) facilitates analytical tractability as the capital stock affects the principal's value function multiplicatively. [1]

Notwithstanding the technical considerations above, we view the primary contribution of this paper as being the implementation of the first (to our knowledge) structural approach to *quantitatively examine* how the interplay between asymmetric beliefs and agency conflicts affects venture capital financing: the manner in which VC investment is staged over time,dynamic contracts between VCs and ENs, and the duration and economic value of VC projects. Sorensen (2007) develops a structural two-sided matching model to quantitatively assess the relative impacts of deal sourcing and investment selection versus VC value-added on VC project returns, and concludes that both matter. Our analysis shows that entrepreneurial optimism is a critical driving factor that affects incentives and value-added in VC projects. In her survey of recent research in behavioral corporate finance, Malmendier (2018) highlights the usefulness of structural approaches to quantify the effects of behavioral biases. There are, however, relatively few studies that develop structural models to analyze the impacts of behavioral distortions and none (to the best of our knowledge) in the context of venture capital. Alti and Tetlock (2014) structurally estimate the impact of investor overconfidence on firms' investment decisions. Jung and Subramanian (2013) quantify the effects of heterogeneous beliefs on firms' dynamic capital structure decisions. Ma et al. (2020) estimate losses in aggregate productivity and output from systematic managerial biases in forecasting. Our structural analysis complements reduced-form or survey-based approaches to analyzing various aspects of the performance of VC relationships (e.g., Gompers (1995), Gompers and Lerner (1997, 1999), Kortum and Lerner (2000), Kaplan and Stromberg (2003), Ljungvist and Richardson (2003),

[1] A number of recent studies apply the principal-agent approach with heterogeneous beliefs in the corporate finance arena with implications for capital structure and payout policy (see, Jung and Subramanian, 2013, Yang, 2013, Bayar et al.,2015a, Bayar et al, 2015b)

Cochrane (2005), Kaplan and Schoar (2005), Korteweg and Sorensen (2011), Driessen et al. (2012), Ewens et al. (2013), Gompers et al. (2020)).

The paper proceeds as follows. We formally present the model in Section 2. In Section 3, we outline the derivation and properties of the optimal contract. In Section 4, we present the quantitative analysis of the model. Section 5 concludes. We provide all supplementary materials in the Appendices.

2 The Model

We develop a discrete-time model with a finite horizon T and dates, 0,1,2...T that are equally spaced for notational convenience. Each period $[t, t+1]$ or period t corresponds to a "round" of financing. Hence, we alternatively refer to a period as a round. A cash-constrained entrepreneur (EN) with a project approaches a venture capitalist (VC) (who could represent a group of investors in general) for financing at date zero. The project generates earnings through physical capital investments by the VC and human capital (effort) investments by the EN. The project requires an initial or "startup" capital investment Q_0. If the VC decides to invest in the project, she provides the initial capital Q_0 and offers the EN a long-term contract that stipulates her subsequent capital investments in the project in each round (conditional on the project's continuation) and the EN's compensation. Depending on the project's interim performance, termination could correspond to a "negative" outcome such as liquidation or a "positive" outcome such as an IPO or sale to outside investors.

2.1 Project Evolution

The project's capital stock, Q_t, at date t, which determines the scale of the project, evolves stochastically as a lognormal process.

$$Q_{t+1} - Q_t = \Delta Q_t = Q_t[\mu_Q + c_t + \sigma_Q N_{t+1}^Q]. \tag{2.1}$$

As we see shortly, the lognormality of the capital stock process generates a skewed distribution of project payoffs, which is consistent with the characteristics of observed VC projects (e.g., Cochrane

(2005)). In the above, $c_t Q_t$ is the VC's investment in round t (that is, c_t is the VC's investment as a proportion of the capital stock); N_{t+1}^{Q} is a standard normal random variable; and μ_Q, σ_Q are the constant drift and volatility of the capital stock process. The random variables, $\left\{N_{t+1}^{Q}; t = 0, ..., T-1\right\}$ are independently and identically distributed across time. We can also interpret Q_t as the "effective" or "installed" capital stock. For expositional convenience, we refer to Q_t as the capital stock since the distinction is irrelevant to our analysis.

In addition to the capital stock, the second key state variable in the model is the project's *termination payoff*, V_t, at date t, which represents the payoff from the project if the VC-EN relationship is terminated at date t. The termination payoff, V_t, is the net present value (NPV) of future earnings from the project if there are no further physical and human capital investments by the VC and EN, respectively. The termination payoff can be interpreted as the "outside value" of the project at date t. Similar to Neher (1999), the VC and the EN possess project-specific skills that are not transferrable. Thus, the outside value of the project diifers, in general, from its "inside value", which includes the value generated by future physical and human capital investments by the VC and EN, respectively. Put differently, only the outside value is pledgeable to outside investors at any date. ,,As in Holmstrom and Milgrom (1987), we assume for simplicity that there are no interim cash flows from the project or, alternatively, any interim cash flows are reinvested and, thereby, reflected in the project's termination payoff. We assume that $V(0) \geq Q(0)$, which simply reflects that the project's termination payoff at date zero is at least equal to the initial capital stock.

The evolution of the project's termination payoff is driven by the physical capital investments by the VC and human capital (effort) investment by the EN, and is given by

$$\delta V_{t+1} - V_t = \Delta V_t = Q_t \left[\overbrace{\Theta}^{\text{intrinsic quality}} + \overbrace{A {c_t}^{\alpha} \eta_t^{\beta}}^{\text{discretionary output}} + \overbrace{s N_{t+1}^{V}}^{\text{project risk}} \right], \tag{2.2}$$

In the above, $\delta \in (0,1)$ is the period discount factor that all agents use to discount future payoffs. Recall that the termination payoff is the NPV of future earnings from the project if the project is terminated, where δ is the factor used to discount future earnings. Hence, the termination payoff changes over the period simply due to the effects of compounding at the periodic interest rate, $\delta^{-1} - 1$, which is reflected in the term, δV_{t+1}, on the L.H.S. The R.H.S. of (2.2), therefore, represents the impact of the VC and EN on the change in the termination payoff *in addition* to the effects of compounding. Hereafter, we refer to the term, $\Delta V_t = \delta V_{t+1} - V_t$, as the change in the *discounted* termination payoff over period t for convenience. We now discuss each of the components of the change in the discounted termination payoff.

2.2 VC-EN Beliefs

As indicated by (2.2), three sources influence the change in the discounted termination payoff. The first component is the project's *intrinsic quality* Θ, which determines the expected change in the discounted termination payoff *independent* of actions by the VC and EN. The VC and EN both have imperfect information about the project's intrinsic quality, Θ, and possibly differing beliefs. The VC's and EN's initial priors on Θ are normally distributed with potentially different means, but a common variance, that is,

$$\Theta^{VC} \sim N(\mu_0^{VC}, \sigma_0^2), \Theta^{EN} \sim N(\mu_0^{EN}, \sigma_0^2) \tag{2.3}$$

Their respective beliefs about the project's intrinsic quality are common knowledge. In other words, they *agree to disagree* about their respective initial assessments.[2] The VC and EN rationally update their assessments of the project's intrinsic quality in a Bayesian manner based on observations of the project's termination payoff process, V, and the capital stock process, Q.

The difference, $\Omega_0 = \mu_0^{EN} - \mu_0^{VC}$ is the initial *degree of asymmetry in beliefs* or the *degree of entrepreneurial (EN) optimism.* As we will show later, the equilibrium contract does not depend on how the EN's and VC's mean assessments of project quality relate to its *true* mean. We, therefore, make no assumption here about the true mean in our theoretical analysis. For expositional convenience, we assume that the EN is optimistic relative to the VC in our theoretical analysis, that is, $\mu_0^{EN} \geq \mu_0^{VC}$. Our structural estimation of the model in Section 5 confirms that entrepreneurs are, indeed, significantly optimistic relative to VCs in the data. Although the VC and EN disagree on the mean of the project's intrinsic quality, they agree on its variance, σ_0^2. [3] We refer to σ_0^2 as the

[2] See, for example, Morris (1995) and Allen and Gale (1999) for discussions of the theoretical foundation for the assumption of heterogeneous priors that are common knowledge, and Ito (1990), Kandel and Pearson (1995), Chemmanur et al. (2010) and Edmonds et al. (2015) for empirical evidence in support of the assumption.

[3] The literature on behavioral economics(see Baker et al., 2005) distinguishes between *optimism* and *overconfi-*

projects initial *transient risk.* The transient risk declines over time as the VC and EN update their priors on Θ in a Bayesian manner from the observations of the project's earnings process.

The second component of the change in the discounted termination payoff is the *discretionary output* that is generated by the VC's capital investment c_tQ_t and the EN's effort η_t, and is described by a Cobb-Douglas production function, in which the total factor productivity, A, and the parameters, α and β are observable. As in the literature on incomplete contracting (see, for example, Hart and Moore, 1998, Aghion and Bolton, 1992), the discretionary output is *observable* but *non-verifiable*, hence *non-contractible.*[4] Because the discretionary output is non-contractible, the EN must be provided with appropriate incentives to exert effort through an explicit contract with the VC that can only be contingent upon the observable and verifiable termination payoff process.

The third component of the change in the discounted termination payoff, sN^V_{t+1}, represents a random shock to the evolution of the termination payoff, where $\left\{N^V_{t+1}; t = 0, ..., T-1\right\}$ are i.i.d. standard normal random variables that are independent of the variables $\left\{N^Q_{t+1}; t = 0, ..., T-1\right\}$ that drive the evolution of the capital stock process (see (2.1)). We refer to the variance, s^2, as the project's *intrinsic risk* that remains constant through time.

We now describe how the VC's and EN's posterior assessments of project quality as well as the transient risk vary over time due to Bayesian updating by the VC and EN. Define the process

$$\Psi_t = \sum_{i=0}^{t-1} (\Theta + sN^V_{i+1}) \tag{2.4}$$

The process, Ψ_t, is effectively observable by the VC and EN because they observe the capital stock process, Q_t, the termination payoff process, V_t, as well as c_t and η_t. By Bayes' law, therefore, the posterior distribution on Θ at each date t is $N(\mu^\ell_t, \sigma^2_t), \ell = VC, EN$, where:

$$\sigma^2_t = \frac{s^2\sigma^2_0}{s^2 + t\sigma^2_0}, \mu^\ell_t = \frac{s^2\mu^\ell_0 + \sigma^2_0(\Psi_t)}{s^2 + t\sigma^2_0} \tag{2.5}$$

---

*dence.*The EN is "optimistic "if his assessment of the mean of the project quality is *higher* than that of the VC, while he is "overconfident "if his assessment of the variance of the project quality is *lower* than that of the VC. In the terminology of the behavioral economics literature, therefore, the EN could be optimistic, but not overconfident in our framework.

[4] The periodic observation or monitoring of the EN's effort by the VC is supported by the evidence in Tian (2001).

The variance of the posterior, σ_t^2, is the project's *transient risk* at date t that declines over time in contrast with the project's intrinsic risk, s^2. The degree of asymmetry in beliefs or the degree of EN optimism, Ω_t in period t is given by

$$\Omega_t := \mu_t^{EN} - \mu_t^{VC} = \frac{s^2\Omega_0}{s^2 + t\sigma_0^2} = \frac{\Omega_0}{\sigma_0^2}\sigma_t^2. \tag{2.6}$$

The degree of EN optimism declines over time as the VC's and EN's respective mean posterior assessments of the project's intrinsic quality converge.

2.3 VC-EN Preferences and Contracting

At date 0, the VC offers the EN a long-term contract that specifies her capital investments in each round (conditional on the project's continuation), the EN's recommended effort, the EN's promised payoff at termination, and the project's termination time. Let $\{\mathcal{F}_t\}$ denote the information filtration generated by the project's capital stock, termination payoff and discretionary output processes. The EN's long-term contract can be expressed as (P_τ, c, η, τ), where c and η are $\{\mathcal{F}_t\}$-adapted stochastic processes denoting, respectively, the VC's capital investments and the EN's effort; τ is an $\{\mathcal{F}_t\}$- stopping time denoting the project's termination time; and P_τ is a nonnegative $\{\mathcal{F}_\tau\}$-measurable random variable that represents the EN's contractually promised payoff upon the project's termination so that $V_\tau - P_\tau$ represents the VC's payoff. Following the traditional principal-agent literature, there is no loss of generality in requiring that the contract specify the EN's effort and be incentive compatible with respect to the specified effort. That is, given the termination date τ, the agent's contractually promised payoff P_τ, and the principal's investments c, it is optimal for the agent to choose the effort levels η specified by the contract (see Holmstrom and Milgrom (1987)).

The VC is risk-neutral and discounts her future cash flows using the discount factor δ. For analytical tractability, we assume that the EN too has linear inter-temporal preferences, but with a subjective *stochastic discount factor* that reflects his costs of bearing risk (see Alvarez and Jermann, 2000). The EN incurs an additive monetary disutility of effort. More precisely, the EN's subjective valuation of his future payoffs (including his disutility of effort) at any date $t \le \tau$ from a contract (P_τ, c, η, τ) is

$$\mathbb{E}_t^{EN}\left[\left[\delta^{\tau-t}e^{-\frac{1}{2}\lambda^2(\tau-t)-\lambda(N_\tau^V-N_t^V)}P_\tau-\sum_{s=t}^{\tau-1}\delta^{s-t}e^{-\frac{1}{2}\lambda^2(s-t)-\lambda(N_s^V-N_t^V)}kQ_s\eta_s^\gamma\right]\right] \tag{2.7}$$

where the notation $\mathbb{E}_t^{EN}$ denotes the EN's expectation conditioned on the information available at time t (that is the σ-field $\mathcal{F}_t$), and the superscript on the expectation denotes that it is with respect to the EN's beliefs.

In (2.7), $\delta^u e^{-\frac{1}{2}\lambda^2 u-\lambda N_u^V}$ is the stochastic discount factor by which the EN discounts a payoff at date u, where $\lambda>0$ measures his cost of bearing risk. A nonzero cost of risk¸ λ, in the EN's subjective valuation (2.7) of his future payoffs (or his stake in the firm) reflects his imperfect access to outside credit markets. In analogy with fundamental results in asset pricing theory (Duffie, 2001), we directly model the EN's subjective valuation of his stake in the firm as the expectation of his future total payoffs weighted by a subjective stochastic discount factor or *valuation kernel.* In Appendix A, we show that the EN's objectivce as described by (2.7) actually belong to the general class of recursive or "stochastic differential" utilities (Duffie and Epstein, 1992).

In Section 5, we estimate λ by matching the model's predictions to data. In (2.7), $kQ_t\eta_t^\gamma;\ k>0$, is the EN's disutility from exerting effort, η_t in round t. As in Holmstrom and Milgrom (1987), we express the EN's disutility of effort in monetary terms. The EN's disutility of effort increases with the scale of the project represented by its capital stock. This is consistent with the fact that the expected change in the discounted termination payoff in (2.2) due to investment and effort is also proportional to the capital stock.

In our subsequent analysis, it is useful to express the EN's valuation (2.7) of his future payoffs in an alternate form. The process $e^{-\frac{1}{2}\lambda^2 t-\lambda N_t^V}; t\in\{1,2,...,T\}$ is a square-integrable $\{\mathcal{F}_t\}$ $-$martingale, and is the Radon-Nikodym derivative process of a new probability measure equivalent to the original one (see Chapter 6 of Duffie, 2001). The EN's valuation (2.7) can then be expressed as

$$\overline{E}_t^{EN}\left[\left[\delta^{\tau-t}P_\tau-\sum_{s=t}^{\tau-1}\delta^{s-t}kQ_s\eta_s^\gamma\right]\right], \tag{2.8}$$

where the expectation above is under the new probability measure. From (2.8), we can express the EN's objective by assuming that he is risk-neutral, but under a *risk-adjusted probability*—the EN's valuation probability—that refects his costs of bearing risk. For future reference we note that by

Girsanov's theorem (see Chapter 6 of Duffie, 2001), the termination payoff process evolves under the EN's valuation probability as

$$\delta V_{t+1} - V_t = \Delta V_t = Q_t \left[\Theta + A{c_t}^{\alpha} \eta_t^{\beta} - \lambda s + s\overline{N}_{t+1}^{V} \right], \tag{2.9}$$

where

$$\overline{N}_t^V = N_t^V + \lambda \tag{2.10}$$

is a normal random variable under the EN's valuation probability. By (2.4), (2.5) and (2.10), the posterior mean of the project's intrinsic quality under the EN's valuation probability is

$$\overline{\mathbb{E}}_t^{EN}\left[\Theta\right] = \overline{\mu}_t^{EN} = \frac{s^2 \mu_0^{\ell} - \lambda t \sigma_0^2 + \sigma_0^2 (\sum_{i=0}^{t-1} (\Theta + s\overline{N}_{i+1}^V))}{s^2 + t\sigma_0^2}; \tag{2.11}$$

The posterior variance of the EN's assessment under the EN's valuation probability is the same as under the physical probability and is given by

$$\sigma_t^2 = \frac{s^2 \sigma_0^2}{s^2 + t\sigma_0^2}$$

For future reference in the derivation of the equilibrium, we define the EN's *promised payoff*, P_t, from the contract at any date t as

$$P_t := \overline{E}_t^{EN} \left[\delta^{\tau - t} P_\tau - \sum_{s=t}^{\tau-1} \delta^{s-t} k Q_s \eta_s^{\gamma} \right] \tag{2.12}$$

From (2.8) and (2.12), the promised payoff at any date is the EN's *certainty equivalent payoff* at that date. The relative bargaining power between the VC and EN is determined by the promised payoff, P_0, that the EN must be guaranteed at date zero. A contract (P_τ, c, η, τ) is *incentive feasible* if and only if it is incentive compatible for the EN and meets his participation constraint at date zero, which specifies that his initial promised payoff must be delivered by the contract. Hence, the contract must satisfy

$$\eta = \arg\max_{\eta'} \overline{E}_0^{EN} \left[\delta^{\tau} P_\tau - \sum_{t=0}^{\tau-1} \delta^t k Q_t \eta_t'^{\gamma} \right] \tag{2.13}$$

$$\overline{E}_0^{EN}\left[\delta^\tau P_\tau - \sum_{t=0}^{\tau-1}\delta^t k {\eta_t}^\gamma\right] = P_0 \tag{2.14}$$

The optimal contract is then an incentive feasible contract that maximizes the VC's expected discounted payoff net of her investments. More precisely, an incentive feasible contract (P_τ, c, η, τ), is optimal if and only if it solves the optimization problem

$$(P_\tau, c, \eta, \tau) \quad \in \quad \arg\max_{(P_{\tau'}, c', \eta', \tau')} E_0^{VC}\left[\left(\delta^{\tau'}\left[V_{\tau'} - P_{\tau'}\right] - \sum_{t=0}^{\tau'-1}\delta^t c_t Q_t)\right)\right] \tag{2.15}$$

where E_0^{VC} denotes the expectation with respect to the VC's beliefs at date t and the maximization is over all incentive feasible contracts that deliver the EN's promised payoff, P_0, at date zero.

3 The Equilibrium

We make the following standing assumption on the model parameters, α, β, γ.

Assumption 1 $(1-\alpha)\gamma/\beta \geq 2$

Assumption 2 $\Omega_0 < \lambda s$

These conditions, which are easily met in the estimated model, ensure that an equilibrium contract between the VC and the EN exists. Assumption 1 ensures that the curvature of the EN's disutility of effort is above a threshold relative to the sensitivity of output to his effort. In other words, the EN's disutility from his effort is sufficiently pronounced relative to his positive contribution to output to ensure the existence of an equilibrium. Assumption 2 guarantees that EN optimism is not high enough to outweigh the costs of risk-sharing.

3.1 Optimal Contractual Parameters

As the EN has linear inter-temporal preferences with a subjective discount factor, and the termination payoff process is Gaussian, we can restrict consideration to contracts in which the change in the EN's discounted promised payoff in any round is an affine function of the change in the termination payoff. Specifically,

$$\delta P_{t+1} - P_t = \Delta P_t = a_t + b_t(\delta V_{t+1} - V_t), \tag{3.1}$$

where the quantity, $\delta P_{t+1} - P_t = \Delta P_t$, is the change in the EN's *discounted promised payoff* over period t. The contractural parameters a_t,b_t are $\{\mathcal{F}_t\}$-measurable. The parameter, b_t, is the *pay-performance sensitivity* of the EN's contract as it determines the change in the EN's discounted promised payoff with the change in the project's discounted termination payoff, $\delta V_{t+1} - V_t$. The parameter, a_t, is the *performance-invariant* component of the change in the EN's promised payoff that we hereafter refer to simply as the EN's performance-invariance compensation in period t. Iterating on (3.1), we see that the EN's contractual payoff at the terminal date, P_τ is

$$P_\tau = \delta^{-\tau} P_0 + \sum_{t=0}^{\tau-1} [\delta^{t-\tau} a_t + \delta^{t-\tau} b_t(\delta V_{t+1} - V_t)] \tag{3.2}$$

The derivation of the equilibrium proceeds in four steps.

Step 1. The EN's incentive compatible effort in each period.

In any period t, conditional on the project's continuation and given the contractual parameters, a_t, b_t, c_t, the EN's effort, η_t, is incentive compatible iff it maximizes his expected future utility, that is,

$$\eta_t = \arg\max_{\eta} \overline{\mathbb{E}}_t^{EN} \left[[(\delta P_{t+1} - P_t) - kQ_t \eta^\gamma] \right] \tag{3.3}$$

From (2.2) and (3.1), the objective function on the R.H.S. above can be rewritten as

$$\begin{aligned} &\overline{\mathbb{E}}_t^{EN} \{(a_t + b_t Q_t(\Theta + A c_t^\alpha \eta^\beta + s N_{t+1}^V) - kQ_t \eta^\gamma]\} \\ = &\overline{\mathbb{E}}_t^{EN} \{(a_t + b_t Q_t(\Theta + A c_t^\alpha \eta^\beta + s(\overline{N}_{t+1}^V - \lambda)) - kQ_t \eta^\gamma]\} \end{aligned}$$

$$= a_t + b_t Q_t(\overline{\mu}_t^{EN} + A c_t^\alpha \eta^\beta) - kQ_t \eta^\gamma - \lambda b_t s Q_t]\}. \tag{3.4}$$

The first equality above follows from (2.10). In the second equality above, we use the fact that

$\overline{\mathbb{E}}_t^{EN}[\overline{N}_{t+1}^V] = 0$, and the fact that

$$\overline{E}_t^{EN}[\Theta] = \overline{\mu}_t^{EN}, \tag{3.5}$$

where $\overline{\mu}_t^{EN}$ is given by (2.11). Therefore, the EN's incentive compatible effort is given by

$$\eta_t = \arg\max_\eta a_t + b_t Q_t(\overline{\mu}_t^{EN} + A c_t^\alpha \eta^\beta) - k Q_t \eta^\gamma - \lambda b_t s Q_t]\}$$

From the above, we have

$$\eta_t = \arg\max_\eta b_t A c_t^\alpha \eta^\beta - k\eta^\gamma \tag{3.6}$$

Assumption 3 guarantees a unique solution to (3.6). Thus, given the EN's pay-performance sensitivity b_t the VC's investment rate c_t and the level of effective capital stock Q_t at date t, the EN's incentive compatible effort is:

$$\eta(b_t, c_t) = \Big(\frac{A\beta c_t^\alpha b_t}{\gamma k}\Big)^{\frac{1}{\gamma-\beta}}, \tag{3.7}$$

where we explicitly indicate the dependence of the incentive compatible effort level on b_t, c_t for clarity.

Step 2. The EN's performance-invariant compensation.

By the definition of the EN's promised payoff in (2.12), the following *promise-keeping constraint* must be satisfied by an incentive feasible contract:

$$\overline{\mathbb{E}}_t^{EN}\left[(\delta P_{t+1} - P_t) - k Q_t\left(\eta_t\right)^\gamma - \lambda b_t s\right] = 0 \tag{3.8}$$

Plugging (3.1) into the above, we have

$$a_t + b_t Q_t(\overline{\mu}_t^{EN} + A c_t^\alpha(\eta_t)^\beta) - k(\eta_t^*)^\gamma - \lambda b_t s Q_t = 0$$

We, thereby, obtain

$$a_t(b_t, c_t, Q_t) := k Q_t(\eta_t)^\gamma - b_t Q_t(A c_t^\alpha(\eta_t)^\beta + \overline{\mu}_t^{EN}) + \lambda b_t s Q_t, \tag{3.9}$$

where we explicitly indicate the dependence of the EN's performance-invariant compensation on (b_t, c_t, Q_t).

Step 3. The optimal investment

By (2.15), the VC's continuation value at the beginning of each period t satisfies the following recursion:

$$\begin{aligned} CV_t &= E_t^{VC}\left[\delta\left(V_{t+1} - P_{t+1}\right) - \left(V_t - P_t\right) - c_t Q_t + \delta CV_{t+1}\right], \\ &= E_t^{VC}\left[\left(\Delta V_t - \left(a_t + b_t(\Delta V_t) - c_t Q_t\right) + \delta CV_{t+1}\right]\right. \end{aligned} \tag{3.10}$$

which can be re-expressed as

$$E_t^{VC}\left[\{(1-b_t)Q_t[(\Theta + Ac_t^{\alpha}(\eta_t)^{\beta}) + sN_{t+1}^{V}] - a_t - c_t Q_t\} + \delta CV_{t+1}\right] \tag{3.11}$$

The EN's pay-performance sensitivity and the VC's optimal investment maximize the VC's continuation value. Substituting (3.9) into (3.11), and after some algebra, we obtain

$$(b_t^*, c_t^*) = \arg\max_{(b_t,c_t)} \Lambda_t(b_t, c_t) := \phi(b_t)c_t^{\frac{\alpha\gamma}{\gamma-\beta}} - c_t + \mu_t^{VC} + b_t\left(\Omega_t - \lambda\left[s + \frac{t\sigma_0^2}{s^2 + t\sigma_0^2}\right]\right); \tag{3.12}$$

where, $\Omega_t = \mu_t^{EN} - \mu_t^{VC}$, the degree of asymmetry of beliefs at date t, and

$$\phi(b) = \begin{cases} A^{\frac{\gamma}{\gamma-\beta}}\left(\frac{\beta b}{\gamma k}\right)^{\frac{\beta}{\gamma-\beta}}(1 - \frac{b\beta}{\gamma}), \text{ if } 0 \leq b \leq \gamma/\beta, \\ 0, \text{ otherwise} \end{cases} \tag{3.13}$$

The function $\phi(b) = 0$ for $b > \gamma/\beta$. Given that Ω_t decreases with t, it follows directly from Assumption 3 and (3.12) that the VC will never choose a pay performance sensitivity $b \geq \gamma/\beta$. For $b \in (0, \gamma/\beta)$, $\phi(b) > 0$ and Assumption 3 guarantees that $\Lambda_t(b, \cdot)$ is strictly concave in c since the exponent on c is less than 1. Consequently, given the pay-performance sensitivity, b_t, there is a unique optimal investment that is given by

$$c(b_t) = \left[\frac{\alpha\gamma}{\gamma-\beta}\phi(b_t)\right]^{\frac{\gamma-\beta}{(1-\alpha)\gamma-\beta}}, \tag{3.14}$$

Plugging the above into (3.12), we finally obtain

$$b_t^* = \arg \max_{0 \leq b_t < \gamma/\beta} \quad \Lambda_t(b_t, c(b_t)), \text{ where} \tag{3.15}$$

where Λ_t is defined in (3.12) and is given by

$$\Lambda_t(b, c(b)) = \mu_t^{VC} + b_t \left(\Omega_t - \lambda \left[s + \frac{t\sigma_0^2}{s^2 + t\sigma_0^2} \right] \right) + Kc(b), \tag{3.16}$$

where K is a constant that depends on A, α, β, γ. We hereafer refer to the function, $c(.)$, as the *optimal investment function.* The following proposition summarizes useful properties of this function.

Proposition 1. *[Optimal Investment Function]*

(a) The optimal investment function $c(\cdot)$ *is positive, increasing and strictly concave on* $(0, 1]$, *is decreasing on* $[1, \gamma/\beta]$ *and therefore achieves its maximum at* $b = 1$.

(b) There is a unique solution $b_t^* \in (0, 1)$ *to (3.15).*

Step 4. The optimal termination time.

Finally, by (2.15), the VC continues the project at date t iff the expected change in her continuation value is nonnegative. By the earlier steps, the optimal termination time τ^*, therefore, satisfies the following:

$$\Lambda_t(b_t^*, c_t^*) \geq 0 \text{ for } t < \tau^*; \Lambda_t(b_t^*, c_t^*) < 0 \text{ for } t = \tau^*; \tag{3.17}$$

where

$$\begin{aligned}
\Lambda_t(b_t^*, c_t^*) &:= \mu_t^{VC} + \Omega_t b_t^* - \lambda \left[s + \frac{t\sigma_0^2}{s^2 + t\sigma_0^2} \right] b_t^* + \phi(b_t^*) \, (c_t^*)^{\frac{\alpha\gamma}{\gamma - \beta}} - c_t^* Q_t \\
&= \mu_t^{VC} + \overbrace{\Omega_t b_t^*}^{\text{rent from EN's optimism}} \overbrace{- \lambda \left[s + \frac{t\sigma_0^2}{s^2 + t\sigma_0^2} \right] b_t^*}^{\text{cost of risk sharing}} + \overbrace{Kc(b_t^*)}^{\text{return from investment}} \\
&= \mu_t^{VC} + F_t(b_t^*)
\end{aligned} \tag{3.18}$$

For further reference, we refer to the function $F_t(b)$ above as the *VC's periodic flow function.*

Proposition 2. *[Equilibrium] Consider any period* $[t, t + 1)$. *Conditional on the project not being terminated before date* t, *we have the following.*

(a) The EN's pay-performance sensitivity parameter, b_t^*, solves (3.15).

(b) The VC's equilibrium investmentis given by $c_t^* = c(b_t^*)$ defined by (3.14).

(c) The EN's performance-invariant compensation is given by $a_t(b_t^*, c_t^*, Q_t)$ defined by (3.9).

(d) The EN's optimal effort is given by $\eta_t^* = \eta(b_t^*, c_t^*, Q_t)$ defined by (3.7).

(e) The optimal termination time of the contract, τ^* satisfies (3.17).

3.2 VC's Periodic Flow Function

As shown by Proposition 2, the equilibrium is determined by the VC's periodic flow function (3.18) that has three components:

(1) *Economic rent (cost) from the EN's optimism (pessimism).* When $\Omega_t > 0$, the term indicates the rents that the VC extracts from the EN's optimism on the project's intrinsic quality. When $\Omega_t < 0$, it indicates the cost that the VC must bear to compensate the EN for his pessimism about the project's intrinsic quality. In our model development, we are agnostic on whether the EN is optimistic or pessimistic compared to the VC. We do, however, confirm the EN's optimism relative to the VC when we take the model to the data on VC projects in the next section. Hence, empirical and acecdotal evidence support the view that the VC extracts economic rents from EN optimism.

(2) *Cost of risk sharing.* The second term in (3.18) reflects the VC's cost of risk-sharing with the risk-averse EN. It can be readily seen that an increase in the intrinsic risk s or the risk aversion coefficient λ raises the risk-sharing costs between the VC and the EN.

(3) *Return from investment.* The last term in (3.18) indicates the VC's expected return as a result of her capital investments and the EN's effort in the project.

In the standard moral hazard model with homogeneous beliefs on the project quality, the EN's optimal incentives trade off the benefits of more powerful incentives on effort and investment (the last term in (3.18)) against the cost of increasing risk-sharing that the risk-averse EN must bear(the second term in (3.18)). The incorporation of heterogeneous beliefs in the model adds another dimension to the theme. The optimal incentives also reflect the rents that the VC could extract from the EN's optimism. The optimism of the EN leads to his higher expectation of future earnings than that of the VC's. Consequently, he overvalues the performance-sensitive component of his compensation relative to the performance invariant one. The optimal contract exploits the EN's optimism by making the pay-performance dependent on his optimism.

3.3 Comparisons with Observed Contractual Structures

The equilibrium contract between the VC and the EN has many of the features observed in actual venture capital contractual structures. First, the evolution of the EN's promised payoff, (3.1), immediately implies that the EN's stake in the project vests over time. Second, by (3.1), the change in the EN's stake in the project over any round $[t, t+1]$ has a component a_t^*, which is known at date t, and a risky component $b_t^* \Delta V_t$, which is a random variable whose value is realized at date $t+1$. Because the termination payoff process grows in expectation if the project is continued, the parameter a_t^* is generally negative (as we verify in the estimated model). Hence, the component a_t^* is similar to a "debt" or payment made by the EN while the component $b_t^* \Delta V_t$ is the "equity" portion of the EN's compensation over the period. Recall, however, that all payoffs occur upon termination so that no payments are actually made by the EN prior to termination. The component $\sum_{t=0}^{\tau^*-1} \delta^{t-\tau^*} a_t^*$ of the EN's termination payoff, P_{τ^*} (see (3.2)) could be viewed as a *cumulative* debt or dividend payment from the EN to the VC fund while the component $\sum_{t=0}^{\tau^*-1} \delta^{t-\tau} b_t^* \Delta V_t$ is the cumulative outcome of the changes in the EN's equity stake over each period.

Because the VC's stake in the project at any date t is $V_t - P_t$, the VC's payoff at termination is

$$\text{VC Payoff} = -\,\delta^{-\tau^*} P_0 - \sum_{t=0}^{\tau^*-1} \delta^{t-\tau^*} a_t^* + \sum_{t=0}^{\tau^*-1} \delta^{t-\tau} (1 - b_t^*) \Delta V_t. \tag{3.19}$$

The VC, therefore, holds a contract that has debt as well as equity components. These features of the optimal contract are consistent with data on observed VC contracts reported by Sahlman (1990) and Kaplan and Stromberg (2003). They document that the most commonly observed security held by VCs is preferred stock in which VC investors hold a claim to a preferred dividend stream (that could be deferred) as well as an equity claim to any residual value of the venture. The complex path-dependence of the VC fund's and EN's payoffs, however, implies that the equilibrium contract between the VC and the EN can only be implemented (or approximated) using combinations of different financial securities, which is also consistent with the evidence in Kaplan and Stromberg (2003).

4 Equilibrium Properties

We now investigate the properties of the equilibrium contract between the VC and the EN. Before analyzing the general scenario with asymmetric beliefs, we briefly discuss the benchmark scenario in which beliefs are symmetric.

4.1 Symmetric Beliefs

In this scenario, the VC's periodic flow function (see (3.18)) is

$$F_t^{symm}(b) = -\lambda \left[s + \frac{t\sigma_0^2}{s^2 + t\sigma_0^2} \right] b + Kc(b). \tag{4.1}$$

The optimal PPS, b_t^* solves

$$Kc^{'}(b_t^*) = \lambda \left[s + \frac{t\sigma_0^2}{s^2 + t\sigma_0^2} \right]. \tag{4.2}$$

The optimal investment function, $c(b)$, achieves its maximum at $b = 1$ by Proposition 1, which implies that $c'(1) = 0$. It then follows from (4.1) that $F_t'(1) < 0$. Since $F_t'(b_t^*) = 0$, the strict concavity of $F_t(\cdot)$ now guarantees that $b_t^* < 1$. Further, b_t^* decreases over time. It then follows that both c_t^* and η_t^* also decrease over time. The PPS, effort and investment are all lower than in the benchmark "no-agency" scenario. It follows immediately from (4.2) and the Theorem of the Maximum that, in the limit as $t \to \infty$, the optimal PPS converges to b_∞^* that solves

$$Kc^{'}(b_\infty^*) = \lambda(s + 1). \tag{4.3}$$

The optimal investment and effort then similarly approach limits that are given by

$$c_\infty^* = c(b_\infty^*); \eta_\infty^* = \eta(b_\infty^*, c_\infty^*). \tag{4.4}$$

4.2 Imperfect Information and Asymmetric Beliefs—The Actual Scenario

In the actual scenario, the VC's periodic flow function may be expressed as

$$F_t(b) = \frac{\Omega_0}{\sigma_0^2} \sigma_t^2 b + F_t^{symm}(b). \tag{4.5}$$

Since $\sigma_t \to 0$, it follows from the above and the Theorem of the Maximum that $b_t^* \to b_\infty^*$, and thus $(c_t^*, \eta_t^*) \to (c_\infty^*, \eta_\infty^*)$ by continuity where $(b_\infty^*, c_\infty^*, \eta_\infty^*)$ are given by (4.3) and (4.4). The following proposition characterizes the dynamics of these economic variables (conditional on the project's continuation).

Proposition 3. *[Dynamics of Contractual Variables] Conditional on the project surviving beyond date t, the EN's pay-performance sensitivity, b_t^*, the VC's investment rate, c_t^*, and the EN's effort, η_t^*, all decrease monotonically with t and respectively approach b_∞^*, c_∞^* and η_∞^* as $t \to \infty$.*

The results of Proposition 3 hinge on the interplay among the value-enhancing effort by the EN that is positively affected by his optimism, the costs of risk-sharing due to the EN's risk aversion that are negatively affected by the project's intrinsic risk, and the effect of both the VC's physical capital investment and the EN's effort on output. Since the EN is optimistic, he is willing to accept a greater portion of the project's risk so that his pay performance sensitivity and effort are initially high. The passage of time lowers the degree of EN optimism as he revises his initial assessment of project quality. The EN's pay performance sensitivity and effort, therefore, decline over time. The decreasing effort of the EN makes it optimal for the VC to also lower her capital investments.

4.3 Sensitivity of Equilibrium Dynamics

We now show how the EN's pay performance sensitivities, the VC's investments and the aEN's effort are affected by changes to underlying parameters.

Proposition 4. *[Sensitivities of Contractual Parameters] The paths of the EN's pay performance sensitivity, the VC's investments and the EN's effort are each pointwise (a) decreasing functions of the EN's cost of risk λ; (b) decreasing functions of the initial transient risk σ_0; (c) decreasing functions of the intrinsic risk s; (d) increasing functions of the initial degree of asymmetry of beliefs Ω_0; and (e) decreasing functions of the EN's cost of effort k.*

The EN's pay performance sensitivity declines with his cost of risk because an increase in the EN's cost of bearing risk increases the costs of risk-sharing between the VC and the EN. An increase in the transient risk lowers the degree of EN optimism at each date because the "signal to noise ratio" is increased so that the EN "learns faster". Hence, the economic rents to the VC in each period from the EN's optimism are lowered relative to the costs of risk-sharing so that the EN's pay-performance sensitivity declines. An increase in the intrinsic risk, on the other hand, increases the degree of asymmetry in beliefs at each date because the EN "learns more slowly" but also increases the costs of risk-sharing. Under Assumption 3, the costs of risk-sharing outweigh the benefits of the EN's optimism so that the EN's pay-performance sensitivity also decreases with intrinsic risk. As a consequence, the investment rates and EN's efforts also decline.

An increase in the EN's optimism leads to a corresponding increase in the economic rents to the VC in each period. The VC exploits this in each period by increasing the pay-performance sensitivity and investment, thereby leading to an increase in effort by the EN. Under Assumption 3, the economic rents the VC can potentially capture due to the EN's exaggerated assessment of project quality are high compared with the costs of risk-sharing and inducing effort from the EN.

4.4 Project Duration

We now investigate the optimal termination decision of the project, which determines the project's duration. At any date t, we show that the VC's continuation value is an increasing, lower semi-continuous function of her current assessment, μ_t^{VC}, of the project's quality. Since the VC continues the project if and only if her continuation value is positive, there exists a trigger level at each date such that she continues the project if and only if her current assessment of the project's quality exceeds the trigger.

Proposition 5. *[Optimal Termination Policy] The optimal termination policy for the VC is a trigger policy. There exist μ_t^* such that the VC continues the project if and only if $\mu_t^{VC} > \mu_t^*$.*

An increase in the EN's initial degree of optimism about project quality increases the rents that the VC is able to extract by exploiting the EN's optimism thereby increasing her expected continuation value at each point in time. Hence, it is optimal for the VC to prolong the project's duration. An increase in the EN's cost of risk or cost of effort, however, increases the costs of risk-sharing for the VC, thereby lowering her continuation value at each point in time. Hence, the VC terminates the project earlier. The following result summarizes the effect of the EN's initial assessment of project quality, his cost of risk, and his cost of effort on the duration of the project.

Proposition 6. *[Project Duration Sensitivity] The project duration τ^* monotonically increases* *with the initial degree of asymmetry in beliefs, decreases with the EN's cost of risk, and decreases with the EN's cost of effort.*

5 Quantitative Analysis

We now take our structural model to the data by estimating its parameters using the simulated method of moments. We then numerically analyze the estimated model to derive quantitative implications for how entrepreneurial optimism affect VC-EN contracts and the value generated by venture capital projects.

5.1 Numerical Implementation and Estimation

Model Parameters

We categorize the parameters of our structural model into into two groups: "direct "parameters that we directly calibrate using suitable statistics in the data, and "indirect" parameters whose values we infer using our simulated method of moments estimation procedure. Gompers (1995) reports that the average length of a round of VC financing is approximately one year. Accordingly, we set the time period between successive dates in the mode, which represents a single round of financing, to one. Cochrane (2005) documents that the average expected return on VC investments is 15%. We accordingly set the discount rate, δ, of the VC and EN to 0.15.

In our implementation, we assume that the VC's assessment of the project is correct so that the VC's mean assessment of the project's intrinsic quality is the true mean quality. Although we can implement a general model with the VC's assessment deviating from the project's true quality, we choose this more parsimonious implementation because of two principal reasons. First, VCs' assessments of project prospects are more likely to be correct on average given their experience and knowledge in investigating, evaluating and managing multiple projects. Second, our paper focuses on the impact of entrepreneurial optimism relative to VCs. We, therefore, lower the dimensionality of the parameter space by not including the VC's mean assessment of project quality as an additional parameter to be estimated. Table 1 lists the "indirect" parameters that we estimate using the simulated method of moments.

Table 1: Structural Parameters

α	Cobb-Douglas output elasticity with respect to investment
β	Cobb-Douglas output elasticity with respect to effort
A	Total factor productivity
γ	EN disutility of effort elasticity
k	Linear coefficient of EN disutility of effort
λ	EN risk aversion
s	Project's intrinsic risk
σ_0	Project's initial transient risk
μ_0^{VC}	VC's initial mean assessment of project quality
Ω_0	Initial degree of entrepreneurial optimism
σ_Q	Volatility of capital stock process
μ_Q	Drift of capital stock process

By (3.13) and (3.18), all equilibrium variables depend only on the ratio, γ/β so parameters γ and β are not separately identifiable. Hence, from the above table, we have eleven free parameters that we need to estimate.

Numerical Implementation

We use the simulated method of moments to estimate the indirect parameters by matching selected moments predicted by the model to their corresponding values in the data. We choose the moments to be informative about the model's parameters so that they can be properly identified by the estimation procedure. We briefly summarize the key steps of the numerical implementation of the model and estimation procedure below. We provide additional details in Appendix C.

By Proposition 2, the VC's investmen, c_t^*,the EN's pay-performance sensitivity, b_t^*, and effort, η_t^*, are all deterministic functions of time conditional on the project's continuation, while the EN's performance-invariant compensation, a_t^*, is stochastic and is affected by the effective capital stock, Q_t, and the VC's posterior mean assessment of project quality, μ_t^{VC}. (Recall from (2.6) that the degree of entrepreneurial optimism is a deterministic function of time.) By (3.17) and (3.18), the termination time of the project is determined by the VC's posterior mean assessment of project quality, μ_t^{VC}. It follows then that the equilibrium is fully determined by the state vector, (Q_t, μ_t^{VC}).

As the termination time is determined solely by μ_t^{VC}, and is unaffected by Q_t, our numerical implementaion proceeds in two steps. In the first step, we approximate the evolution of the state variable μ_t^{VC} on a discrete lattice. We then use Proposition 5 to determine the termination threshold, μ_t^*, at each date t such that the project is terminated iff $\mu_t^{VC} < \mu_t^*$. In the second stage of the numerical implementation, we simulate the evolutions of the effective capital stock, Q_t, the termination payoff, V_t, and the VC's mean posterior assessments of project quality, μ_t^{VC}. Using the termination thresholds obtained in the first stage, we compute the moments we employ in the estimation of the model.

Estimation Procedure

We employ the simulated method of moments in our estimation of the model. to compute the moments of interest. The moments that we match are based study by Cochrane (2005) of the risk and return of VC projects. His study is built on the VentureOne database with a comprehensive dataset on round-by-round VC venture financing spanning more than a decade. He reports various statistics that characterize the distributions of round-by-round returns of VC investments. Specifically, in Table 4 of his paper, he reports the means and standard deviations of the returns of the first four rounds of financing for VC projects as well as the mean and standard deviation of the overall returns of VC investments. In Table 2 of his paper, he also reports the exit probabilities of VC projects in each of the first four rounds. In addition, Cochrane (2005) reports an average duration of VC projects of 2.1 years. We accordingly match the fifteen statistics reported in Table 3. We estimate the baseline parameters by minimizing the squared distance between the model-predicted and actual moments. We compute the standard errors of the parameters and moments via bootstrapping as we describe in the Appendix C.

In our model, V_t, represents the project's termination payoff or outside value at date t. Accordingly, the return from the VC project in round t in the model is the change in the outside value of the project during the round net of the investment divided by the outside value at the beginning of the round, that is,

$$\text{Round-by-Round Return } = (V_{t+1} - V_t - c_t Q_t)/V_t, \tag{5.1}$$

Since Cochrane (2005) uses log returns, we match the means and standard deviations of log returns reported by Cochrane (2005) to the means and standard deviations of the round by round log returns

Table 2: Baseline Parameter Values

α	λ	γ/β	A	k	s	σ_0	μ_0^{VC}	Ω_0	σ_Q	μ_Q
0.54	12.48	5.48	4.18	6.67	0.88	0.15	0.60	2.54	0.90	0.54
(0.032)	(0.560)	(0.198)	(0.099)	(0.231)	(0.051)	(0.007)	(0.061)	(0.123)	(0.042)	(0.038)

Table 3: Predicted and Observed Moments

	Means of Round Returns				Stdevs of Round Returns			
	Rnd 1	Rnd 2	Rnd 3	Rnd 4	Rnd 1	Rnd 2	Rnd 3	Rnd 4
Observed	0.26	0.20	0.15	0.09	0.90	0.83	0.77	0.84
Predicted	0.31	0.16	0.15	0.12	0.87	0.83	0.77	0.78
Std err	0.033	0.036	0.049	0.054	0.026	0.040	0.032	0.032
95% C.I.	[0.25,0.37]	[0.09,0.23]	[0.05.0.24]	[0.01.0.22]	[0.82,0.92]	[0.75,0.91]	[0.71,0.84]	[0.79,0.85]

	Mean Duration	Exit Probabilities				Overall Mean Return	Overall
		Rnd 1	Rnd 2	Rnd 3	Rnd 4		
Observed	2.1	0.41	0.45	0.50	0.59	0.2	0.7
Predicted	2.56	0.47	0.49	0.51	0.52	0.18	0.8
Std err	0.14	0.030	0.032	0.033	0.034	0.025	0.01
95% C.I.	[2.29,2.83]	[0.41,0.53]	[0.43,0.55]	[0.44,0.57]	[0.45,0.59]	[0.14,0.24]	[0.79,0

predicted by our model. We calculate the exit probabilities in our model by the probabilities of termination in each round from our simulations.

Baseline Parameter Values

We report the baseline parameter values obtained from our estimation in Table 2. The point estimates of the parameters and their standard errors computed via bootstrapping are in the second and third rows, respectively.

Table 3 reports the predicted and observed statistics along with the corresponding standard errors as well as the 95% confidence intervals. As we can see from the table, the model does a good job matching the moments with most of the observed moments falling within the 95% confidence intervals of the predicted moments.

Table 2 shows that entrepreneurs are, indeed, significantly optimistic relative to VCs. The initial degree of entrepreneurial optimism is $\Omega_0 = 2.54$. The VC's initial mean assessment of project quality is 0.6 Hence, the EN's initial mean assessment of project quality is $\mu_0^{EN} = \mu_0^{VC} + \Omega_0 = 3.14$,which is more than five times the VC's initial mean assessment. The estimates suggest that EN optimism is a major driver of the value generated by VC projects, which we now explore quantitatively.

Table 4: Comparison with Benchmark “Symmetric Beliefs” Model

	b_1^*	b_2^*	b_3^*	b_4^*	c_1^*	c_2^*	c_3^*	c_4^*	η_1^*	η_2^*	η_3^*	η_4^*
Actual Model	0.100	0.913	0.0836	0.077	0.992	0.937	0.889	0.772	0.368	0.358	0.349	0.34
Symmetric Beliefs	0.059	0.0547	0.0512	0.0482	0.714	0.684	0.656	0.632	0.314	0.308	0.302	0.29

5.2 Impact of Entrepreneurial Optimism

We now compare the model with the parameters set to their baseline values obtained in our estimation with the benchmark model in which beliefs are symmetric discussed in Section 4.1. We set all parameters to their baseline values in the benchmark model except for the degree fo EN optimism, which we set to zero. Table 4 compares the EN's pay-performance sensitivities, efforts, the VC's investments, and the project termination probabilities in the two scenarios.

We can see from the table that the VC exploits the EN's optimism by setting significantly higher pay-performance sensitivities in the actual scenario with EN optimism. She thus induces the EN to exert greater effort by providing more powerful incentives confirming our theoretical discussion in Section 4. The project value in the actual model is more than twice its value in the benchmark model, and the VC's stake is almost twice its value. The mean project duration is also longer in the actual scenario. As shown by Proposition 4, the EN's pay-performance sensitivities, efforts, and the VC's investments decline over time as the degree of EN optimism declines. Overall, Table 4 shows that EN optimism has a very large impact on venture capital projects.

5.3 Implied Discount Rate

A salient aspect of venture capital finance is the prevalence of strikingly high discount rates used by VCs to value projects that usually range between 30% and 70% (Sahlman, 1990, Gladstone and Gladstone, 2002, Fuerst and Geiger, 2005). Yet, the average expected return of VC projects is only around 15% (Cochrane, 2005). Several commentators suggest that the high discount rates offset over-optimistic pro-forma projections by ENs. To the best of our knowledge, however, there is quantitative analysis to rigorously ascertain whether EN optimism in the data is, indeed, high enough to explain the discrepancy. Using structural estimation approach is well suited to address this question. We define the *implied discount rate*(IDR) as the rate that the VC would apply to the EN's projection of the project's payoffs so that the resulting valuation of her stake in the project would be equal to her own *true* valuation of her stake. In other words, the IDR is the discount rate the VC would use to obtain her valuation of her stake if the project's intrinsic quality were hypothetically distributed according to the EN's beliefs. The IDR R_{VC}, therefore, solves

$$\text{VC value}{=}\mathbb{E}_0\left[\left(e^{-R_{VC}\tau^{EN}}\left[V_{\tau^{EN}}-P_{\tau^{EN}}\right]-\sum_{t=0}^{\tau^{EN}-1}e^{-R_{VC}t}c_t^{EN}Q_t)\right)\right];\theta\sim N(\mu_0^{EN},\sigma_0^2) \quad (5.2)$$

In the above, τ^{EN} is the project's optimal termination time of the project in the hypothetical scenario in which the VC and EN agree that the project's true intrinsic quality is distributed according to the EN's beliefs. Hence, the optimal contractual parameters, $(a^{EN}, b^{EN}, c^{EN}, \tau^{EN})$ are those in the "symmetric beliefs" scenario discussed in Section 4.1 with $\mu_0^{VC}=\mu_0^{EN}=\mu_0$. The VC value in (5.2) is the VC's valuation of her stake in the project in the baseline estimated model where the project's intrinsic quality is distributed according to the VC's beliefs, and the EN is optimistic.

$$\text{VC value }{=}\mathbb{E}_0^{VC}\left[\left(\delta^{\tau^*}\left[V_{\tau^*}-P_{\tau^*}\right]-\sum_{t=0}^{\tau^*-1}\delta^t c_t^* Q_t)\right)\middle|\theta\sim(\mu_0^{VC},\sigma_0^2)\right] \quad (5.3)$$

Consistent with the average expected annual return of VC projects reported in Cochrane (2005), $\delta = e^{-0.15}$ above, and the optimal contractual parameters are those obtained in our baseline estimated model. We find that the implied discount rate (IDR) is 42% in our baseline model, which is within the range of IDRs reported anecdotally. Our structural estimation exercise, therefore, implies that average EN optimism in the data is, indeed, high enough to explain the discrepancy between the discount rates used by VCs and the average expected return of VC projects. In the next section, we further explore the comparative statics of IDRs with respect to model parameters along with other key outputs of the model.

5.4 Comparative Statics

We now explore use our estimated model to explore the comparative statics of four key outputs of the model—the project value, the value of the VC's stake, the project duration, and the implied discount rate—with respect to various underlying parameters of the model. We, thereby, derive testable implications of our theory.

Entrepreneurial Optimism

Figure 5.1 shows the impact of the degree of EN optimism. As discussed earlier, an increase in EN optimism mitigates the costs of risk sharing between the VC and the EN thereby enhancing the power of incentives that could be provided by the EN. Consequently, the project value, the VC value, and the expected project duration all increase with EN optimism. The IDR also naturally increases because an increase in EN optimism increases the discount rate that the VC must apply to adjust for the EN's payoff projects. These results along with our analytical results in Section 4.2 lead to the following testable implications:

Implication 1. *a) The project value, VC value, the implied discount rate, and expected duration all increase with the degree of EN optimism.*
b) The EN's pay-performance sensitivity and the VC's proportional investment in any round increase with the initial degree of EN optimism.

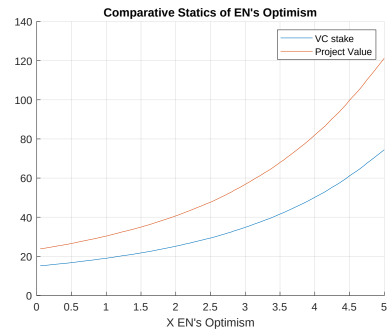
Previously reported empirical evidence is indirectly consistent with the implication a) above. The positive effects of EN optimism on project value are consistent with the evidence reported in Gelderen et al (2005). The positive relation between duration and the degree of EN optimism is consistent with the evidence in Kaplan and Stromberg (2003) that experienced entrepreneurs, who are likely to have more realistic beliefs, receive fewer rounds of financing.

Intrinsic Quality

Figure 5.2 shows the effects of varying the projects true mean intrinsic quality, μ_0^{VC}. As expected, the project value, the VC value and the expected duration all increase with the mean intrinsic quality. The IDR declines because an increase in μ_0^{VC}, ceteris paribus, increases the VC's valuation of the project, thereby lowering the discount rate that she uses to pare down the EN's optimistic projections. These results, combined with our analytical results in Section 4.2, lead to the following testable implications:

Implication 2. *a) The project value, VC value, and expected duration all increase with the project's true mean intrinsic quality, while the implied discount rate declines.*
b) The EN's pay-performance sensitivity and the VC's proportional investment in any round are unaffected by the project's true mean intrinsic quality.

Figure 5.1: Effects of Entrepreneurial Optimism



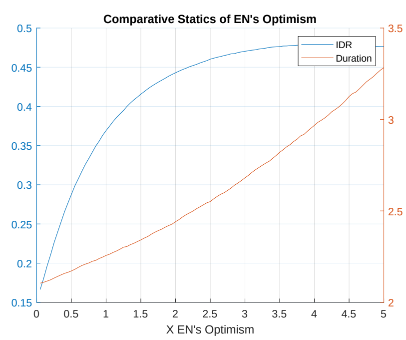
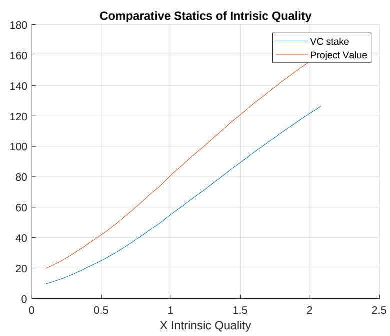


Figure 5.2: Effects of Project's Mean Intrinsic Quality



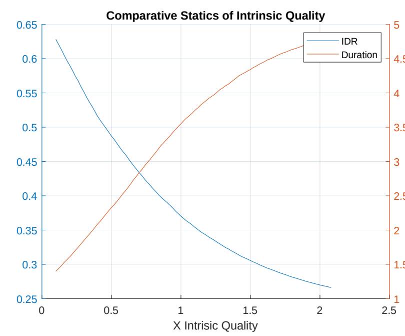
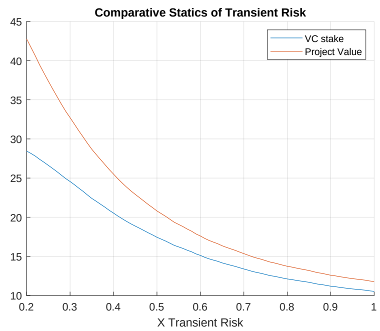
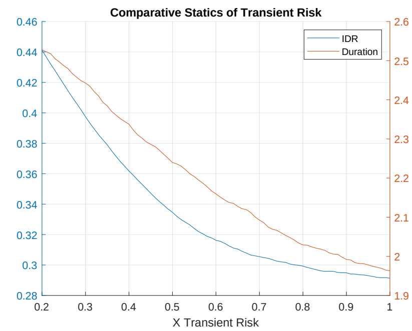


Figure 5.3: Effects of Initial Transient Risk





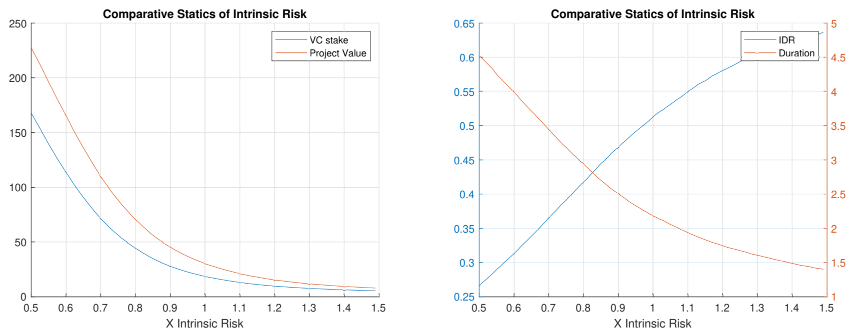
Transient Risk

Figure 5.3 shows that the project value, VC value, expected duration, and the IDR all decline with the initial transient risk σ_0. As shown by Proposition 4, the EN's pay-performance sensitivities, effort choices, and the VC's investments all decrease with the initial transient risk. because of the increased costs of risk-sharing. As discussed after the proposition, an increase in the initial transient risk lowers the degree of EN optimism because the EN learns faster and also increases the costs of risk-sharing. The decline in EN optimism with the initial transient risk causes the IDR to also decline.

Implication 3. *a) The project value, VC value, expected duration, and the implied discount rate all decrease with the project's initial transient risk.*

b) The EN's pay-performance sensitivity and the VC's proportional investment in any round decline with the project's initial transient risk.

Figure 5.4: Effects of Intrinsic Risk



Intrinsic Risk

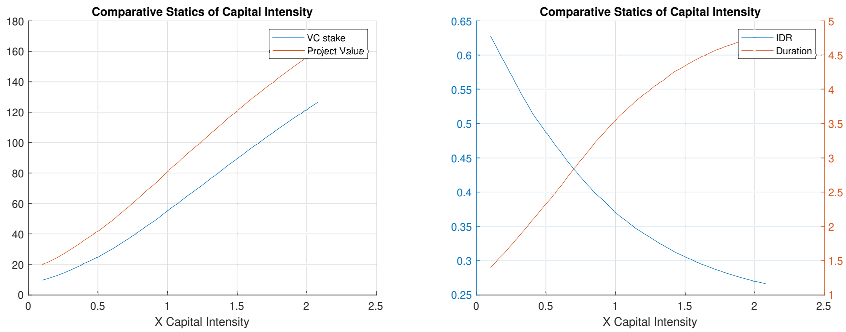
Figure 5.4 shows that the effects of the intrinsic risk, s, on the project value, VC value, and expected duration are similar to those of the initial transient risk. Indeed, as shown by Proposition 4.3, the EN's pay-performance sensitivities and effort as well as the VC's proportional investments in each round all decline with the intrinsic risk. Interestingly, however, the IDR increases with the intrinsic risk. The reason is is that an increase in the intrinsic risk increases the "signal to noise" ratio so that the EN learns more slowly. Hence, the EN's optimism declines more slowly with the intrinsic risk. The slower decline in EN optimism coupled with the decline in the VC value with intrinsic risk causes the discount rate that the VC uses to pare down the EN's optimistic projections to increase with the intrinsic risk.

Implication 4. *a) The project value, VC value and expected duration decrease with the project's intrinsic risk, while the implied discount rate increases.*
b) The EN's pay-performance sensitivity and the VC's proportional investment in any round decline with the project's intrinsic risk.

Physical Capital Intensity

Figure 5.5 shows that an increase in the project's physical capital intensity, α, increases the project value, VC value and expected project duration. An increase in α increases the productivity of EN effort so that the EN's pay-performance sensitivities, effort choices and the VC's proportional investments in each round increase. The IDR, however, declines because the increase in the valuation of the project from the VC's standpoint lowers the discount rate that the VC needs to apply to the

Figure 5.5: Effects of Project's Physical Capital Intensity



EN's projections to match her own valuation.

Implication 5. *a) The project value, VC value and expected duration increase with the project's physical capital intensity, while the implied discount rate decreases.*

b) The EN's pay-performance sensitivity and the VC's proportional investment in any round increase with the project's physical capital intensity.

6 Conclusions

We develop a dynamic principal-agent model to qualitatively and quantitatively analyze the impact of entrepreneurial optimism on the values and durations of venture capital projects |as well as the properties of dynamic contracts between venture capitalists (VCs) and entrepreneurs (ENs). We derive the equilibrium in which the VC's investments, the EN's compensation and effort choices, and the project's duration are derived endogenously. Consistent with observed contractual structures, the equilibrium dynamic contract features staged VC investments, progressive vesting of the EN's stake, and the presence of inter-temporal performance targets that must be realized for the project to continue.

We employ the simulated method of moments to structurally estimate the model parameters by matching the round-by-round returns, risks and termination probabilities as well as the overall expected return, risk and duration of VC projects. Our model does a good job matching the data with the predicted moments all lying within the 95% confidence intervals of the observed moments. The degree of EN optimism that we indirectly infer from our estimation reconciles the discrepancy between the discount rates used by VCs to value projects (˜ 40%), which adjust for optimistic payoff projections by ENs, and the average expected return of VC projects (˜ 15%) (e.g. Fuerst and Geiger (2005)). EN optimism offsets the agency costs of risk-sharing between VCs and EN's, thereby leading to substantially more powerful EN incentives, and increases VC project values by over 80%. Optimism is, therefore, a major determinant of the durations and economic values of VC relationships. We derive novel testable implications for the effects of the degree of EN optimism, the true intrinsic qualities of projects, and the permanent and transitory components of their risks on project values and durations, the values of VC stakes, the implied discount rates of VCs, and the characteristics of contracts between VCs and ENs. The tractability of the model and its ability to match disparate statistics associated with VC projects suggest that it could be a useful tool to value risky ventures.

Appendix A: Representation of Entrepreneur's Objective as a Recursive Utility

We show that the EN's objective as described by (2.7) or (2.8) actually belong to the general class of recursive utilities (see Duffie and Epstein, 1992). For analytical convenience, we work in continuous time. If

$$U_t = \overline{E}_t^{EN} \left[\left[\delta^{\tau - t} P_\tau - \sum_{s=t}^{\tau-1} \delta^{s-t} k Q_s \eta_s^\gamma \right] \right] \tag{6.1}$$

is the EN's conditional expected utility at date t, then U_t satisfies the following *backward stochastic difference equation* or BSDE (see Ma and Yong, 1999):

$$\begin{aligned} \delta U_{t+1} - U_t &= -kQ_t \eta_t^\gamma + Z_t \overline{N}_{t+1}^V = (-k\eta_t^\gamma Q_t + \lambda Z_t)dt + Z_t N_{t+1}^V, \\ U_\tau &= P_\tau, \end{aligned} \tag{6.2}$$

The second equality above follows from (2.10). The process Z_t is $\{\mathcal{F}_t\}$-adapted and the pair of processes (U, Z) is the solution of the BSDE (6.2). From (6.2), the EN's conditional expected utility U_t has the general recursive representation

$$
\begin{aligned}
U_t &= U_\tau + \sum_{s=t}^{\tau-1} f(U_s, Z_s) - \sum_{s=t}^{\tau-1} Z_s N_{s+1}^V, \\
f(U_s, Z_s) &= k\eta_s^\gamma Q_s - \lambda Z_s,
\end{aligned} \tag{6.3}
$$

where the function f is the *aggregator* (see Duffie and Epstein, 1992, Ma and Yong, 1999).

Appendix B: Proofs

Proof of Proposition 1

The marginal optimal investment is given by

$$
c'(b) \propto b(\gamma - b)^{r_2}(1 - b) \tag{6.4}
$$

where

$$
r_1 := \frac{2 - (1-\alpha)\frac{\gamma}{\beta}}{(1-\alpha)\frac{\gamma}{\beta} - 1} \text{ and } r_2 := \frac{\alpha\frac{\gamma}{\beta}}{(1-\alpha)\frac{\gamma}{\beta} - 1},
$$

and where the symbol $\propto$ means "equal up to a positive multiplicative constant". Under Assumption 3, the parameter r_2 is positive and the parameter r_1 is negative. Since $\frac{\gamma}{\beta} > 1$ (Assumption 3), the strong unimodality of $c(\cdot)$ easily follows from (6.4). Since $c(0) = c(\gamma) = 0$ and $c'(0) = +\infty$, it also follows from (6.4) that $c(\cdot)$ achieves its maximum at $b = 1$. Part (a) has been established.

To establish part (b), we note that the second derivative of the optimal investment function is

$$
c''(b) \propto b^{r_1 - 1}(\frac{\gamma}{\beta} - b)^{r_2 - 1}[r_1(\frac{\gamma}{\beta} - b)(1 - b) - r_2 b(1 - b) - b(\frac{\gamma}{\beta} - b)].
$$

The expression inside the brackets is a strictly convex quadratic function whose value at 1 is negative, whose value at $\frac{\gamma}{\beta} > 1$ is positive, and whose value at 0 is negative since $r_1 < 0$. Consequently, there is exactly one root b_M of the quadratic in the interval $(1, \frac{\gamma}{\beta})$ such that $c''(b_M) = 0$. At b_M the marginal investment is at its minimum. Moreover, since $c''(\cdot)$ is negative on $[0, b_M)$ and is positive on $(b_M, \frac{\gamma}{\beta})$, the function is strictly concave on $[0, b_M]$ and strictly convex on $[b_M, \frac{\gamma}{\beta}]$.

Suppose $\Omega_0 \geq 0$. It directly follows from (3.16) that

$$\frac{d\Lambda_t}{db}(b, c(b)) \leq \Omega_0 - \lambda s b + \frac{\gamma - \beta - \alpha\gamma}{\alpha\gamma} c'(b),$$

since $\Omega_t \leq \Omega_0$ for all t. Our previous arguments show that $c'(1) = 0$ and $c'(b) < 0$ for all $b \in (1, \frac{\gamma}{\beta})$. It then follows that $\frac{d\Lambda_t}{db}(b, c(b)) < 0$ for all $b > 1$ because $\Omega_0 < \lambda s$ by Assumption 3. Since $c(b)$ is strictly concave for $b \in (0, 1)$, $\Lambda_t(b, c(b))$ is also strictly concave. It then follows that there exists a unique $b_t^* \in (0, 1)$ that maximizes $\Lambda_t(b, c(b))$ when $\Omega_0 \geq 0$.

Suppose $\Omega_0 < 0$ so that $\Omega_t < 0$ for all t. Then

$$\frac{d\Lambda_t}{db}(b, c(b)) < -\lambda s b + \frac{\gamma - \beta - \alpha\gamma}{\alpha\gamma} c'(b),$$

By the previous arguments, the right hand side above is less than zero for $b \geq 1$ so that we again have a unique $b_t^* \in (0, 1)$. Q.E.D.

Proof of Proposition 2

We proceed by backward induction starting with the last possible period/round $[T-1, T]$. Suppose that the project has not been terminated as of date $T-1$. The VC's and EN's priors on the project's intrinsic quality at date $T-1$ are given by $N(\mu_{T-1}^{\ell}, \sigma_{T-1}^2), \ell = VC, EN$, where $\left(\mu_{T-1}^{\ell}, \sigma_{T-1}^2\right)$ are given by (2.5) with $t = T-1$. For subsequent convenience, we use the index t to denote the date, which is set to $T-1$ for now, but later denotes an arbitrary date when we establish the inductive step in our analysis.

The Optimal Contractual Parameters in Period $[T-1, T]$.

Suppose that in period $[t, t+1]$ (recall that $t = T-1$), the VC's investment is c_t and the agent's contractual parameters are (a_t, b_t); see Equation (3.1). Following the analysis in Section 3.1, the contract is incentive compatible for the EN if and only if the EN's effort η_t specified by the contract solves (3.3). By the arguments in Step 1 of Section 3.1, the EN's effort, $\eta_t(b_t, c_t)$ is then given by (3.7).

Next, by (3.1), the EN's promise-keeping constraint (3.8) must be satisfied by the contract where $t = T-1$. Following the arguments in Step 2 in Section 3.1, the EN's performance-invariant compensation, $a_t(b_t, c_t, Q_t)$ is then given by (3.9).

The VC's continuation value at date $T-1$ satisfies (3.10) with $CV_{t+1} = 0$ because T is the last possible date. Following the analysis in Step 3 of Section 3.1, the VC's optimal investment $c_t(b)$ is given by (3.14), and the EN's optimal PPS, b_t^*, is given by (3.15).

Appendix C

Numerical Implementation

The dynamic process of our model is driven by two underlying state variables: μ_t and Q_t. The stopping time τ depends on the realization of μ_i^{VC} and Q_i.

Define the process $\Psi_i = \sum_{t=1}^{i}(\Theta + S_i)$. By the Bayesian updating rule, VC's μ_i process follows:

$$\mu_i = \frac{s^2\mu_0^j + \sigma_0^2 \sum_{t=1}^{i}(\Theta + S_t)}{s^2 + i\sigma_0^2} = \frac{s^2\mu_0^j + \sigma_0^2(\Psi_i)}{s^2 + i\sigma_0^2} \tag{6.5}$$

Lattice design We approximate the evolutions of μ_i and Q_i using discrete lattices and derive the triggers that determines whether the contract will be terminated at each period. We use a discrete lattice to model the μ_t processes. Let $n(i)$ denote the number of states at date i and let $\mu_{i,j}$ denote the μ at the j^{th} state at date $i, j = 1, ..., n(i)$ with increasing value. The minimal and maximal states at date $i, \mu_{i,1}$ and $\mu_i, n(i)$ are κ standard deviations below and above the minimal and maximal states at date $i-1$, respectively.

$$\mu_{i,n(i)} = \mu_{i-1,n(i-1)} + \kappa\sigma_{i-1}^{\mu} \tag{6.6}$$

$$\mu_{i,1} = \mu_{i-1,1} - \kappa\sigma_{i-1}^{\mu} \tag{6.7}$$

$$\sigma_{i,1}^{\mu} = \frac{\sigma_{i-1}^2}{\sqrt{s^2 + \sigma_{\mu_{i-1}}^2}} \tag{6.8}$$

by the Bayesian updating formula. The value for the intermediate states between the minimum and maximum are set equally spaced. The number of states in the lattice increased linearly from period to period. That is $n(i) = Mi$ for $i > 0$. We set $M = 25$ and $\kappa = 2.5$.

The VC's Continuation Value and the Termination Triggers

Starting from the last investment period T and working backwards, we use dynamic programming to compute the CV for all states and dates. At date $T-1$,the continuation value is simply (**??**) + present value of expected Q_T.We compute the CV for all possible states in the two-dimensional tuple (Q_{T-1}, μ_{T-1}). At earlier dates, the continuation values are given by:

$$CV_{i,q} = \Lambda_{i,j,q} + exp(-R) \sum p_{i,j,q}^{i+1,k,r} max(CV_{i+1,k,r}, Q_{i+1}). \tag{6.9}$$

In which, $p_{i,j,q}^{i+1,k,r}$ is the transition probability from the state at date i with μ at $\mu_{i,j}$ and Q at $Q_{i,q}$ to the state at date $i+1$ with μ at $\mu_{i+1,k}$ and Q at $Q_{i+1,r}$. Σ sums over all possible 2-dimensional transitions.

To implement the computation we take the following steps:

1) Marginal transition probabilities for μ process:
if $\mu_{i+1,k}^{pr}$ is within $\pm\kappa_\mu \sigma_i^\mu$ from $\mu_{i,j}^{pr}$, we set

$$p_{\mu(i,j)}^{(i+1,k)} := \Phi\big[(\frac{1}{2}(\mu_{i+1,k}^{pr} + \mu_{i+1,k+1}^{pr}) - \mu_i^{pr})\frac{1}{\sigma_i^\mu}\big] - \Phi\big[(\frac{1}{2}(\mu_{i+1,k}^{pr} + \mu_{i+1,k-1}^{pr}) - \mu_i^{pr})\frac{1}{\sigma_i^\mu}\big] \tag{6.10}$$

2) Marginal transition probabilities for Q process:
if $Q_{i+1,r}$ is within $\pm\kappa_Q \sigma_Q$ from $Q_{i,q}$, we set

$$p_{Q(i,q)}^{(i+1,r)} := \Phi\big[(\frac{1}{2}(ln(Q_{i+1,r}) + ln(Q_{i+1,r+1})) - ln(Q_i))\frac{1}{\sigma_Q}\big] - \Phi\big[(\frac{1}{2}(ln(Q_{i+1,r}) + ln(Q_{i+1,r-1})) - ln(Q_i))\frac{1}{\sigma_Q}\big] \tag{6.11}$$

we set $\kappa_\mu = \kappa_Q = \kappa$.

3) Since μ and Q processes are independent,$p_{i,j,q}^{i+1,k,r} = p_{\mu(i,j)}^{(i+1,k)} p_{Q(i,j)}^{(i+1,q)}$

4) Fix a $Q_{i,q}$ from the lattice, by the continuous function argument,numerically determine if for simulated μ_i and Q_i, $CV_i(\mu_i, Q_i) > Q_i$

In determining the trigger values, we use linear extrapolation. To be more specific, for any point on the simulation path with (μ, Q) , find the corresponding period lattice points,(μ_1, Q_1) and (μ_2, Q_2), such that $\mu \in (\mu_1, \mu_2)$ and $Q \in (Q_1, Q_2)$. Denote

$|d_1|$ = distance between (μ, Q) and (μ_1, Q_1) and

$|d_2|$ = distance between (μ, Q) and (μ_2, Q_2),

then the extrapolated $CV(\mu, Q)$

$$CV(\mu, Q) = \frac{|d_1|CV_2 + |d_2|CV_1}{|d_1| + |d_2|} \tag{6.12}$$

Simulation Procedure

For each set of candidate parameters, π,

$$\pi = \left(\alpha, \lambda, \gamma/\beta, A, k, s, \sigma_0, \mu_0^{true}, \Omega_0, \mu_Q, \sigma_Q\right) \tag{6.13}$$

We simulate a large number $N = 5000$ of firms as a sample. In principle, we choose the number N that balances the needs for small simulation error as well as for speedy calculation. For the simulation, we generate a sample path with the main focus on the first four rounds of financing(Cochrane, 2005). We set the period length to one year, approximately the period of one round of VC financing. The matching statistics are from Cochrane(2005):

* The mean returns of four rounds of VC financing.

* The standard deviations of four-round returns of VC financing.

* The mean and standard deviation of all returns and the mean of project duration.

* The survival probabilities for each round of financing.

Therefore, we have 11 calibration parameters to match 14 statistics to achieve sufficient degree of over-indentification for the model. From the simulated sample, we calculate the above statistics. Denote $d_i(V_i(\pi), O_i)) = V_i - O_i$ as the difference between the simulated and observed value of the i_{th} matching statistics, and $\mathbf{d}$ as the vector of the differences. Define

$$f(\pi) := \mathbf{d}^T \Sigma \mathbf{d} \tag{6.14}$$

in which Σ is the weighting matrix. We may use the simplest diagonal matrix. The calibrated base parameters solves:

$$\pi^* = \arg\min_{\pi} f(\pi) \tag{6.15}$$

The simulation process follows the two-dimensional evolution of the μ_t and Q_t processes. In our model,

$$Q_{t+1} - Q_t = \Delta Q_t = Q_t[\mu_Q + c_t + \sigma_Q N_1], \tag{6.16}$$

$$V_{t+1} = Q_t \left[\left(\Theta + A(c_t)^\alpha \eta_t^\beta \right) + sN_2 \right], \tag{6.17}$$

At each date t on the simulation path, Q_t and μ_t are the two state variables and governed by the above process dynamics. For the Q_t simulation, the noisy "shock" N_1 is drawn from the standard normal distribution. θ is drawn from a normal distribution with mean μ_0^{true} and standard deviation σ_0. The noisy "shock" N_2 is also drawn form the standard normal distribution.The discretion output $Ac_t^\alpha \eta_t^\beta$ is deterministic once Q_t and μ_t are determined.

Thus, along the sample path at each time point t,

1) simulate Q_t.

2) μ_t follow the Bayesian updating rule from simulated V:

$$\mu_i = \frac{s^2 \mu_0^j + \sigma_0^2 \sum_{t=1}^{i} (\Theta + S_t)}{s^2 + i\sigma_0^2} = \frac{s^2 \mu_0^j + \sigma_0^2 (\Psi_i)}{s^2 + i\sigma_0^2} \tag{6.18}$$

3) calculate the CV from simulated point using linear extrapolation from the lattice CVs and determine whether the project will be liquidated(if $CV < Q$) or not.

4) calculate the value of the project at t(if liquidated, then liquidation value)

5) calculate the round by round returns for the project.

References

[1] Adrian, T. and M.M. Westerfield, 2009, "Disagreement and Learning in a Dynamic Contracting Model," Review of Financial Studies, 22(10), 3873-3906.

[2] Allen, F. and D. Gale, 1999,"Diversity of Opinion and the Financing of New Technologies, " Journal of Financial Intermediation, 8, 68-89.

[3] Alti, Aydogan, Tetlock, Paul C, 2014. Biased beliefs, asset prices, and investment: a structural approach. *Journal of Finance* 69 (1), 325–361.

[4] Bayar, O., T. Chemmanur, and M.H. Liu, 2015, "A Theory of Capital Structure, Price Impact and Long-Run Stock Returns under Heterogeneous Beliefs " Review of Corporate Finance Studies, forthcoming.

[5] Chemmanur, T., J. Michel, D.Nandy, and Yan, A, 2010, "Capital Structure and Security Issuance under Heterogeneous Beliefs," SSRN Working Paper.

[6] Cochrane, 2005, "The Risk and Return of Venture Capital," Journal of Financial Economics, 75, 3-52.

[7] DeMarzo, Peter M. and Yuliy Sannikov (2017), "Learning, Termination, and Payout Policy in Dynamic Incentive Contracts." *Review of Economic Studies* 84, 182–236.

[8] Driessen, Joost & Lin, Tse-Chun & Phalippou, Ludovic, 2012. "A New Method to Estimate Risk and Return of Nontraded Assets from Cash Flows: The Case of Private Equity Funds," Journal of Financial and Quantitative Analysis, Cambridge University Press, vol. 47(03), pages 511-535

[9] Duffie, D. and Epstein, L. (1992), "Stochastic Differential Utility," Econometrica, 60, 353-394.

[10] Edmonds, C. T., J.E. Edmonds, R. Fu, and D.S. Jenkins, 2015, "Price Momentum, Heterogeneous beliefs, and the Premium for Meeting or Beating Analysts' Forecasts of Earnings," Working Paper.

[11] Ewens, Michael, Charles M. Jones, Matthew Rhodes-Kropf, 2013. “The Price of Diversifiable Risk in Venture Capital and Private Equity, The *Review of Financial Studies*,” Volume 26, Issue 8, Pages 1854–1889

[12] Fuerst, Oren and Uri Geiger, 2005, *“*The Discount Rate Used by Venture Capital Funds,*”* Chapter 8 in *From Concept to Wall Street: A Complete Guide to Entrepreneurship and Venture Capital.* Financial Times-Prentice Hall–Pearson Education.

[13] Gompers, P.A., 1995,“Optimal Investment, Monitoring, and the Staging of Venture Capital ” Journal of Finance, 50(5), 1461-1489.

[14] Gompers, Paul A, William Gornall, Steven N Kaplan, and Ilya A Strebulaev. 2020. “How do venture capitalists make decisions?" *Journal of Financial Economics*, 135(1): 169–190.

[15] Gompers, Paul, and Josh Lerner. "An Analysis of Compensation in the U.S. Venture Capital Partnership." *Journal of Financial Economics* 51, no. 1 (January 1999): 3–44.

[16] Giat,Y.,Hackman, S.T., and Subramanian, A., 2010,“Investement under Uncertainty, Heterogeneous Beliefs, and Agency Conflicts ” Review of Financial Studies,70:281-315.

[17] Giat, Y and Subramanian, A., (2013). Dynamic Contracting Under Imperfect Public Information and Asymmetric Beliefs. *Journal of Economic Dynamics and Control* 37, 2833-2861.

[18] Gladstone,D. and L. Gladstone,2003,“Venture Capital Investing, ” FT press.

[19] Gompers,1995,“Optimal Investment, Monitoring, and the Staging of Venture Capital, ” Journal of Finance, 50(5), 1461-1489.

[20] Gompers, P., and J. Lerner. 1997. Risk and Reward in Private Equity Investments: The Challenge of Performance Assessment. Journal of Private Equity 1:5-12.

[21] He, Zhiguo, Bin Wei, Jianfeng Yu and Feng Gao, 2017, “Optimal long-term contracting with learning,” *Review of Financial Studies* 30, 2006–2065.

[22] Holstrom, B. and P. Milgrom, 1987, “Aggregation and Linearity in the Provision of Intertemporal Incentives, ” Econometrica, 56, 1177-1190.

[23] Ito, T.,1990,“Foreign Exchange Rate Expectations: Mirco Survey Data, ” American Economic Review, 80(3), 434-449.

[24] Jung, H.W., A. Subramanian,2003,“Capital Structure under Heterogeneous Beliefs, ” Review of Finance.

[25] Kaplan, S. and A. Schoar. 2005. Private Equity Performance: Returns, Persistence, and Capital Flows. *Journal of Finance* 60:1791-1823.

[26] Kaplan, S. and P.Stromberg, 2003, “Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts, ” Review of Economic Studies,70, 281-315.

[27] Korteweg, Arthur and Morten Sorensen, 2011, “Risk and Return Characteristics of Venture Capital-Backed Entrepreneurial Companies,” *Review of Financial Studies* 23(10):3738-3772.

[28] Kortum, Samuel, and Josh Lerner, 2000, "Assessing the Impact of Venture Capital to Innovation." *RAND Journal of Economics* 31, no. 4: 674–692.

[29] Ljungqvist, A., and M. Richardson. 2003. The Cash Flow, Return and Risk Characteristics of Private Equity. Working paper, NYU Stern School of Business

[30] Ma, Yueran, Tiziano Ropele, David Sraer, and David Thesmar, 2020. A Quantitative Analysis of Distortions in Managerial Forecasts. Working Paper.

[31] Neher, D. (1999), “Staged Financing: An Agency Perspective,” *Review of Economic Studies*, 66, 255-274.

[32] Prat, Julien and Boyan Javanovic, 2014, “Dynamic contracts when the agent’s quality is unknown,” *Theoretical Economics,* 9, 865–914.

[33] Sahlman, 1990, “The Structure and Governance of Venture-Capital Organizations ” Journal of Finanical Economics, 27(2), 473-521.

[34] Schattler, H. and J. Sung, 1993, “The First-Order Approach to the Continuous-Time Principal-Agent Problem with Exponential Utility ” Journal of Economic Theory, 61, 331-371.

[35] Sorensen, Morten. 2007, “How smart is smart money? A two-sided matching model of venture capital,” *Journal of Finance* 62(6), 2725{2762.

[36] Tian, X., 2011, "The Causes and Consequences of Ventual Capital Stage Financing, " *Journal of Financial Economics*, 101, 132-159.

[37] Yang, Baozhong, 2013, "Dynamic Capital Structure with Heterogeneous Beliefs and Market Timing," *Journal of Corporate Finance* 22, 254–277.