Dynamic Macroeconomic Implications of Immigration*

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Abstract

International immigration flows are large, volatile and have recently increased. This paper is the first to study the dynamic effects of immigration shocks on the economy within a search and matching framework. Since the microdata indicates that some central macroeconomic effects of immigration are largest in the short run, a steady state analysis would be insufficient. To quantify the effects in our general equilibrium framework, we use Swedish population registry data on labor market status. We then study the effect of a large immigration shock on various macroeconomic aggregates. Due to compositional effects, there is a substantial negative effect on GDP per capita and the employment rate on impact that then decreases over time.

Keywords: Immigration, dynamics, search and matching.

JEL classification: J21, J31, J61.

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1 Introduction

International immigration flows are large and volatile and have been growing in recent decades. A prominent example is the refugee crisis that reached its peak in 2015. Although there is a large literature analyzing the implications of immigration in many dimensions (e.g., Borjas, 2014), this literature mainly focuses on micro-level effects of immigration. Aggregate effects of immigration have been studied less, and, in particular, the dynamic effects of immigration shocks on macroeconomic aggregates is little studied in the literature. Compared to most macroeconomic time series the volatility of immigration is staggering - changes in annual growth rates of $\pm 50\%$ are not unusual for large European countries like Spain or Germany (Eurostat, 2020). This indicates that, while steady state analysis is interesting, it might not be sufficient to spell out all implications of immigration.

In this paper, we attempt to fill this gap by building a theoretical model and use rich microdata to guide the quantification. Some other papers study this issue broadly from a steady state perspective, e.g., Ottaviano and Peri (2012), Dustmann et al. (2013), Chassamboulli and Palivos (2014) and Battisti et al. (2018). However, none of these papers analyzes the dynamic effects from an immigration shock on macroeconomic aggregates in a structural model.

To study the dynamic effects of immigration, we thus use a structural model that quantifies the effects of several immigration scenarios on the paths of per capita GDP, unemployment, labor force participation (LFP), real wages and aggregate labor productivity. Our modelling approach allows us to capture the relevant general equilibrium effects. We incorporate public-finance implications of immigration but keep the fiscal dimension simple compared to, e.g., Storesletten (2000).

An important driver of fiscal effects of immigration present in the model is age differences between natives and migrants, where, as long as migrants arrive early in their working age, immigration have a positive fiscal effect, typically referred to as a "demographic dividend". Such age differences also have positive effects on other aggregates, e.g., GDP per capita. Indeed, the fact that immigration can improve the old-age dependency ratio is an important contributor to the positive welfare effects from immigration that are found in Busch et al. (2020). A potential offsetting effect, however, comes from the gradual and slow-moving integration into the labor market: it is a well-documented fact that employment rates for immigrants in both the U.S. and Western Europe start below the employment rates of natives and are increasing in the number of years since immigration, see Brell et al. (2020) and Busch et al. (2020). Lubotsky (2007) has documented a similar pattern for the relative earnings of immigrants. The speed of the integration process depends on, among other things, the level of education and the relative productivity of the migrants, but a slow integration process tends to imply negative macroeconomic effects and may, in fact, overturn the demographic dividend. An obvious implication from gradual integration is that an immigration shock will have its largest direct negative effect on employment rates and, presumably, GDP per capita, on impact. This is also a key reason why a dynamic approach is warranted. A steady state analysis of this problem gives an incomplete answer and underestimates one key (short- and medium-run) negative economic effect of immigration. Our main exercise is to quantify the effects on the macroeconomy of a change in the immigration flow and, in particular, to quantify which of these opposite forces dominates at various horizons.

The exercises that we carry out in this paper are useful for several reasons. First of all, we will be able to analyze the effects on macroeconomic aggregates following large changes in immigration. Furthermore, we can quantify to what degree recent sluggish wage and labor productivity growth is due to immigration flows and how much equilibrium unemployment rates change in response to immigration. Another important aspect is that our framework is useful for illustrating the macroeconomic implications of a change in the composition of immigrants.

We construct a real general equilibrium model with a strong focus on the labor market to compute the effect of a large immigration shock on various macroeconomic aggregates. We make use of detailed Swedish data on the entire population to document native and immigrant labor force participation and unemployment as a function of years since immigration when calibrating the model. An important advantage of using Sweden as our "laboratory" is that it enables us to use unique microdata estimates of differences in labor productivity by country of birth obtained using rich matched employee-employer datasets documented in Ek (2018). Note also that Sweden has a large foreign-born population: around 20%.

The productivity differences documented in Ek (2018), naturally leads to observed differences between natives and migrants in e.g., wages and unemployment. Also, in a model, such productivity differences can generate wage and unemployment differences, including structural unemployment. The labor market in the model is characterized by search and matching, with substantial structural unemployment. We consider two skill (education) groups, high skill and low skill, which look for work in separate markets. In addition, workers within each skill group differ with respect to individual productivity (efficiency units of labor). Moreover, as mentioned above, to capture differences in unemployment and wages, the productivity distributions differ between natives and immigrants, allowing for structural unemployment. In particular, it allows for different structural unemployment rates for immigrants and natives as well as differences across skill groups. Our model also allows for gradual integration of immigrants in terms of productivity improvements: the longer a migrant stays in the country the more his/her individual productivity increases and the probability of structural unemployment decreases. This implies, in line with the data, that unemployment is highest for immigrants' right after immigration and then gradually and slowly declines. Allowing for structural unemployment is thus important since it captures the gradual nature of integration. Any excessive frictional unemployment dissipates quickly and thus has a hard time explaining unemployment that remains elevated for at least a decade.

Our main exercise is to our model to analyze the effects of an immigration shock corresponding to one percent of the total population, similar in size to the increase in immigration in Sweden around the refugee crisis of 2015. This shock leads to a reduction in GDP per capita of 1.0 percent and an increase in aggregate unemployment of 0.9 percentage points on impact. The effect on GDP per capita is persistent—half of the initial effect remains after five years, and it takes twenty years for 80% of the effect to dissipate. These are large effects, and, in addition to the high unemployment and low labor force participation rates of the newly arrived immigrants, they are explained by the fact that unemployment of low-skilled natives increases by 0.3 percentage points in response to the immigration shock. This is mainly due to the resulting increase in taxes and to a lesser degree to the increased fraction of low-productivity workers in the low-skilled unemployment pool, both of which discourage job creation. The effect of the shock on high-skilled natives' unemployment is instead negligible.

In line with the intuition provided above, we find that the effect on most variables is largest on impact and then gradually decreases over time. The effects of an immigration shock on aggregate quantities differ substantially from the steady state effects. In particular, the dynamic effects are generally roughly one order of magnitude larger than the steady state effects. In the intermediate term, two opposing forces are at play regarding the employment-to-population ratio. This ratio is pushed upwards since a larger fraction of migrants than natives is of working age, reducing the old-age dependency ratio. However, the dominating force is that immigrants have lower employment rates than natives within the working age population. In our model, and in contrast to the result in Busch et al. (2020), immigration decreases the employment-population ratio. The reason for our result is the gradual nature of the integration process. Under the assumption of a balanced budget, taxes increase by 0.8 percentage points on impact and then decline but remain elevated for an extended period of time. Net transfers from natives to immigrants increase by 0.5 percent of GDP on impact and then decline gradually. Finally, the effects on aggregate productivity and wages are very limited. In line with the empirical literature, the effects on wages of natives are even smaller, basically negligible, except for the first few quarters.

We find that the degree of tax smoothing is important for the magnitude of the effects. In our baseline exercise, we assume that the government budget is balanced, implying that taxes increase substantially in response to an immigration shock. This reduces job creation incentives, leading to large effects on GDP and unemployment. In particular, the increase in unemployment for low-skilled natives is considerable. With tax smoothing, so that the government finances the extra costs over several decades, when the shock hits, the effects on GDP and aggregate unemployment are substantially reduced. In particular, the increase in unemployment for low-skilled natives is less than half as large as in the baseline scenario.

Compositional effects are also important for the size and sign of the effects. In our baseline exercise, immigrants have substantially lower productivity than natives do when they first arrive to the country. If we keep the age difference fixed but assume that immigrants are identical to natives in terms of productivity, we can isolate a substantial demographic dividend. Immigration then results in a substantial increase in the employment-population ratio and GDP per capita in the medium run. Moreover, taxes fall and transfers to immigrants decrease.

The most closely related theoretical work to our paper is limited to steady state analysis. For example, Battisti et al. (2018) analyze the effects on natives' welfare using a search and matching framework. In terms of modelling, their work is related to Chassamboulli and Palivos (2014) that quantifies the steady state effects on skilled and unskilled natives' wages. Neither of these two papers allow for structural unemployment or the gradual integration of immigrants in the labor market, both of which are important when modelling aggregate dynamics. There are a couple of papers modelling immigration in a dynamic macroeconomic setting, but differently from us emphasizing immigration (mainly within EU) as a channel for the response to other shocks, see e.g. Bandeira et al. (2018). Smith and Thoenissen (2019) analyze effects of shocks to skilled immigration in a structural model in a setting abstracting from unemployment and labor force participation. Canova and Ravn (1998, 2000) study immigration shocks of low-skilled workers in a framework also abstracting from unemployment. Stähler (2017) studies shocks to refugee immigration but emphasizes demand effects and also abstracts from search frictions in the labor market. Similarly to us, Busch et al. (2020) study the refugee wave around 2015, but with a focus on welfare implications for various groups of natives, and abstract from modelling unemployment.

Recent empirical work by Furlanetto and Robstad (2019) uses an SVAR approach to document the substantial importance of within-EU immigration shocks for the variation in aggregate unemployment and GDP, indicating the importance to study related questions in a structural model. Dustmann, Fabbri and Preston (2005) document the effects of immigration on labor market outcomes of the native population. Their focus on the native population (as opposed to the aggregate) is shared by a large literature, and they find limited or negligible negative effects on natives' wages from immigration, in line with the literature. The implications of our model are consistent with this finding.

The paper is outlined as follows. We start in section 2 by outlining a simple search and matching model where some of the main mechanisms can be described analytically. Section 3 describes the micro data we use and some salient facts of the labor-market status of migrants. Section 4 presents the model, section 5 documents the calibration and section 6 provides the quantitative results. Finally, section 7 concludes.

2 The Main Mechanisms

We first illustrate two key mechanisms for our analysis in a simple search and matching model, where effects can be computed analytically in steady state. The two mechanisms we document in the simple model are, first, that the productivity distribution is affected by immigration, and second, that immigration can have fiscal effects through higher unemployment or a demographic premium from immigration. The fiscal effects of course depend on the composition of immigrants, and, depending on this, the net effect on taxes can be both positive and negative.

Workers have heterogenous productivities ε_i and are distributed according to the cumulative distribution function G with probability density g with support I. Letting G^d and G^m (Ω^d and Ω^m) denote the cumulative distribution function (population) of individual productivities for natives and immigrants, respectively, the cumulative distribution function of the entire population is $G(\varepsilon) = \frac{\Omega^d G^d(\varepsilon)+\Omega^m G^m(\varepsilon)}{\Omega^d+\Omega^m}$. To be able to derive closed-form solutions, we here just consider steady state variations of taxes and the productivity distribution G. This simple model is described in detail in Appendix A.3 while the more general model allowing for shocks to immigration flows, an explicit demographic premium and integration of immigrants is described in section 4.

Assuming a Cobb-Douglas meeting function with elasticity ξ with respect to unemployment gives job and vacancy meeting rates $f = \theta^{1-\xi}$ and $q = \theta^{-\xi}$, where θ is labor market tightness.

The value of the firm when employing a worker with productivity ε_i and paying the wage w_i is given by

$$J_i = \varepsilon_i - w_i + \beta(1-\delta) J_i,$$

where β is the discount factor and δ the exogenous probability that a match is destroyed. The surplus of an employed worker with productivity ε_i is

$$S_i = (1-\tau) w_i - b + \beta\left(1-\delta-\tilde{f}_i\right) S_i,$$

where τ is a tax on labor income, b the flow payoff of a worker when unemployed, and $\tilde{f}_i = f\mathbb{I}(J_i \geq 0)$ the probability of finding a job, recalling that f is the job meeting rate and $\mathbb{I}$ is an indicator function that captures whether a worker is employable or not. Wages are determined by the Nash bargaining solution $(1-\tau)\eta J_i = (1-\eta) S_i$, where η is the worker bargaining power.[1] Finally, job creation is

[1] This follows from noting that the wage maximizes $S_i^{\eta} J_i^{1-\eta}$ and using the definitions of J_i and S_i.

given by

$$c = q\beta \int_I \frac{u_i}{u} \max\{J_i, 0\}\, di,$$

where c is the vacancy cost and $u = \int_I u_i di$.

There exists a cutoff value for the individual productivity, ε^c, for which the firm is indifferent between employing and not employing a worker ($J_i = 0$). Using the solution for the wage described in Appendix A.3, this value is given by

$$\varepsilon^c = \frac{b}{1-\tau} \equiv \tilde{b}. \tag{1}$$

Thus, the cutoff productivity is equal to the flow value of unemployment, net of tax. The share of employable workers is thus $1 - G(\tilde{b})$, and structural unemployment is given by $G(\tilde{b})$. Frictional unemployment is then $\int_{i:\varepsilon_i \geq \varepsilon^c} u_i di = u - G(\tilde{b}) = 1 - n - G(\tilde{b})$. Since frictional unemployment is similar for any $\varepsilon_i \geq \varepsilon^c$, the probability density function (PDF) for workers with productivity ε_i, conditional on employability, is $g_i/(1 - G(\tilde{b}))$. Using this and the solution for firm values that can be derived using the Nash solution for wages, job creation can be written as

$$c = \frac{q\beta(1-\eta)\left(\bar{\varepsilon} - \frac{b}{1-\tau}\right)}{1 - \beta(1-\delta) + \beta\eta f} \frac{\delta\left(1 - G\left(\tilde{b}\right)\right)}{\delta + fG\left(\tilde{b}\right)} = \Psi(\theta, \tau) \underbrace{\frac{\delta\left(1 - G\left(\frac{b}{1-\tau}\right)\right)}{\delta + fG\left(\frac{b}{1-\tau}\right)}}_{=\Upsilon(\tau)},$$

where $\bar{\varepsilon} = \int_{i:\varepsilon_i \geq \varepsilon^c} \frac{g_i}{1-G(\tilde{b})} \varepsilon_i di$ is average productivity among employed workers. Here, the first term, denoted by Ψ, is standard in the search and matching model. The second term is the additional effect from idiosyncratic productivity and hence structural unemployment on job creation. It can easily be shown that the partial derivatives of Ψ and Υ satisfy the following: $\Psi_\theta(\theta, \tau) < 0$, $\Psi_\tau(\theta, \tau) < 0$ and $\Upsilon_\tau < 0$.

We first consider fiscal effects. Generally, such effects could depend on several factors, e.g., the replacement rate b. Here, the fiscal effects are due to either the demographic dividend that leads to a reduction in taxes or compositional effects. How the composition of migrants affects taxes depends on whether the change in inflow consists of relatively high- or low-productivity workers. From (1), the tax rate directly affects the cutoff productivity ε^c and, thus, the structural unemployment level.

Consider now the effect of an increase in taxes in a standard search and matching model without structural unemployment (when $\Upsilon(\tau) = 1$). Then, since $\Psi(\theta, \tau) = c$ and $\Psi_\tau < 0$, it follows that tightness decreases. As a result, the job finding rate decreases, and this increases (frictional) unemployment.

In the model with structural unemployment, there is an additional effect. Specifically, taxes affect the surplus and in turn the cutoff value for when it is profitable to employ workers in (1); $\tilde{b}$ increases. This leads to a fall in Υ, leading to additional effects on job creation. Specifically, since $\Psi(\theta, \tau)\Upsilon(\tau) = c$, this effect leads to an additional force pushing tightness down. Since $\tilde{b}$ increases, structural unemployment increases, and since the job finding rate decreases, unemployment for high-productivity workers also goes up.

We now study the effects of compositional changes in immigration, affecting the distribution G while keeping distortionary taxes fixed.[2] Note that the cutoff $\tilde{b}$ is unaffected by this. Depending on the individual productivities of the immigrants, the share of workers below the employability cutoff $\tilde{b}$ can both increase and decrease. Similarly, the average productivity among employed workers can also increase or decrease. Here, we focus on two distinct cases highlighting the two different channels.

First, consider an inflow of relatively unproductive migrants, modelled as a $\bar{\varepsilon}$-preserving spread with the new distribution being denoted by G', i.e., keeping the average productivity of the employable workers fixed, $\bar{\varepsilon}' = \bar{\varepsilon}$. Thus, structural unemployment increases, $G'(\tilde{b}) > G(\tilde{b})$. Ψ remains unaffected, while Υ decreases. Consequently, labor market tightness and the job meeting rate decrease. Intuitively, since a larger share of workers gives no surplus to the firms, it is less profitable to post vacancies, leading to a decrease in vacancies and job creation.

Second, consider an increase in relatively productive migrants while keeping the fraction of employable workers fixed, modelled as a $G(\tilde{b})$-preserving productivity increase so that $G'(\tilde{b}) = G(\tilde{b})$ while $\bar{\varepsilon}' > \bar{\varepsilon}$. Then Υ is unchanged, while Ψ increases and $\tilde{b}$ is unaffected. Since Ψ increases (for a given tightness θ), from the job creation condition labor market tightness and the job meeting rate both increase. The intuition is that, since employable workers are more productive on average, firms post more vacancies, which in turn leads to an increase in job creation and a reduction in frictional unemployment.

Both of these changes in G described above can have fiscal effects along the lines described above if taxes are distortionary. Specifically, in the first (second) case, taxes increase (decrease) due to the reduction (increase) in employment, leading to additional effects on job creation and, in turn, employment.

Our full model, described in detail in section 4, has a detailed formulation of immigration flows and the population of natives. Also, a general technology with (imperfectly substitutable) high- and low-skilled workers is used instead of the simplified technology used above.[3] As we will see in section 6, immigration in the baseline calibration of the full model triggers a fall in the fraction of employable workers, which increases structural employment $G(\tilde{b})$. The resulting fiscal effects lead to an increase

[2] In this experiment, any shortfall or increase in revenue due to immigration can be thought of as being financed by/reimbursed to households using lump sum transfers/taxes.

[3] In an extension, natives and immigrants are also assumed to be imperfectly substitutable.

in taxes, in turn increasing both frictional and structural unemployment. In an experiment with an alternative composition of migrants that has higher average productivity, we find a substantial demographic dividend that leads to an increase in GDP in the medium run as well as a reduction in unemployment and taxes.

3 Data

We use data from Statistics Sweden, and specifically the STATIV/LISA database, to calibrate the model. This is a rich dataset on the entire Swedish population from which we have obtained data for individuals in the age range 20–64 years. The sample period is 2000–2017, and the variables include region of birth, the date and reason for immigration, labor market status, labor income and various demographic variables, e.g., education attainment. Importantly, we also have estimates of productivity for immigrants from different regions of birth based on Ek (2018). These estimates are based on a rich matched employer-employee population dataset.

3.1 Some facts about immigration and labor market status in Sweden

Our detailed micro data regarding immigrants to Sweden is fairly representative for immigration in continental Europe in terms of the employment rate as a function of the number of years in the country; see Brell et al. (2020). In addition, they document that for the first 10 years since immigration, employment rates of immigrants in both the US and all European countries studied fall short of that of natives. Regarding Sweden, refugee residence permits have varied between around 5,000 and 70,000 per year in the period 1980–2016, with peaks in 1994, 2007, and 2016; see Ruist (2018). Other types of immigration, such as family re-unification and work-based residence permits, are generally larger and less volatile with an increasing trend over time. Specifically, during the period 2000–2017, refugees and their families accounted for one-third of the immigrants living in Sweden according to STATIV.

Immigrants are different from natives in many dimensions. Some of these differences are most pronounced the first few years after immigration. In Figure 1, we document labor force participation and unemployment rates for immigrants as a function of the number of years since immigration.[4] For now, we will limit our attention to the black dashed lines that show the values of these two rates in the data. The left panel of Figure 1 documents that the unemployment rate is very high in the first few years after the immigration date but slowly and partially drops toward the level of natives, which is 6.75% in the data. The right panel of Figure 1 instead documents the labor force participation rate of immigrants over time. The initial difference compared to native-born individuals (87%) is also very

[4] To be specific, number of years since the residence permit was issued.

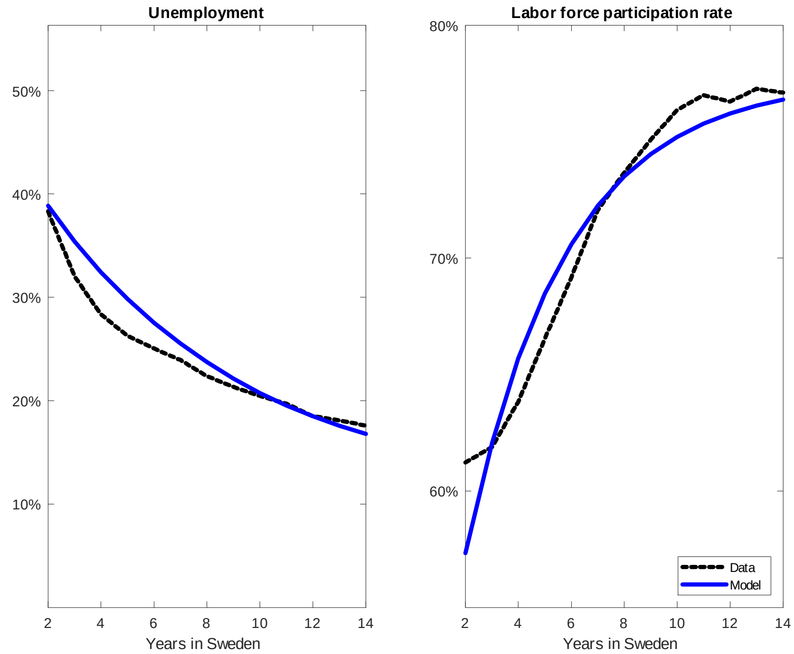

Figure 1: Unemployment rate and labor force participation rate of immigrants as a function of number of years in the country.

large but shrinks over time.

The facts documented in this figure are important for understanding the aggregate consequences of immigration. For example, they indicate that the maximum effect on economic outcomes like employment rates occur on impact. Below, we build a model that captures these facts and illustrate their macroeconomic implications quantitatively.

4 The model

Agents are risk neutral and can either be of working or non-working age. We simplify the modelling of age by using the "Model of Perpetual Youth" approach of Blanchard-Yaari (Blanchard, 1985, and Yaari, 1965). In this approach, there is a constant probability of transition to retirement and death, respectively, that is independent of age. This approach captures what we care about - the public finance implications of immigration on the age-dependency ratio - equally well as less tractable OLG frameworks would.

The labor market is characterized by search and matching and allows for both frictional and structural unemployment in order to fit the data. We consider two skill groups: high (H) and low (L) skilled, which corresponds to workers with and without a college degree in the data. Unemployed workers search for jobs within their skill-group-specific labor market. In addition, workers within each skill group differ with respect to individual productivity (efficiency units of labor). This generates variation across natives and immigrants and high- and low-skilled in, respectively, unemployment rates, wages, and labor productivities.[5] The individual productivity of a worker with type $i \in \{1, 2, ..., I\}$ is then denoted by ε_i. For natives, denoted by superscript d as in domestically born, the (discrete) PDF of the productivity distribution is approximated by a log-normal distribution, and its parameters vary across skill (education) levels.

For immigrants, denoted by m, the productivity distribution is slightly more complicated. When entering the country, immigrant productivity also follows a log normal distribution with a potentially different mean and standard deviation than the native distribution and across skill level. The productivity distribution for immigrants is more complicated since we assume gradual integration of (newly arrived) immigrants so that the productivity of an individual immigrant increases the longer he or she stays in the country. This is intended to capture improvements in local language skills, improved matching of other skills to job requirements, and a growing network of potential employers resulting in better job matches. We model this integration by assuming that productivity follows a Markov process, where productivity remains the same with probability $1 - \pi$ and increases by some small amount, from ε_i to ε_{i+1}, with probability π. In addition, this integration process ends with probability ϕ, which is introduced in order to match how unemployment decreases convexly with the number of years since immigration; see the left panel in Figure 1. This means that the worker productivity distribution in the model is determined by the four (log normal) means, μ_g^o, and standard deviations, σ_g^o, where $g \in \{H, L\}$ and $o \in \{d, m\}$, as well as the integration parameters π and ϕ. In addition, labor force participation rates for immigrants are modelled as an exogenous process that increases in the number of years an individual has stayed in the country.

4.1 Relationship to Battisti et al. (2018)

It might be illuminating to spell out the differences in assumptions compared to the existing theoretical literature. A key distinction between our model and the model in Battisti et al. (2018) is how we generate different unemployment rates between immigrants and natives. In our model, this is due to heterogeneity in terms of productivity across workers, which is in line with the empirical results in Ek (2018). This generates structural unemployment, because some workers are unemployable. In particular, when taking the model to the data, structural unemployment is higher among immigrants than natives, due to this mechanism. In addition, our approach implies lower average wages for immigrants, as in the data. In contrast, Battisti et al. (2018) only allow for frictional unemployment. They assume the same productivity for immigrants and natives in their baseline, differently from the

[5] For empirical differences in productivity, see Ek (2018).

results in Ek (2018). To generate lower wages for immigrants, they assume that the outside option is lower for immigrants than natives. In order to generate a higher unemployment rate for immigrants, Battisti et al. (2018) assume that immigrants have higher job separation rates than natives. Labor market frictions are higher for immigrants in our model as well, but instead modelled as a lower average job finding rate. Another key difference in assumptions is that their model abstracts from the demographic dividend, i.e., the positive effect on government finances from immigrants arriving young but mainly in working age, reducing expenditures on, e.g., schooling.[6]

While the distinction between structural and frictional unemployment might be of less importance when analyzing steady state effects on e.g., unemployment, it does matter when analyzing aggregate dynamics as well as for individual outcomes. Also, allowing for heterogeneity in structural unemployment that gradually fall over time for immigrants, as we do, appear to be a natural way to match the empirical fact that individual unemployment is much higher in the first couple of years after an immigrant arrives to a country. This pattern would be hard to capture using only frictional unemployment.

4.2 Search and matching

There is random search within each skill group, and the job meeting rate for skill group g can be written as

$$f_g = \frac{M_g}{u_g},$$

where we assume that the meeting function is Cobb-Douglas, modified to take into account the fact that meeting probabilities are at the most one,

$$M_g = \min\left\{A\left(u_g\right)^{\xi}\left(v_g\right)^{1-\xi}, u_g\right\},$$

and unemployment within each skill group is

$$u_g = \sum_{i \in I} u_{i,g},$$

[6] A final difference is that Battisti et al. (2018) assumes that all households participate in the labor force, or interpreted more loosely, that there are no participation differences between natives and immigrants. This assumption is inconsistent with the data as indicated by the low labor force participation for first the 5-10 years after immigration as documented in Figure 1. Abstracting from heterogeneity in labor force participation when analyzing dynamics would generate counterfactual implications.

where $u_{i,g}$ is unemployment for workers with productivity i in skill group g. The vacancy meeting rate and labor market tightness are, respectively, given by

$$q_g = \frac{M_g}{v_g} \quad \text{and} \quad \theta_g = \frac{v_g}{u_g}.$$

Finally, exogenous separations vary across markets and are denoted by δ_g, where $g \in \{L, H\}$. Firms post vacancies in the market for skilled or unskilled workers at cost c_g.

4.3 Technology

Workers are assumed to be perfectly substitutable within each skill group. In appendix A.6.1, we consider an extension of the model to the case with imperfect substitutability between natives and migrants and show that imperfect substitutability is not very for our results. Here, in our baseline specification, perfect substitutability implies that the effective (productivity-adjusted) labor supply of skill group g is independent of country of origin and is given by

$$n_g = \sum_i \varepsilon_i n_{i,g}, \tag{2}$$

where $n_{i,g}$ is the employment for workers with skill g and productivity ε_i.

The two skill groups are imperfectly substitutable, and the production function is of the Cobb-Douglas type

$$Y \equiv F(n_H, n_L, K) = A^{tfp} K^{\alpha} Z(n_H, n_L)^{1-\alpha}, \tag{3}$$

where A^{tfp} is total factor productivity, K is capital, α the capital share and Z is a CES aggregate over the two types of labor

$$Z = \left(a n_H^{\frac{\rho-1}{\rho}} + (1-a) n_L^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}.$$

The marginal product of skill labor of type k is then $\partial Y / \partial n_{i,k} = (1-\alpha) A^{tfp} K^{\alpha} Z^{-\alpha} \partial Z / \partial n_{i,k}$. Capital is freely mobile internationally, so the return is determined on world markets.[7] Let the stock of capital owned by natives be denoted by $\bar{K}$, which is fixed over time, and the risk free rate by r and depreciation by ς.

[7] Specifically, we follow Battisti et al. (2018), and assume that each individual owns an equal share of capital and the marginal return of capital is equal across individuals. We further assume that the amount of capital belonging to natives does not change over time and is independent of immigration. Note however, that allowing for gradual adjustments of the capital stock would amplify the negative initial effects of increased immigration as the marginal product of labor temporarily would fall and result in lower employment and wages during the transition.

This implies that the marginal products of high- and low-skilled labor, respectively, are given by

$$\frac{\partial Y}{\partial n_{i,H}} = (1-\alpha) A^{tfp} K^{\alpha} Z^{-\alpha} a \left(\frac{Y}{n_H}\right)^{\frac{1}{\rho}} \varepsilon_i \text{ and } \frac{\partial Y}{\partial n_{i,L}} = (1-\alpha) A^{tfp} K^{\alpha} Z^{-\alpha} (1-a) \left(\frac{Y}{n_L}\right)^{\frac{1}{\rho}} \varepsilon_i. \quad (4)$$

4.4 Values

Recall that there are two types of individuals, natives and migrants, denoted by d and m, respectively. Moreover, migrants can either be newly arrived or established. An individual starts life when entering working age. Labor force participation is determined exogenously. Working age individuals transit into non-working age with some fixed probability $p^o, o \in \{d, m\}$. When calibrating the model, we choose these probabilities to match the time in non-working age (both retirement and childhood, although we use retirement as a shorthand for non-working agents below), which differs between natives and immigrants. This is important if we want to capture the demographic dividend from immigration that is due to many immigrants arriving young but of a working age. Individuals of working age who are outside the labor force receive z_l in government assistance, and retirees receive z_{ret}. Finally, retirees die with the exogenous probability Θ^o.

4.4.1 Worker values

The value of being retired is, for group $o \in \{d, m\}$, given by

$$R^o = z_{ret} + \beta \left(1 - \Theta^o\right) R^{o\prime},$$

where β denotes the discount factor.

Established migrant workers are identical to native workers, except that the parameters for retirement and labor force participation differ. Consequently, their values of being employed and unemployed are also conceptually similar. Note that when an unemployed worker gets a job, the worker can end up (randomly) at any of the firms in the model. Denoting the vector of employment levels of the firm by $\boldsymbol{n} \equiv \{n^d_{1,L} n^m_{1,L}, \ldots, n^m_{I,L}, n^d_{1,H}, \ldots, n^m_{I,H}\}$, the value of being unemployed for natives and established workers is given by

$$U^o_{i,g} = b_{i,g} + rk^o_{i,g} + \beta \left(1 - p^o\right) \left[\tilde{f}^o_{i,g} \mathbb{E}_{\boldsymbol{n}'} W^{o\prime}_{i,g}\left(\boldsymbol{n}'\right) + \left(1 - \tilde{f}^o_{i,g}\right) U^{o\prime}_{i,g}\right] + \beta p^o R^{o\prime}, \text{ with } o \in \{d, e\}, \quad (5)$$

where $rk^o_{i,g}$ is capital income, $\mathbb{E}_{\boldsymbol{n}'}$ is the expectation over firms across employment $(\boldsymbol{n})$, reflecting the fact that workers can end up randomly at each of the firms. Also, the term $\tilde{f}^o_{i,g} \equiv f_g I(J^o_{i,g}\left(\boldsymbol{n}'\right) \geq 0)$ is the job finding probability, i.e., the job meeting rate, f_g times an indicator function, $\mathbb{I}$, that takes the value one if the firm value of hiring the specific worker is positive and zero otherwise.[8] Intuitively, the value of unemployment depends on the unemployment insurance benefit and the continuation value that, in turn, depends on the future values of being employed, unemployed, and retired as well as the probabilities of ending up in each of these states.

Similarly, the value of being employed for natives and established workers is given by

$$W_{i,g}^{o}(\boldsymbol{n}) = (1-\tau) w_{i,g}^{o} + rk_{i,g}^{o} + \beta(1-p^{o})\left[(1-\delta_g) W_{i,g}^{o\prime}(\boldsymbol{n}') + \delta_g U_{i,g}^{o\prime}\right] + \beta p^{o} R^{o\prime}, \tag{6}$$

where $w_{i,g}^{o}$ is the wage and τ is a labor income tax.

Motivated by the findings shown in Figure 1 that the unemployment rate as a function of the number of years since immigration is decreasing and convex, we assume gradual integration so that the individual productivity of an immigrant is increasing in the time that the person has stayed in the country. The timing of the possible transitions for immigrants is as follows. In a given time period, agents first retire with probability p^m. Newly arrived immigrants then become established with probability ϕ. The potential productivity improvements of newly arrived immigrants are then realized, i.e., their individual productivities increase by one grid point with probability π. Similar to native-born workers, established immigrants have constant productivity.[9] The value of employing a newly arrived worker is then

$$\begin{aligned} W_{i,g}^{na}(\boldsymbol{n}) \;=\;& (1-\tau) w_{i,g}^{na} + \beta(1-p^m)(1-\phi)\left[(1-\delta_g)\left((1-\pi) W_{i,g}^{na\prime}(\boldsymbol{n}') + \pi W_{i+1,g}^{na\prime}(\boldsymbol{n}')\right)\right] \\ & + \beta(1-p^m)(1-\phi)\left[\delta_g\left((1-\pi) U_{i,g}^{na\prime} + \pi U_{i+1,g}^{na\prime}\right)\right] \\ & + \beta(1-p^m)\phi\left[(1-\delta_g) W_{i,g}^{e\prime}(\boldsymbol{n}') + \delta_g U_{i,g}^{e\prime}\right] + \beta p^m R^{m\prime}. \end{aligned} \tag{7}$$

Even though the above expression is extensive, it is intuitive. Compared to (6), the continuation value now also includes the possibility that individual productivity increases between periods (with probability π) as well as the possibility that the newly arrived goes on to become established (with probability ϕ).

Proceeding as for $U_{i,g}^{o}$, the value of unemployment for a newly arrived worker is given by

$$\begin{aligned} U_{i,g}^{na} \;=\;& b_{i,g} + \beta(1-p^m)(1-\phi)\left[(1-\pi)\tilde{f}_{i,g}^{na}\mathbb{E}_{\boldsymbol{n}'} W_{i,g}^{na\prime}(\boldsymbol{n}') + \pi \tilde{f}_{i+1,g}^{na}\mathbb{E}_{\boldsymbol{n}'} W_{i+1,g}^{na\prime}(\boldsymbol{n}')\right] \\ & + \beta(1-p^m)(1-\phi)\left[(1-\pi)\left(1-\tilde{f}_{i,g}^{na}\right) U_{i,g}^{na\prime} + \pi\left(1-\tilde{f}_{i+1,g}^{na}\right) U_{i+1,g}^{na\prime}\right] \\ & + \beta(1-p^m)\phi\left[\tilde{f}_{i,g}^{e}\mathbb{E}_{\boldsymbol{n}'} W_{i,g}^{e\prime}(\boldsymbol{n}') + \left(1-\tilde{f}_{i,g}^{e}\right) U_{i,g}^{e\prime}\right] + \beta p^m R^{m\prime}. \end{aligned} \tag{8}$$

[8] Obviously, the value needs to be positive for a match to take place.

[9] Allowing for productivity improvements for established immigrants only has small effects on the calibration, and the resulting probability of productivity improvement is very close to zero (0.01).

4.4.2 The firm values

Firms are large and employ several workers. Let employment of group $o \in \{d, na, e\}$ be denoted $n^o_{i,g}$. The value of a firm is then given by

$$V(\boldsymbol{n}) = \max_{\{v_L, v_H, K\}} F(n_H, n_L, K) - \sum_{o \in \{d,na,e\}} \sum_{i=1}^{I} \sum_{g \in \{H,L\}} w^o_{i,g} n^o_{i,g} - \sum_{g \in \{H,L\}} c_g v_g - (r + \varsigma) K + \beta V(\boldsymbol{n}'), \quad (9)$$

where v_g is the number of vacancies and $r + \varsigma$ the user cost of capital. Naturally, the value is increasing in output and decreasing in factor payments and the costs associated with posting vacancies.

The value to the firm of an additional worker of group o, type g, and productivity i is denoted by $J^o_{i,g}(\boldsymbol{n})$. This value can be computed by differentiating (9) with respect to $n^o_{i,g}$.[10] The firm surplus of an additional native worker can then be shown to be given by

$$J^d_{i,g}(\boldsymbol{n}) = \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) - w^d_{i,g} + \beta \left(1 - p^d\right)(1 - \delta_g) J^d_{i,g}(\boldsymbol{n}'). \quad (10)$$

With the marginal products of labor given by (4), it follows that the marginal value to the firm of a worker with productivity ε_i in skill group g only depends on F, n_g and i. This is convenient in that it implies that the state space can be reduced to $\{F, n_g, i\}$ instead of the full employment vector $\mathbf{n}$.

The value to the firm of employing an established and a newly arrived worker each with productivity level ε_i are, respectively, given by

$$J^e_{i,g}(\boldsymbol{n}) = \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) - w^e_{i,g} + \beta (1 - p^m)(1 - \delta_g) J^e_{i,g}(\boldsymbol{n}') \quad (11)$$

and

$$\begin{aligned} J^{na}_{i,g}(\boldsymbol{n}) = {} & \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) - w^{na}_{i,g} \\ & + \beta (1 - p^m)(1 - \delta_g) \left[(1 - \phi)\left((1 - \pi) J^{na}_{i,g}(\boldsymbol{n}') + \pi J^{na}_{i+1,g}(\boldsymbol{n}')\right) + \phi J^e_{i,g}(\boldsymbol{n}')\right]. \end{aligned} \quad (12)$$

4.5 Wage determination

The wage is determined by Nash bargaining between the representative firm and each worker of group $o \in \{d, na, e\}$, type g and productivity ε_i:

$$(1 - \tau)\eta J^o_{i,g}(\boldsymbol{n}) = (1 - \eta)\left(W^o_{i,g}(\boldsymbol{n}) - U^o_{i,g}(\boldsymbol{n})\right), \quad (13)$$

[10] Note that $\partial n^{o\prime}_{i,g} / \partial n^o_{i,g} = (1 - \delta_g)(1 - p^o)$ from the employment transition equation (see (16) below).

where η is the worker bargaining power.

4.6 Immigration

The total labor force of workers of type g and productivity ε_i is given by

$$L_{i,g} = \sum_{o \in \{d,na,e\}} l_{i,g}^{o}.$$

Aggregate employment is analogously

$$N = \sum_{o \in \{d,na,e\}} \sum_{i=1}^{I} \sum_{g \in \{L,H\}} n_{i,g}^{o}.$$

4.6.1 Evolution of population and labor force

As mentioned above, we use the "Model of Perpetual Youth" approach of Blanchard-Yaari, and hence there are constant transition probabilities to retirement and death. Accordingly, the measure of working age population $\omega_{i,g}^{o}$ of type g, productivity i for $o \in \{d, m\}$ follows stochastic processes that are governed by inflows and outflows.

For natives, there is an outflow into retirement and an inflow of newborn agents.[11] For established immigrants, there is also an outflow into retirement and an inflow consisting of newly arrived immigrants that become established. Regarding newly arrived migrants, as mentioned above, there is gradual integration in that individual productivities may increase over time. In addition, newly arrived immigrants can become established immigrants. Thus, there is an outflow of newly arrived immigrants into retirement as well as into established immigrants. Moreover, due to fresh arrivals in the country, there is also an inflow of additional newly arrived immigrants. For expositional reasons, these transitional equations are laid out in Appendix A.1. The working age population of high- and low-skilled natives and immigrants, respectively is given by:

$$\Omega_g^o = \sum_{i=1}^{I} \omega_{i,g}^{o},\ o \in \{d, m\}$$

The total working age population of natives and immigrants, respectively, is then given by $\Omega^o = \Omega_H^o + \Omega_L^o$, $o \in \{d, m\}$, and the total working age population is defined as $\Omega = \Omega^d + \Omega^m$.

The labor force measures $l_{i,g}^{d}$, $l_{i,g}^{na}$ and $l_{i,g}^{e}$ are stochastic processes that follow from population processes and labor force participation assumptions. We also assume that, after joining the labor force,

[11] This implies that all agents born in the country are of the same type, "native". In other words, second generation immigrants are assumed to be identical to children of natives.

the worker remains a participant until retirement. Labor force participation rates are set exogenously to match values in the data for natives and immigrants, allowing for time since immigration to affect the participation rates. In particular, for natives we assume

$$l_{i,g}^{d\prime} = \left(1 - p^d\right) l_{i,g}^d + \lambda_{i,g}^{d\prime}, \tag{14}$$

where $\lambda_{i,g}^d$ denotes inflow into the labor force $l_{i,g}^d$. In the calibration, we choose $\lambda_{i,g}^{d\prime}$ so that it is equal to the outflow from the labor force, i.e., $p^d l_{i,g}^d$. Finally, let κ^d (κ^m) denote the share of new natives (migrants) that participate in the labor force in the long run. Then, in steady state, $l_{i,g}^d = \kappa^d \omega_{i,g}^d$.

For immigrants, the dynamics are slightly more complicated since we assume that new immigrants have a lower labor force participation than immigrants that have been in the country for some time.

Actual labor supply dynamics for newly arrived and established immigrants, respectively, evolve according to the following equations:

$$\begin{aligned} l_{i,g}^{na\prime} &= (1-p^m)\Big((1-\phi)\left[(1-\pi)\left[l_{i,g}^{na} + \kappa^{new}\left(\hat{l}_{i,g}^{na} - l_{i,g}^{na}\right)\right]\right] \\ &\quad + (1-\phi)\left(\pi\left[l_{i-1,g}^{na} + \kappa^{new}\left(\hat{l}_{i-1,g}^{na} - l_{i-1,g}^{na}\right)\right]\right)\Big) + \lambda_{i,g}^{na\prime}, \end{aligned}$$

and

$$\begin{aligned} l_{i,g}^{e\prime} &= (1-p^m)\Big(\left[l_{i,g}^{e} + \kappa^{new}\left(\hat{l}_{i,g}^{e} - l_{i,g}^{e}\right)\right] + \phi\left[(1-\pi)\left[l_{i,g}^{na} + \kappa^{new}\left(\hat{l}_{i,g}^{na} - l_{i,g}^{na}\right)\right]\right] \\ &\quad + \phi\left[\pi\left[l_{i-1,g}^{na} + \kappa^{new}\left(\hat{l}_{i-1,g}^{na} - l_{i-1,g}^{na}\right)\right]\right]\Big) + \lambda_{i,g}^{e\prime}, \end{aligned}$$

where $\hat{l}$ denotes potential labor supply (i.e., the labor supply that results after an immigrant is fully integrated). Here, κ^{init} and κ^{new} capture initial labor supply of newly arrived immigrants and how quickly the labor force of immigrants approaches its long-run level, respectively.[12]

Finally, the population of retirees and the total population are, respectively, given by

$$\begin{aligned} ret &= ret^d + ret^m \text{ and} \\ \Pi &= \Omega + ret. \end{aligned} \tag{15}$$

[12] For details about the potential labor supply and the definition of the various κ's, see Appendix A.1.

4.7 Employment transition equations

Noting that a job only is created when a worker meets a firm and the match has a positive value to the firm, the law of motion for native employment, $n_{i,g}^{d\prime}$, is given by

$$n_{i,g}^{d\prime} = (1-\delta_g)\left(1-p^d\right) n_{i,g}^d \mathbb{I}\left(J_{i,g}^d\left(n'\right)\geq 0\right) + \left(1-p^d\right) f_g\left(l_{i,g}^d - n_{i,g}^d\right) \mathbb{I}\left(J_{i,g}^d\left(n'\right)\geq 0\right). \tag{16}$$

For newly arrived immigrants,

$$\begin{aligned} n_{i,g}^{na\prime} \quad = \quad & (1-p^m)(1-\phi)(1-\delta_g)\,\mathbb{I}\left(J_{i,g}^{na}\left(n'\right)\geq 0\right)\left[(1-\pi)\,n_{i,g}^{na} + \pi n_{i-1,g}^{na}\right] \\ & + (1-p^m)(1-\phi)\,\tilde{f}_{i,g}^{na}\left[(1-\pi)\left(l_{i,g}^{na} - n_{i,g}^{na}\right) + \pi\left(l_{i-1,g}^{na} - n_{i-1,g}^{na}\right)\right], \end{aligned} \tag{17}$$

and, for established immigrants,

$$\begin{aligned} n_{i,g}^{e\prime} \quad = \quad & (1-p^m)(1-\delta_g)\,\mathbb{I}\left(J_{i,g}^{e}\left(n'\right)\geq 0\right) n_{i,g}^{e} + (1-p^m)\,\tilde{f}_{i,g}^{e}\left(l_{i,g}^{e} - n_{i,g}^{e}\right) \\ & + (1-p^m)\,\phi\,(1-\delta_g)\,\mathbb{I}\left(J_{i,g}^{e}\left(n'\right)\geq 0\right)\left[(1-\pi)\,n_{i,g}^{na} + \pi n_{i-1,g}^{na}\right] \\ & + (1-p^m)\,\phi\tilde{f}_{i,g}^{e}\left[(1-\pi)\left(l_{i,g}^{na} - n_{i,g}^{na}\right) + \pi\left(l_{i-1,g}^{na} - n_{i-1,g}^{na}\right)\right]. \end{aligned} \tag{18}$$

Letting $n_{i,g}^m = n_{i,g}^{na} + n_{i,g}^e$, we have

$$n_{i,g} = n_{i,g}^d + n_{i,g}^m.$$

Finally, unemployment $u_{i,g}^o$ is given by

$$u_{i,g}^o = l_{i,g}^o - n_{i,g}^o. \tag{19}$$

Total unemployment for workers with productivity i and skill g is $u_{i,g} = u_{i,g}^d + u_{i,g}^m$. Note that (19) implies that newborn natives and newly arrived immigrants enter the labor force as unemployed.

4.8 Job creation

A vacancy that is filled today turns into a productive match tomorrow. The optimal choice of vacancies in (9) then gives the following job creation conditions for skill groups $g \in \{L, H\}$:

$$
\begin{aligned}
c_g &= q_g \beta \mathbb{E}_{\boldsymbol{n}'} \left[\left(1-p^d\right) \sum_{i=1}^{I} h_{i,g}^{d} \max\left\{ J_{i,g}^{d}\left(\mathbf{n}'\right), 0\right\} \right. \\
&+ \left(1-p^m\right)\left(1-\phi\right) \sum_{i=1}^{I} h_{i,g}^{na}\left(\pi \max\left\{J_{i+1,g}^{na}\left(\mathbf{n}'\right),0\right\} + \left(1-\pi\right)\max\left\{J_{i,g}^{na}\left(\mathbf{n}'\right),0\right\}\right) \\
&\left. + \left(1-p^m\right)\phi \sum_{i=1}^{I} h_{i,g}^{na}\left(\max\left\{J_{i,g}^{e}\left(\mathbf{n}'\right),0\right\}\right) + \left(1-p^m\right)\sum_{i=1}^{I} h_{i,g}^{e}\left(\max\left\{J_{i,g}^{e}\left(\mathbf{n}'\right),0\right\}\right)\right],
\end{aligned} \tag{20}
$$

where $h_{i,g}^{o}$ is the share of unemployed workers in period t in group $o \in \{d, na, e\}$ in skill group g with productivity ε_i, i.e.,

$$
h_{i,g}^{o} = \frac{u_{i,g}^{o}}{\sum\limits_{o \in \{d,na,e\}} \sum\limits_{i=1}^{I} u_{i,g}^{o}}. \tag{21}
$$

4.9 Government

In the baseline version of the model, we assume that the government budget is balanced period by period. In order to analyze the effects of tax smoothing, this assumption is relaxed in section 6.3. The government spends money on unemployment benefits, government assistance to individuals outside the labor force, and retirees. This is financed by taxing labor income. The government budget constraint is then

$$
\sum_{g \in \{H,L\}} \sum_{i=1}^{I} u_{i,g} b_{i,g} + z_l \sum_{o \in \{d,na,e\}} \sum_{g \in \{H,L\}} \sum_{i=1}^{I} \left(\omega_{i,g}^{o} - l_{i,g}^{o}\right) + z_{ret} \times ret = \tau \sum_{o \in \{d,na,e\}} \sum_{g \in \{H,L\}} \sum_{i=1}^{I} n_{i,g}^{o} w_{i,g}^{o}, \tag{22}
$$

where z_l is government assistance for people of working age not in the labor force and z_{ret} government assistance for retirees.

5 Calibration

In this section we describe the calibration of the model. The model is calibrated for a quarterly frequency. Some parameters, e.g., discounting and matching function elasticity, are set to standard values in the literature. Table 1 documents these parameter values and their sources.

Some other parameters, e.g., the labor force participation rates and the fraction of college educated among natives and immigrants, are set to match empirical values in our data.[13]

[13] We also normalize A^{tfp} so that $\left(1-\alpha\right) A^{tfp}\left(\left(\alpha A^{tfp}/\left(r+\varsigma\right)\right)^{1/(1-\alpha)}\right)^{\alpha} = 1$.

Table 1: Parameters set to standard values in the literature

Parameter	Definition	Value	Motivation
β	Discount factor	$0.98^{1/4}$	Annual rate of 2%
ξ	Match elasticity wrt u	0.5	Pissarides (2009)
η	Bargaining strength	0.5	Standard in the literature
c_H, c_L	Vacancy posting costs	0.17*MPL_g	Fujita & Ramey (2012)
δ_H, δ_L	Job separation rates	0.015	Carlsson & Westermark (2016)
ρ	Elasticity of subs between skill groups	2	Ottaviano & Peri (2012)
α	Capital share	0.25	Christiano et al. (2010)
r	Interest rate	$1.02^{1/4}$	Annual rate of 2%
ς	Depreciation of capital	0.025	Christiano et al. (2010)

We restrict our attention to symmetric steady states, where all firms are identical. This implies that expectations over $\mathbf{n}$ (i.e., $\mathbb{E}_{\boldsymbol{n}}$) in expressions (5) and (20) can be dropped. Furthermore, since firms are large, by the law of large numbers firms continue to be identical following a shock in, e.g., immigration flows.

To capture the fact that immigrants, on average, are older than natives when entering the workforce, we impose $p^m > p^d$. We also set $\Theta^m > \Theta^d$, which implies longer expected time in non-working age for natives. The underlying aspect we want to capture is the fact that immigrants tend to be of working age when they arrive, thereby not causing any cost for the government in terms of childhood-related fiscal expenditures such as childcare and basic skill (although mechanically modelled as pension payments here). Table 2 documents these parameter values and their sources.

Table 2: Parameters set outside the model

Parameter	Definition	Moment/data used	Value
κ^d	LFP rate natives	SCB, Stativ	87%
κ^{init}	LFP rate initial immigr	SCB, Stativ	39.69%
κ^m	LFP rate long-run immigr	SCB, Stativ, LFP rate after >15 yrs	78%
κ^{new}	LFP rate gap closure immigr	SCB, Stativ	0.0636
Ω^m/Ω	Immigrant share	SCB, Stativ	18%
Ω^d_H/Ω^d	Fraction high-skilled natives	SCB, Stativ	36%
Ω^m_H/Ω^m	Fraction high-skilled immigr	SCB, Stativ	34%
p^d	Probability of retirement d	40 years working life	1/160
p^m	Probability of retirement m	33.3 years working life	1/(4*33.3)
Θ^d	Probability of death d	20 yrs youth +18 yrs retirement	1/(4*38)
Θ^m	Probability of death m	0.32*9 yrs youth + 18 yrs retirement	1/83
z_l, z_{ret}	Welfare payment	Fraction of unemployment benefits	0.703

The first four parameters relate to labor force participation in different groups. The parameter κ^d is set to match the participation rate for natives. Three parameters describe immigrant labor supply. First, κ^{init} is set to match the immigrant labor force participation rate in the second year after the immigration date, and, second, κ^m is set to match the participation years after sixteen years in the country. Third, κ^{new} regulates how fast the gap between immigrants' initial and long-run participation rates is closed. We have chosen to match the empirical speed of this gap closing. We set the immigrant share, $\frac{\Omega^m}{\Omega}$, and the skill shares, $\frac{\Omega^d_H}{\Omega^d}$ and $\frac{\Omega^m_H}{\Omega^m}$, to match the shares in the data.

We account for differences in age at the time of labor force entry between natives and migrants in the following way. Working age is considered to be 24–64 years. Using SCB data with this assumption, the average age of immigrants' entry in the labor market is 30.7, and they spend an average of 33.3 years working in Sweden before retirement. We set p^d and p^m accordingly. The death probability of natives is then set to reflect the fact that agents spend 38 years outside working age. We set death probability of immigrants as follows. A fraction, 0.32, of immigrants are below the age of 20 at the immigration date. The average age in this group is about 11.4. Hence, the death probability for immigrants, Θ^m, is adjusted to take into account the lower childhood-related fiscal costs of immigrants. We calibrate z_l by setting it as a ratio relative to the average of unemployment benefit level. In the data, $k_z = \frac{z_l}{b} = 0.703$, computed using welfare payments for single adult households and average unemployment benefit payments, respectively.[14] To compute average benefits in the model, we first proxy model unemployment by the targeted (i.e., empirical) total unemployment levels for high- and low-skilled, denoted by u^{ta}_H and u^{ta}_L. Then, we set z_l as k_z times average benefits in the model so that $z_l = k_z(b_L u^{ta}_L + b_H u^{ta}_H)/(u^{ta}_L + u^{ta}_H)$.

We normalize the mean efficiency units of labor of both high- and low-skilled natives to unity, $\mu^d_g = 1$. Note that the benefit parameters $b_{i,g}$ are independent of individual productivity and are denoted by b_L and b_H, respectively. For the remaining eleven parameters, we search jointly for the parameter values that minimize the square percent deviation of the eleven model moments from the data moments discussed below. The resulting moments and parameter values are displayed in Table 3 and indicate a very good match. Where possible, the rows in Table 3 indicate the main identifying moment for each parameter. A few parameters simultaneously affect multiple moments in a direct way. Specifically, the parameters π and ϕ both influence the unemployment rates for immigrants after any number of years in the country, although more for longer horizons. In addition, μ^m affects all unemployment rates for immigrants as well as their mean relative productivity.

Let us now elaborate slightly on how these empirical moments are obtained. Unemployment rates for the different groups are computed from the LISA database from Statistics Sweden.[15] The target for the skill premium is from OECD (2011). The data for replacement rates are for 2009 and are taken from Bennmarker et al. (2011). The mean relative productivity of employed immigrants is

[14] We assume that government expenditures for retirees and children are the same.

[15] We use the variable ArbSokNov>0 to define whether an individual is unemployed. It measures whether an individual is not working and looking for work in November in a given year. Employment is measured using the RAMS database based on the RAMS definition of an annual earnings threshold, which is in line with the ILO definition of employment. LFP rates are constructed as a sum of employment- and unemployment-to-population rates.

Table 3: Parameters obtained by moment-matching

Parameter	Value	Main targeted moment	Data value	Model value
A	0.4794	Several unemployment rates	see below	see below
σ_H^d	0.1565	Unempl rate high-skill natives	3.34%	3.29%
σ_L^d	0.2765	Unempl rate low-skill natives	8.71%	8.90%
σ_H^m	0.3066	Unempl rate high-skill immigrants	15.34%	15.13%
σ_L^m	0.1332	Unempl rate low-skill immigrants	22.12%	20.89%
a	0.4939	Skill premium	1.26	1.26
b_H	0.3714	Replacement rate, avg in highest quartile	0.425	0.435
b_L	0.3486	Replacement rate, avg in lowest three quartiles	0.649	0.672
μ^m	0.6035	Relative productivity of employed immigrants	0.73	0.77
π	0.1285	Unempl for immigrants in year X=3, 11 & ≥15	38.33% 20.48% 12.96%	38.84% 20.76% 13.44%
ϕ	0.007121	Unempl for immigrants in year X=3, 11 & ≥15	see above	see above

taken from Ek (2018). In that study, matched employer-employee data was used to estimate worker productivity by country of origin. The dataset used include all workers at Swedish firms with at least five employees.[16] The unemployment rates for immigrants who have been in the country 2–3, 10–11 and ≥15 years are computed from the LISA data from Statistics Sweden.

6 Results

6.1 Parameter estimates and implied steady state values

Let us briefly comment on some of the parameter values implied by the moment matching documented in Table 3. First, note that large dispersion in productivity, $\sigma_L^d = 0.28$ and $\sigma_H^m = 0.31$, is needed for unemployment rates of low-skilled natives and high-skilled immigrants to match the moments in the data. While lower, productivity dispersion for high-skilled natives and low-skilled migrants is also fairly large at $\sigma_H^d = 0.16$ and $\sigma_L^m = 0.13$, respectively. These high values of productivity dispersion generate substantial structural unemployment. The mean productivity of immigrants at arrival to the country compared to natives is 0.60, i.e., substantially lower than the relative productivity of employed immigrants compared to employed natives. The difference is driven both by the selection of who is employed and by the fact that immigrant productivity increases over time. In particular, the estimate of π implies a 13% probability that a newly arrived immigrant improves her productivity by one gridpoint in a given quarter. This corresponds to an average productivity improvement of 0.26 percent per quarter for this group. With the very low probability $\phi = 0.7\%$, this integration process

[16] The value for mean relative productivity that we use was calculated as follows. The author generously shared his country-specific productivity estimates with us and we weighted these with the share of immigrants that arrived from each country in 2016-2017. Quantitatively, this weighted average is very close to the values reported in Table 3 in Ek (2018), i.e., the unweighted and the frequency-weighted average in his sample.

ends for a specific individual.

Table 4 reports some key unmatched moments. Aggregate unemployment is 8.8%. The wage of immigrants relative to natives is 78%, i.e., quite close to their relative productivity in the model and well in line with the wage evidence reported in Brell et al. (2020). Net fiscal transfers from natives to immigrants is around 2.1% of GDP, which is above but close to the interval reported in Ekberg (2009) for Sweden. Overall, we note that these untargeted moments are broadly in line with the data. The tax rate of 38% reflects a calibration with substantial transfers/public expenditures on children, retirees and individuals outside the labor force. In this context it is worth mentioning that the lion's share (78%) of the public expenditures in the model are related to non-workers, i.e., pensions and spending on children, indicating a very large role of demographics for public finances. Welfare payments (13%) and unemployment benefits (9%) make up the remaining public expenditures. Finally, we note that the job meeting rate is substantially higher for high-skilled workers than for low-skilled. The structural unemployment (i.e. the unemployability of some workers) implies that job finding rates are substantially lower than job meetings rates–the aggregate job finding rate is 0.23 per month.

Table 4: Some additional key moments in steady state

Moment	Model, baseline
Aggregate unemployment	8.77%
Average wage for immigrants/natives	77.67%
Net transfers from natives to immigrants	2.12%
Labor income tax rate	37.61%
Job meeting rate, low-skilled	0.433
Job meeting rate, high-skilled	0.625
Job finding rate, aggregate	0.234

The left panel of Figure 1 in section 3.1 documents the unemployment rates of immigrants as a function of the number of years in the country both in the data and in the model. The match between model and data is good (unsurprisingly, as unemployment for three different time periods after arrival is targeted in the calibration)[17], and we note that, both in the model and in the data, the unemployment rate is a convex function of the number of years in the country. Correctly matching unemployment as a function of the number of years in the country, jointly with the corresponding LFP rates, is crucial for the quantitative implications of the model in terms of aggregate dynamics from an immigration shock. Together, these two determine the direct effect, whereby immigration dynamically affects the employment-population ratio and is also a determinant of productivity and wages. The right panel in Figure 1 documents the fit of the exogenous processes for LFP used in the model vs. the data. We note that the simple process we use captures the pattern in the data well.

[17]Note that the unemployment rate for migrants who have been in the country for at least fifteen years is not displayed in the figure.

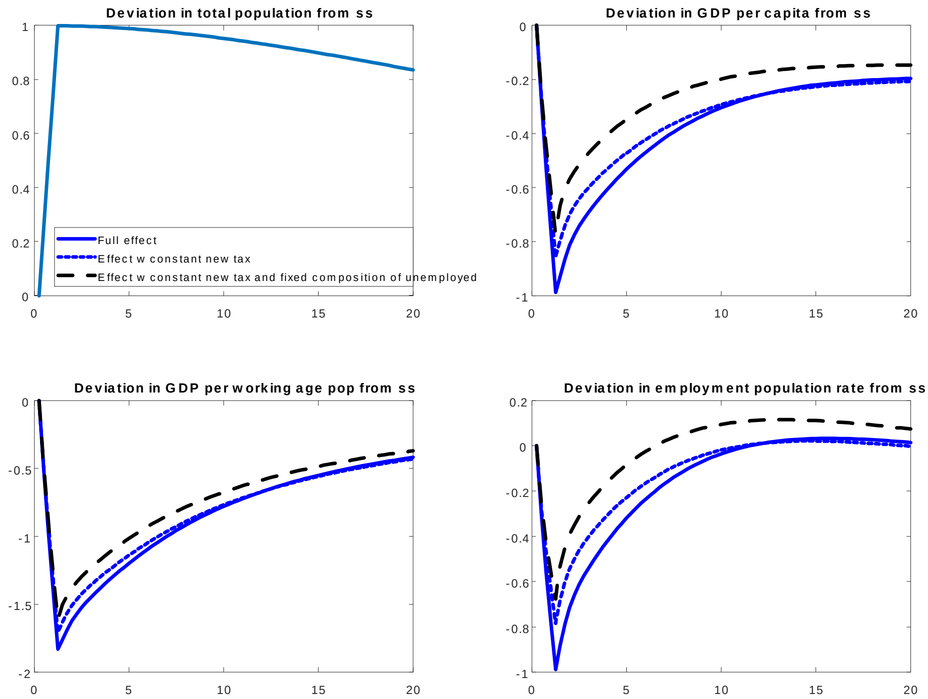

Figure 2: The effect of a one percent migration shock on GDP and employment. Annual scale on x-axis.

6.2 Dynamic effects of an immigration shock

In Figure 2, we illustrate the effect of an immigration shock corresponding to one percent of the population occuring during one year, i.e. spread out over four quarters. We document the effect on the population, GDP and employment over a twenty-year horizon. For now, let us focus on the full effect as depicted by the solid lines. GDP per working age population (per capita) drops by 1.8 (1.0) percent on impact and then slowly recovers to a deviation of 0.4 (0.2) percent at the end of the period. One reason that the initial drop is larger than the increase in the population is that all immigrants (by assumption) are of working age when they arrive in the new country. Hence, the working age population increases more than the total population in percentage terms. The employment-to-population ratio also drops on impact and then only recovers slowly. In the model, since the immigrants have a higher share in the working age population than in the total population, there is a demographic dividend, but it is initially dwarfed by the lower employment rates of (working age) immigrants. As can be seen from the graph of the employment-population rate, the demographic dividend becomes relatively more pronounced over time, but never strong enough to drive the employment-population rate above its steady state.

The solid line in Figure 3 illustrates the full effects of an immigration shock on unemployment. Aggregate unemployment increases by 0.9 percentage points on impact and then decreases slowly.

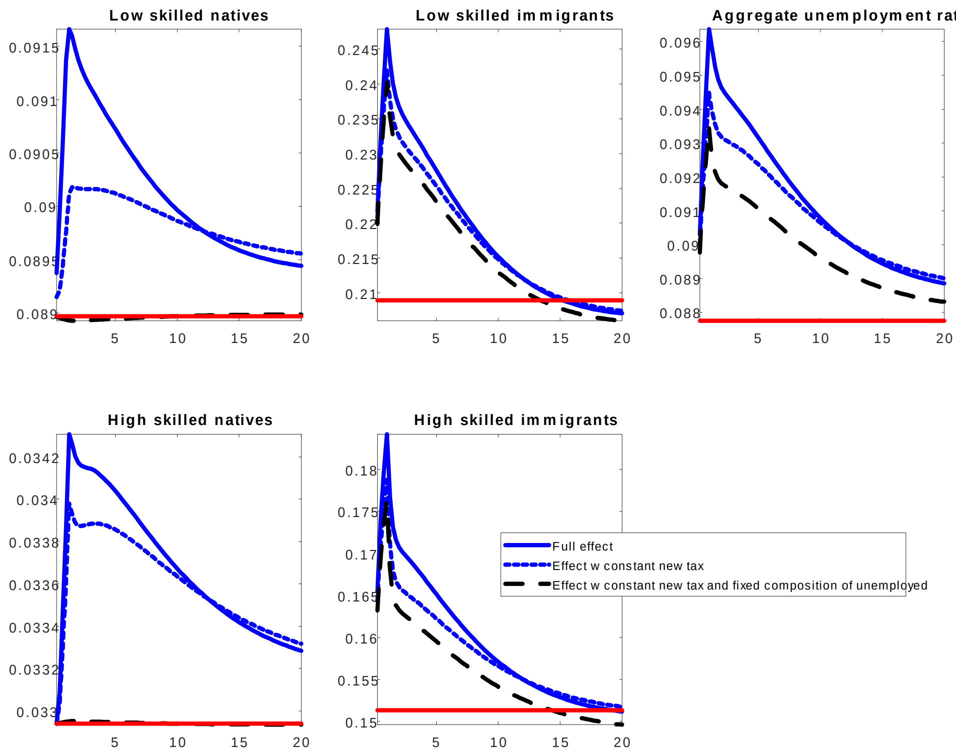

Figure 3: The effect of a one percent migration shock on various unemployment rates. Annual scale on x-axis.

After twenty years, aggregate unemployment is still marginally elevated by roughly 0.1 percentage points. For the immigrants, unemployment increases strongly on impact for both skill groups and only gradually falls back to the steady state level after roughly twenty years. An interesting general equilibrium effect is how unemployment of low-skilled natives surges by 0.3 percentage points in response to the immigration shock. This is due to a combination of the (initial) reduction in the average productivity level for the low-skilled unemployment pool and the increase in taxes, both of which discourage job creation. We quantify these two channels below in sections 6.3 and 6.7. Finally, we note unemployment of high-skilled natives is approximately unaffected by immigration (note the scale on the y-axis in that panel).

Figure 4 documents the effect of the immigration shock on taxes and transfers from natives to immigrants. Both taxes and transfers increase on impact, as immigrants enter either as unemployed or as outside the labor force. Transfers then have to be paid out to them initially. The initial increase in the tax rate is 0.8 percentage points, while the increase in fiscal transfers from natives to immigrants is 0.5 percentage points. As immigrants gradually become employed, both taxes and transfers slowly fall back toward their steady state level, but the sign of the effect of the extra immigration on taxes and transfers never switches.

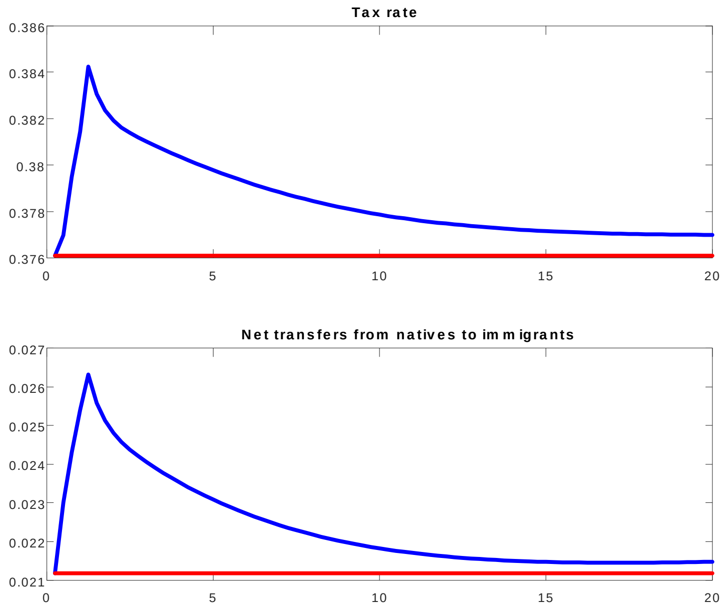

Figure 4: The effect of a one percent migration shock on taxes and fiscal transfers. Annual scale on x-axis.

6.3 Tax smoothing

In the baseline version of the model, the government budget is balanced. As can be seen in Figure 4, this leads to a rapid increase in taxes by almost 1 percentage point, which in turn affects job creation negatively. An alternative would be to have an intertemporally balanced budget with tax smoothing. To study this, we compute the constant tax rate that ensures an intertemporally balanced budget. In Figures 2 and 3, the dotted line documents the dynamic effects on the economy. With a constant distortionary tax rate τ, the maximum decrease of the employment-population ratio is only 79% as large. The tax rate is less important for the GDP dynamics, but also there we note that taxes have substantial effects. With a new constant tax rate, the maximum decrease in GDP per capita is 87% of the full effect. The dotted line in Figure 3 documents the effects of immigration under tax smoothing on unemployment, both in the aggregate and for the various demographic groups. In terms of the aggregate unemployment, we note that for constant tax rates the maximum increase is only 78% as large as the full effect. Among the various demographic groups, we note that constant (smoothed) taxes substantially dampens the effect on unemployment for low-skilled natives, which is reduced by more than half. In terms of absolute effects, the time-varying tax is also important for both of the immigrant skill groups. It accounts for roughly half a percentage point of the maximum increase in unemployment for both immigrant skill groups. Thus, tax smoothing can have substantially beneficial effects on unemployment of both natives and immigrants.

6.4 The importance of the composition of immigrants

In our model, the composition of immigrants is important for the effects of an immigration shock. To illustrate this, we consider a shock that increases the immigrant population by one percent of the population as above but with a productivity distribution equal to that of natives, also turning off integration ($\pi = 0$). In the top row of Figure 5, the effects of this type of immigration on GDP and employment is illustrated by the dashed line. The higher share of immigrants in the working age population leads to a substantial increase in GDP per capita and the employment-to-population ratio, above the steady state level, within two years. These results isolate the effects of the demographic dividend, which turn out to be substantial. They show a qualitative difference compared to our baseline results, which are plotted for comparison. In the baseline scenario, the demographic dividend is dwarfed by (initial) differences in productivity between natives and immigrants, which leads to low employment rates of the newly arrived immigrants and low GDP per capita.

The higher productivity and the accompanying lower (structural) unemployment compared to the baseline experiment also affects taxes and net transfers to immigrants, as can be seen in the bottom row of the same figure. After a brief increase in both taxes and transfers, they both drop substantially below the steady state level for decades.[18]

6.5 Steady state effects of higher immigration

The previous theoretical literature studying the effects of immigration is restricted to steady state analysis. However, as the above analysis indicates, the dynamics can be substantially different from the effects in steady state, especially in the short to medium run. To be more specific on this issue, we now study the effects of a permanent increase in the share of immigrants in the population by the same amount as the shock, i.e., by one percentage point. The first column of Table 5 documents these effects for key variables. We note that all steady state effects are quantitatively limited. There are non-negligible negative effects on GDP per capita and GDP per working age due to the lower productivity and the lower labor force participation of immigrants. The demographic dividend of higher immigration almost exactly offsets the other effects on the public finances and result in basically unchanged taxes (an increase by 0.02 percentage points). The aggregate unemployment is also basically unaffected by higher steady state immigration. Fiscal net transfers from natives to immigrants increase marginally, by 0.08 percentage points of GDP. We find that aggregate native welfare fall by 0.13% in the steady state when there is a permanent increase in immigration. Importantly, natives' average wages account only for a small part of this decrease as they only fall by 0.02%. Instead, the increased

[18] This is in line with fiscal effects of immigration in the 60s and 70s in Sweden, where transfers from migrants to natives were substantial and reached a peak of around 1% of GDP; see Ekberg (2009).

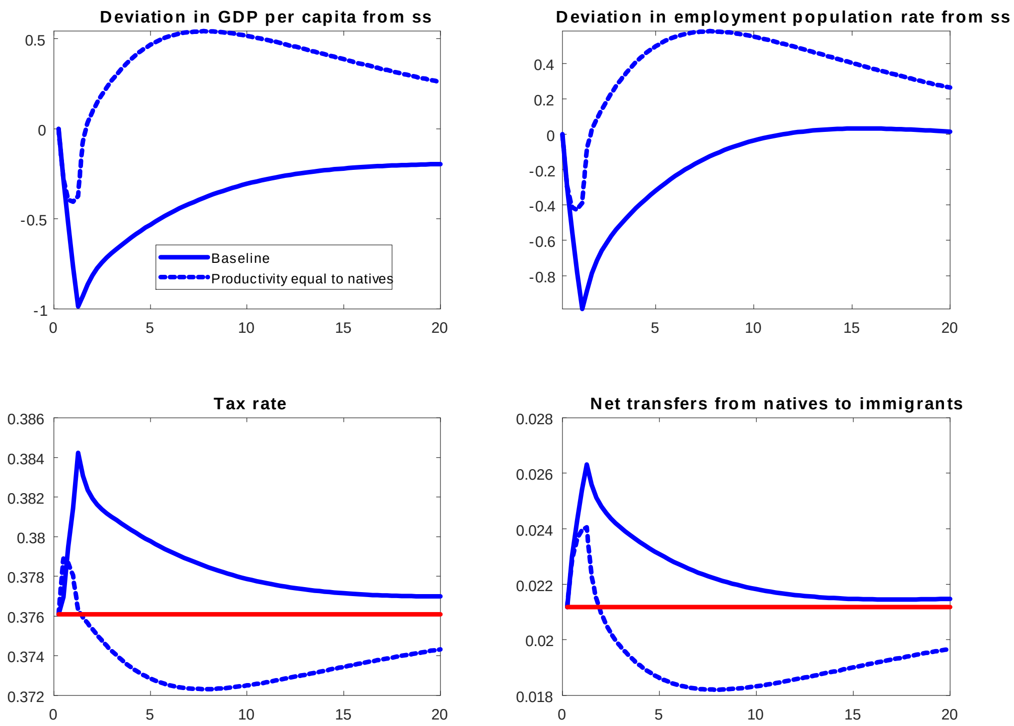

Figure 5: Immigration shock where the productivity distribution of immigrants is the same as for the natives.

fiscal net transfers from natives to immigrants account for the main part of the reduction in welfare. Note also that effects on GDP per capita is not a good proxy for welfare of natives. This is because income of natives differs from income of immigrants and welfare depends both on current and future (discounted) payoffs.

We now compare our steady state results to the existing literature, in particular Battisti et al. (2018). Despite key differences as discussed above in section 4.1, our steady state results in terms of welfare appear not that different from theirs - they report small effects on natives' welfare overall and negative effects for 7 out of 20 countries studied. The similarity of our result to theirs indicates that the differences in assumptions roughly offset each other in terms of the steady state effect on natives' welfare - the demographic dividend that we capture appear to have similar size to what they obtain by abstracting from productivity differences. Note that Battisti et al. (2018) do not report effects of immigration on other macroeconomic aggregates, only on welfare of natives.

The second column of Table 5 summarizes the maximum dynamic effects that we discussed in section 6.2. We note that the dynamic effects generally are roughly one order of magnitude larger than the steady state effects.[19] We conclude that the implications of our model confirm what the

[19] One exception is the effect on natives welfare which is quite similar between steady state and dynamics. To a large

microdata indicates - that the steady state effect generally would be a bad proxy for the short to medium term effects.

Table 5: Steady state vs. dynamic effects of immigration

	Steady state effect	Max dynamic effect
GDP per capita	-0.27%	-0.98%
GDP per working age	-0.44%	-1.83%
Labor income tax rate	+0.02 pp	+0.81 pp
Aggregate unemployment	+0.06 pp	+0.86 pp
Net transfers from natives to immigrants	+0.08 pp	+0.51 pp
Welfare of natives	-0.13%	-0.16%

6.6 Effects across groups

In our exercise, as in the Swedish data, immigrants have approximately the same educational (skill) composition as natives. Nevertheless, immigration implies interesting differential effects across groups. We have already seen that unemployment for low-skilled natives increases more than for high-skilled natives following a shock to immigration. Interestingly, the higher job finding rates (or, equivalently, lower unemployment rate) for high-skilled workers implies that the ratio of high-skilled employment to low-skilled employment increases persistently following an immigration shock. This implies an initial increase in the marginal product of labor (MPL) of low-skilled workers and a corresponding decrease in the MPL of high-skilled workers as documented in Figure 6. Over time, as more low-skilled immigrants become employed, another force dominates this relative supply of high- vs. low-skilled labor: the lower mean productivity of immigrants compared to natives. This implies that after the first 1.5 years the MPL of both low- and high-skilled workers fall below their respective steady state values.

Quantitatively, though, the effects on aggregate productivity and wages are very limited, as shown in Figure 7. After an initial uptick due to the increased fraction of high-skilled employment, aggregate productivity falls very gradually and by at most 0.27 percent as the immigrant share of aggregate employment increases. Aggregate wages follow productivity closely but fall slightly more. The maximum effect on both these variables occurs after roughly ten years. We also note that the immigration shock leads to very moderate increases in the wage of low-skilled natives for most of the first five years while wages of high-skilled natives fall slightly for all horizons except the first three quarters. All of this can be understood by noting the job finding rates and MPL in Figure 6: for the high-skilled (native) workers, lower job finding rates and lower MPL both drag down wages. Instead, for the low-skilled

degree this follows from the fact that welfare is computed as a present value of utility flows over many periods.

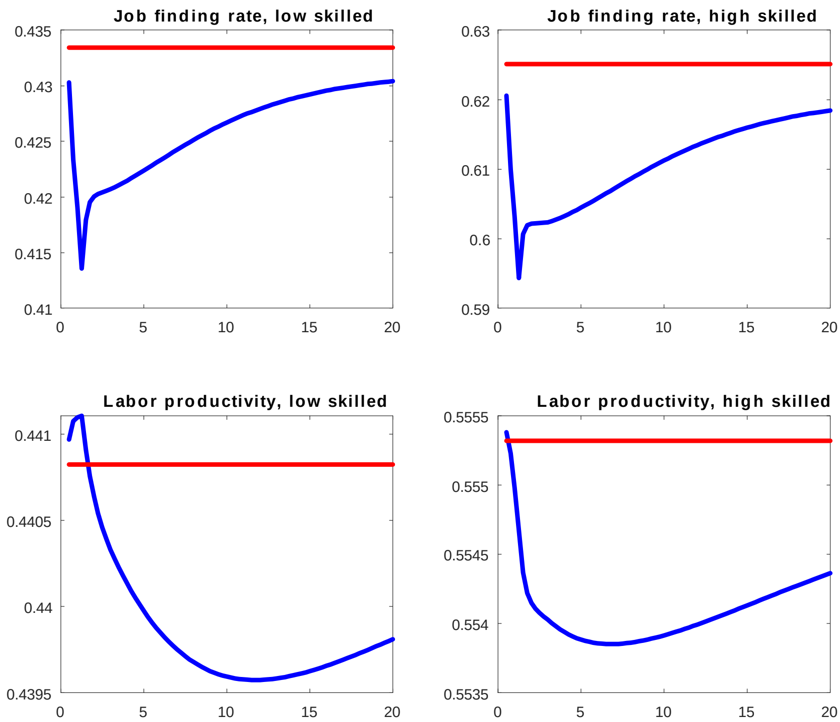

Figure 6: Job finding rate and labor productivity across skill groups.

natives, the increase in the ratio of high-skilled labor to low-skilled labor and the related increase in MPL of low-skilled workers dominates the fall in the job finding rates for the first five years.[20]

Note that, in many countries, immigration of low-skilled is over-represented relative to the fraction of low-skilled natives, and this tend to lead to decreases in (relative) wages for low-skilled native workers in sectors where the inflow is large. For Sweden, the skill composition of immigrants does not differ markedly from the composition of natives, which tends to mute such effects.

6.7 Effects of the composition of the unemployment pool

Let us now look back at Figure 2 and Figure 3, this time to quantify and decompose the mechanisms whereby immigration affects the economy. Recall that the dotted line documents the dynamic effect on the economy if the distortionary labor income tax rate, τ, is set as a constant level that balances the intertemporal budget constraint.

The dashed line in both figures shows a counterfactual exercise with both the (constant) tax rate, τ, and the job creation decision abstracting from any changes in the composition of the unemployment

[20] Another way to see this is through the exercise without frictional unemployment in section 6.8. In that exercise, there is obviously no heterogeneity in job finding rates, and none of the results discussed in this paragraph then occur: there is almost no initial uptick in productivity or wages, the wage for high-skilled natives barely falls, and the increase in wages for low-skilled natives is approximately cut in half.

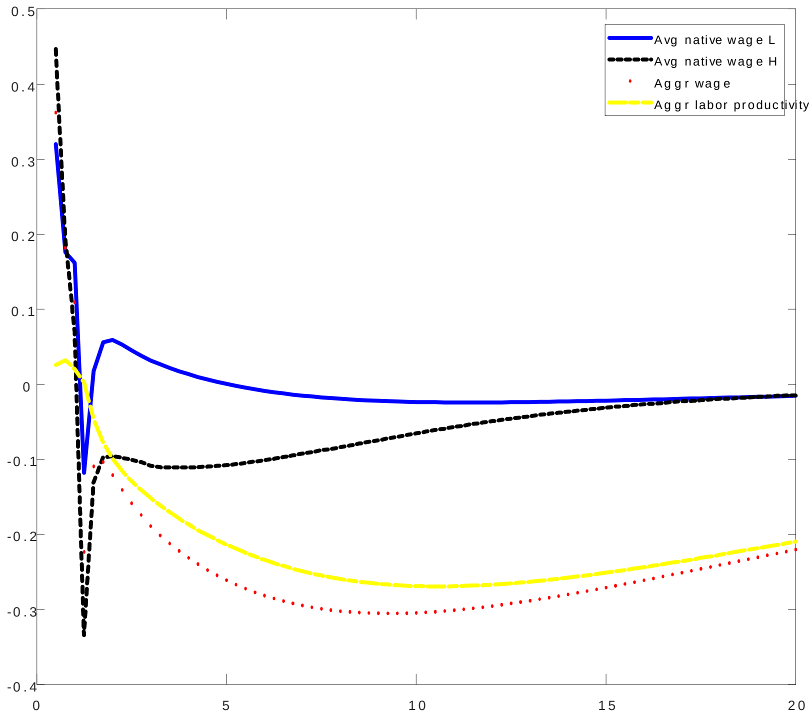

Figure 7: Deviation in wages and labor productivity from steady state (%).

pool. Due to the assumption of linear vacancy posting costs, this counterfactual exercise implies more or less constant tightness as can be seen from the basically constant unemployment rates of natives it implies; see Figure 3.

In both Figure 2 and Figure 3, we observe fairly moderate differences between the dotted and the dashed lines on aggregate variables, indicating that the effect on job creation through the composition of the unemployment pool has limited effects on aggregate output and employment for the immigration shock considered. Another way to read these two figures is to compare the full effect to the effect with the more mechanical partial equilibrium effect, i.e., with the general equilibrium effects turned off (the dashed line). We note that the partial equilibrium effect for most variables differ markedly from the full effect. In other words, a general equilibrium analysis is important when quantifying the effects of immigration on the economy.

6.8 Abstracting from frictional unemployment

In this section we document the importance of frictional unemployment for our results. Alternatively, it is possible to view this exercise as documenting the remaining unemployment—i.e., structural unemployment—and its effects on the economy.

We compute the implications of our model when there are no frictions in the labor market by setting the vacancy posting costs, c_H and c_L, approximately equal to zero and keeping all other parameter values fixed. The job meeting rates for all groups accordingly becomes unity, and only structural employment remains.[21] The fit of all the targeted moments deteriorates, and steady state aggregate unemployment falls from 8.77% to 5.63%. A key dimension of the model where the fit deteriorates substantially is the unemployment rates for immigrants as a function of the number of years in the country; see Figure 8. We note that structural unemployment of immigrants in the model is still high, starting at 30% after two years in the country and falling to 12% after fourteen years. Nevertheless, the structural unemployment of immigrants is well below the actual unemployment in the data. This has first order effects on the macroeconomic implications of immigration that we present next. Figure 9 documents the dynamic implications for GDP and employment abstracting from frictional unemployment and includes the baseline results for comparison. The figure documents that GDP per capita and the employment-population ratio only fall by roughly two-thirds as much as they do in the baseline. Figure 10 documents the implications for unemployment. We note that abstracting from frictional unemployment substantially reduces the impact on unemployment of low-skilled natives and to a lesser degree both skill groups of immigrants. Overall, the exercise documented in this section indicates the importance for the macroeconomic implications of matching the unemployment rates of immigrants well. In addition, it provides a decomposition of the results in terms of structural and frictional unemployment, which indicates that structural unemployment is the main component of unemployment among recent immigrants.

6.9 Robustness of results

In appendix A.6, we provide an exercise documenting the robustness of our results to the assumption that immigrants and natives are perfect substitutes in production. The results from this exercise are similar to those in the baseline specification with perfect substitutability, although the effects are somewhat muted. The details of this exercise are provided in the appendix.

7 Conclusions

The analysis of dynamic effects of immigration on the macroeconomy is an important issue, especially in light of the increasing immigration flows in the recent decades. We construct a general equilibrium model to shed light on this issue. We use the model to quantify the effect of several immigration

[21] To be exact, the heterogeneity in frictional unemployment between high, and low-skilled disappears in this exercise. Given that a match made today does not become productive until tomorrow, there is still a small amount of frictional unemployment, but it is the same for all groups.

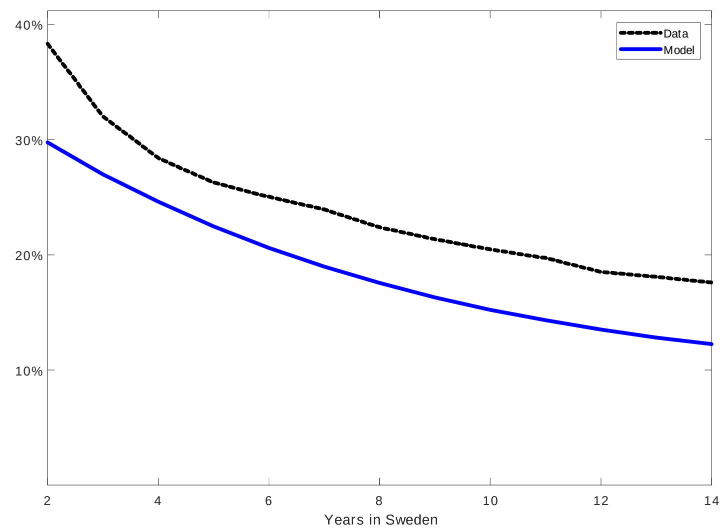

Figure 8: Unemployment rate as a function of years since migration. Version of model without frictional unemployment.

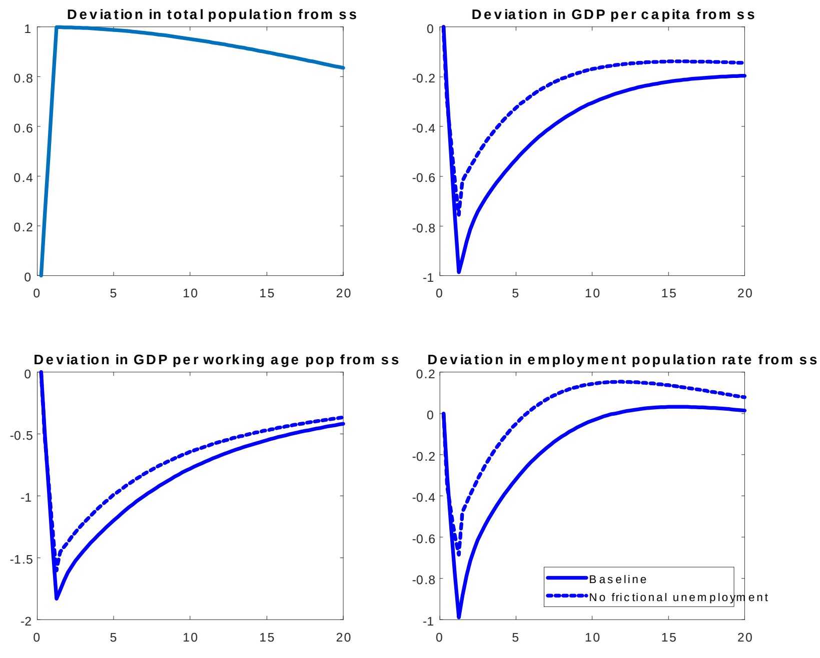

Figure 9: The effect of a one percent migration shock on GDP and employment. Annual scale on x-axis. Version of model without frictional unemployment.

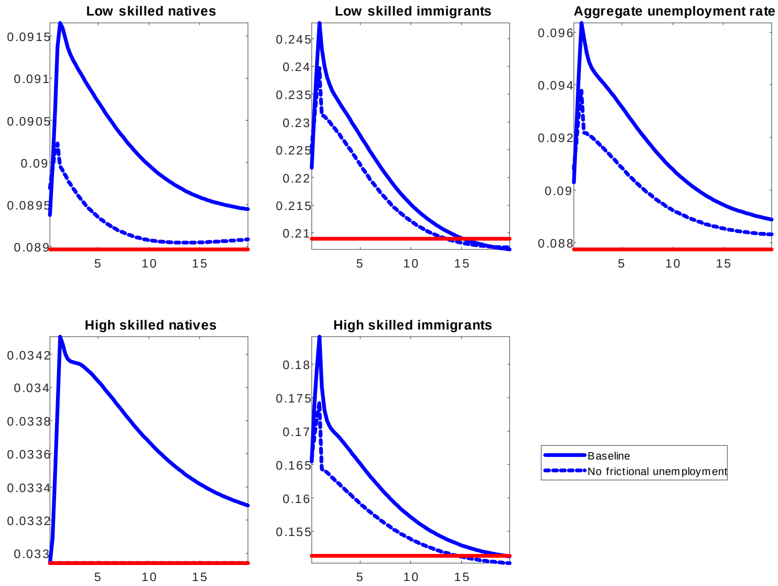

Figure 10: The effect of a one percent migration shock on various unemployment rates. Annual scale on x-axis. Version of model without frictional unemployment. The plot for the case without frictional unemployment has been adjusted to share the starting point with the baseline specification. Annual scale on x-axis.

scenarios on the paths of per capita GDP, unemployment, labor force participation, real wages and average labor productivity. The model also addresses public finance implications of immigration in a simple setting. A salient feature of the model is that we can capture demographic differences between natives and migrants that tend to be important as migrants often arrive early in their working age and thereby reduce the age-dependency ratio. Immigration then has a positive fiscal effect, typically referred to as a demographic dividend. These age differences also have positive effects on other aggregates, e.g., GDP per capita. Differences in employment rates, skill or productivity levels can overturn the demographic dividend if they are large enough. This is indeed what we find in the calibrated version of the model. The model enables us to analyze the macroeconomic dynamics implied by large changes in immigration. Furthermore, we can quantify to what degree recent sluggish wage and labor productivity growth is due to immigration flows and how much equilibrium unemployment rates change in response to immigration. We also use the model to study the effects of changes in the composition of immigration flows.

A key reason for studying the dynamic effects of immigration is that labor market integration is a gradual process, e.g., employment (unemployment) rates are increasing (decreasing) in the number of years since immigration. This implies that the direct effect of an immigration shock on employment rates and GDP per capita is largest on impact. We confirm that this holds true also in our general equilibrium model. A steady state analysis would underestimate the economic effects of immigration, and we find that the effects of an immigration shock on aggregate quantities can differ by more than one order of magnitude when comparing dynamic and steady state responses.

Our main exercise is to analyze the effects of an increase in immigration corresponding to one percent of the population, similar to the increase in immigration in Sweden around the refugee crisis of 2015. This increase leads to a reduction in GDP per capita of 1.0 percent and an increase in aggregate unemployment of 0.9 percentage points on impact. On top of the direct effect of high unemployment of newly arrived immigrants, this substantial effect on aggregate unemployment is generated by the fact that unemployment of low-skilled natives increases by 0.3 percentage points in response to the immigration shock. This is mainly due to the resulting increase in taxes and to a lesser degree due to the sullying of the low-skilled unemployment pool, both of which discourage job creation. Moreover, under the assumption of a balanced budget, taxes increase by 0.8 percentage points on impact and then gradually decline. Net transfers from natives to immigrants increase by 0.5 percent of GDP on impact but then decline fairly rapidly and remain slightly elevated for a long period. Finally, the effects on aggregate productivity and wages are very limited. The effects on wages of natives are even smaller, almost negligible, except for the first few quarters.

The degree of tax smoothing is important for the magnitude of the effects. With tax smoothing, so that the government finances the extra costs over several decades, when the shock hits, the negative effects on job creation of increased taxes are essentially absent and the effects on GDP and aggregate unemployment are substantially reduced.

Compositional effects are also important for the size and sign of the effects. In our baseline exercise, the lower productivity of migrants relative to natives dwarfs the demographic dividend in the short to medium run. On the other hand, if migrants have the same productivity distribution as natives, the dominating effect from increased immigration is a substantial demographic dividend, leading to higher GDP, lower unemployment and lower taxes.

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A Appendix

A.1 Transitional equations

For domestically born individuals, the working age population is

$$\omega_{i,g}^{d} = \left(1 - p^{d}\right) \omega_{i,g}^{d,lag} + \lambda_{i,g}^{\omega,d}, \tag{23}$$

where p^d denotes the retirement probability for domestically born and $\lambda_{i,g}^{\omega,d}$. Here, $\lambda_{i,g}^{\omega,d}$ is drawn from the PDF dH_g^d, which is a distribution of idiosyncratic productivities of natives with skill level g. The measure of working age population is, for newly arrived immigrants,

$$\omega_{i,g}^{na} = (1 - p^{m})(1 - \phi)\left[(1 - \pi)\,\omega_{i,g}^{na,lag} + \pi\omega_{i-1,g}^{na,lag}\right] + \lambda_{i,g}^{\omega,na}, \tag{24}$$

where $\lambda_{i,g}^{\omega,na}$ is drawn from dH_g^{na} and captures immigrants arriving. For established immigrants,

$$\omega_{i,g}^{e} = (1 - p^{m})\,\omega_{i,g}^{e,lag} + (1 - p^{m})\,\phi\left[(1 - \pi)\,\omega_{i,g}^{na,lag} + \pi\omega_{i-1,g}^{na,lag}\right] + \lambda_{i,g}^{\omega,e}, \tag{25}$$

where $\lambda_{i,g}^{\omega,e}$ is drawn from dH_g^e and captures established immigrants arriving.[22]

The long-run or potential labor supply for newly arrived and established immigrants are respectively given by

$$\hat{l}_{i,g}^{na\prime} = (1 - p^{m})(1 - \phi)\left[(1 - \pi)\,\hat{l}_{i,g}^{na} + \pi\hat{l}_{i-1,g}^{na}\right] + \hat{\lambda}_{i,g}^{na\prime},$$

and

$$\hat{l}_{i,g}^{e\prime} = (1 - p^{m})\left[\hat{l}_{i,g}^{e} + \phi\left((1 - \pi)\,\hat{l}_{i,g}^{na} + \pi\hat{l}_{i-1,g}^{na}\right)\right] + \hat{\lambda}_{i,g}^{e\prime},$$

where $\hat{\lambda}_{i,g}^{o}$ is the inflow in potential labor supply of type $o \in \{na, e\}$. As in (14), we can define $\kappa^m = \hat{\lambda}_{i,g}^{m}/\lambda_{i,g}^{\omega,m}$, which is the labor force participation rate of immigrants that have spent an infinitely long period in the country, given that $\hat{\lambda}_{i,g}^{m}$ and $\lambda_{i,g}^{\omega,m}$ attain their steady state values.[23]

The retirement populations follow

$$ret^{o\prime} = (1 - \Theta^{o})\, ret^{o} + p^{o}\Omega^{o} \tag{26}$$

[22] Note that our baseline specification imposes that all immigrants are "newly arrived" on arrival, such that $\varepsilon_{i,g}^{e} = 0$ $\forall i$ always.

[23] Note that by denoting the share of new native individuals that participate in the labor force as $\kappa^d = \lambda_{i,g}^{d}/\lambda_{i,g}^{\omega,d}$, we get in steady state $l_{i,g}^{d} = \kappa^d \omega_{i,g}^{d}$. If κ^d (or κ^m below) varies across time, the steady state has to be defined differently.

A.2 Some auxiliary definitions

We are interested in reporting several variables in per capita terms. The following definitions are therefore useful:

$$GDP/capita \text{ (working age)} = \frac{Y}{\Omega}.$$

An alternative GDP per capita measure considers the entire adult population, including retirees,

$$GDP/capita \text{ (all adults)} = \frac{Y}{\Pi}.$$

Average labor productivity is

$$LP = \frac{Y}{N}.$$

The average wage is

$$\bar{w} = \frac{\sum\limits_{o\in\{d,na,e\}} \sum\limits_{i=1}^{I} \sum\limits_{g\in\{L,H\}} n_{i,g}^{o} w_{i,g}^{o}}{\sum\limits_{o\in\{d,na,e\}} \sum\limits_{i=1}^{I} \sum\limits_{g\in\{L,H\}} n_{i,g}^{o}} = \frac{\sum\limits_{o\in\{d,na,e\}} \sum\limits_{i=1}^{I} \sum\limits_{g\in\{L,H\}} n_{i,g}^{o} w_{i,g}^{o}}{N}.$$

The productivity-adjusted average wage is instead

$$\tilde{w} = \frac{\sum\limits_{o\in\{d,na,e\}} \sum\limits_{i=1}^{I} \sum\limits_{g\in\{L,H\}} n_{i,g}^{o} w_{i,g}^{o}}{\sum\limits_{o\in\{d,na,e\}} \sum\limits_{i=1}^{I} \sum\limits_{g\in\{L,H\}} \varepsilon_i n_{i,g}^{o}}.$$

We define wages per skill group and by natives/migrants analogously.

A.3 Simple model

Assume that workers can have different productivities but otherwise are identical. The productivity of a worker is denoted ε_i. Firms employ one worker. The meeting function is Cobb-Douglas

$$M = Au^{\xi}v^{1-\xi}.$$

Aggregate unemployment is

$$u = \int_I u_i di,$$

where u_i is unemployment for workers with productivity i. The vacancy and job meeting rates are

$$q = \frac{M}{v} \quad \text{and} \quad f = \frac{M}{u}.$$

The value of being employed for a worker with productivity i is

$$W_i = (1-\tau)\, w_i + \beta\left[(1-\delta)\, W_i + \delta U_i\right],$$

where w_i is the wage and U_i is the value when unemployed;

$$U_i = b + \beta\left[\tilde{f}_i W_i + \left(1-\tilde{f}_i\right) U_i\right],$$

where $\tilde{f}_i = f\mathbb{I}\left(J_i \geq 0\right)$ is the job finding rate with $\mathbb{I}$ being an indicator function and J_i the value of a firm employing a worker with productivity ε_i;

$$J_i = \varepsilon_i - w_i + \beta\left(1-\delta\right) J_i.$$

Let $S_i = W_i - U_i$. Wages are determined by the Nash bargaining solution

$$(1-\tau)\,\eta J_i = (1-\eta)\, S_i.$$

Then the wage is

$$w_i = \eta\varepsilon_i + (1-\eta)\left(\frac{b}{1-\tau} + \beta\frac{\eta}{1-\eta}\tilde{f}_i J_i\right).$$

Finally, job creation is given by

$$c = q\beta\int_I \frac{u_i}{u}\max\left\{J_i, 0\right\} di, \tag{27}$$

where c is the vacancy cost.

In the model, there is a cutoff value of idiosyncratic productivity ε^c so that the firm is indifferent between employing and not employing a worker. In particular, $J_i = 0$ implies that

$$\varepsilon^c = \frac{b}{1-\tau} \equiv \tilde{b}.$$

Letting G denote the cummulative distribution function (CDF) of the productivity distribution, the share of employable workers is $1 - G(\tilde{b})$. Since the job finding rate is f for all workers above the threshold, we can write the aggregate employment transition as

$$n = (1-\delta) n_{-1} + f \int_{i:\varepsilon_i \geq \varepsilon^c} u_i di = (1-\delta) n_{-1} + f \left(1 - n_{-1} - G\left(\tilde{b}\right)\right),$$

where $1 - n_{-1} - G(\tilde{b})$ is frictional unemployment. Structural unemployment is $G(\tilde{b})$.

Letting g denote the PDF and $g_i = g(\varepsilon_i)$ unemployment is, noting that $u_i = g_i$ for $\varepsilon_i < \varepsilon^c$,

$$u = \int_{i:\varepsilon_i \geq \varepsilon^c} u_i di + \int_{i:\varepsilon_i < \varepsilon^c} u_i di \iff \int_{i:\varepsilon_i \geq \varepsilon^c} u_i di = u - G\left(\tilde{b}\right) = 1 - n - G\left(\tilde{b}\right).$$

Since frictional unemployment is similar for any $\varepsilon_i \geq \varepsilon^c$, in steady state we have

$$u_i = \frac{g_i}{1 - G\left(\tilde{b}\right)} \left(1 - n - G\left(\tilde{b}\right)\right).$$

Then

$$c = q\beta \int_{i:\varepsilon_i \geq \varepsilon^c} \frac{u_i}{u} \max\{J_i, 0\} di = q\beta \frac{1 - n - G\left(\tilde{b}\right)}{1-n} \int_{i:\varepsilon_i \geq \varepsilon^c} \frac{g_i}{1 - G\left(\tilde{b}\right)} J_i di.$$

Hence, letting

$$\bar{\varepsilon} = \int_{i:\varepsilon_i \geq \varepsilon^c} \frac{g_i}{1 - G\left(\tilde{b}\right)} \varepsilon_i di$$

denote average productivity among employed workers and using

$$J_i = \frac{(1-\eta)\left(\varepsilon_i - \frac{b}{1-\tau}\right)}{1 - \beta(1-\delta) + \beta\eta f},$$

we have

$$c = \frac{q\beta(1-\eta)\left(\bar{\varepsilon} - \tilde{b}\right)}{1 - \beta(1-\delta) + \beta\eta f} \frac{1 - n - G\left(\tilde{b}\right)}{1-n}.$$

Noting that we have, from employment transition,

$$n = \frac{f}{\delta + f}\left(1 - G\left(\tilde{b}\right)\right),$$

and labor market tightness is hence determined by

$$c = \frac{q\beta(1-\eta)\left(\bar{\varepsilon} - \frac{b}{1-\tau}\right)}{1-\beta(1-\delta)+\beta\eta f} \frac{\delta\left(1-G\left(\tilde{b}\right)\right)}{\delta + fG\left(\tilde{b}\right)} = \Psi(\theta,\tau) \underbrace{\frac{\delta\left(1-G\left(\frac{b}{1-\tau}\right)\right)}{\delta + fG\left(\frac{b}{1-\tau}\right)}}_{=\Upsilon(\tau)},$$

where, noting that $q = \theta^{-\xi}$ and $f = \theta^{1-\xi}$, Ψ is decreasing in θ. An increase in structural unemployment through a change in the distribution G (keeping $\bar{\varepsilon}$ unchanged) implies a decrease in Υ. This, in turn, requires that Ψ increases, leading to a fall in tightness and the job finding rate. Letting θ^0 denote the initial value of labor market tightness, an increase in the tax from τ^0 to τ^1 leads to a decrease in Ψ, for a given θ. Moreover, an increase in the tax leads to an increase in $\tilde{b}$. Then,

$$\partial\Upsilon/\partial\tau = -\frac{(\delta+f)\,\delta g\left(\tilde{b}\right)}{\left(\delta + fG\left(\tilde{b}\right)\right)^2} \frac{b}{(1-\tau)^2} < 0.$$

Thus, $\Psi\left(\theta^0,\tau^1\right) < \Psi\left(\theta^0,\tau^0\right)$ and $\Upsilon\left(\tau^1\right) < \Upsilon\left(\tau^0\right)$. Since $\Upsilon(\tau)\Psi(\theta,\tau) = c$ from job creation, $\theta^1 < \theta^0$ and hence the job finding rate decreases. Thus, an increase in the tax increases structural unemployment through the increase in $\tilde{b}$ as well as frictional unemployment through the fall in the job finding rate.

A.4 Algorithm for solving steady state

A.4.1 Interpolation of values

We interpolate employment as follows. When computing firm values for different levels of productivity, there is some grid point ε_g^c such that $J_{\varepsilon_g^c,g} > 0$ and $J_{i,g} < 0$ for $i < \varepsilon_g^c$. Since the firm value is a continuous function, and in practice close to linear, we can find the "true" cutoff along the following lines.

We approximate the value function by the following linear function

$$J^{lin,o} = c^o + s^o * \varepsilon,$$

where

$$\begin{aligned} s^o &= \frac{J^o_{\varepsilon_g^c,g} - J^o_{\varepsilon_{g-1}^c,g}}{\varepsilon_g^c - \varepsilon_{g-1}^c} \\ c^o &= J^o_{\varepsilon_g^c,g} - s^o * \varepsilon_g^c. \end{aligned}$$

By setting $J^{lin} = 0$, this gives a cutoff for productivity

$$\varepsilon_c^{lin} = -\frac{c}{s}.$$

Letting

$$\varepsilon_m = \frac{\varepsilon_g^c + \varepsilon_{g-1}^c}{2},$$

we interpolate as follows. If $\varepsilon_c^{lin} \geq \varepsilon_m$, letting $J_{i,g}^{int,o}$ denote the interpolated value, we set $J_{i,g}^{int,o} = J_{i,g}^o$ for all $i \neq \varepsilon_g^c$. We then set

$$J_{\varepsilon_g^c,g}^{int,o} = \frac{\varepsilon_m^{+1} - \varepsilon_c^{lin}}{\varepsilon_m^{+1} - \varepsilon_m} J_{\varepsilon_g^c,g}^o,$$

where

$$\varepsilon_m^{+1} = \frac{\varepsilon_{g+1}^c + \varepsilon_g^c}{2}$$

is the midpoint between gridpoints ε_{g+1}^c and ε_g^c. For the indicator function, we construct an interpolated version of the indicator function, denoted $\mathbb{I}^{int}$, as follows. First, we set $\mathbb{I}^{int}\left(J_{i,g}^{int,o} \geq 0\right) = \mathbb{I}\left(J_{i,g}^d \geq 0\right)$ for all $i \neq \varepsilon_g^c$. When $i = \varepsilon_g^c$, we set

$$\mathbb{I}^{int}\left(J_{i,g}^{int,o} \geq 0\right) = \frac{\varepsilon_m^{+1} - \varepsilon_c^{lin}}{\varepsilon_m^{+1} - \varepsilon_m}.$$

If $\varepsilon_c^{lin} < \varepsilon_m$ we set $J_{i,g}^{int} = J_{i,g}$ for all $i \neq \varepsilon_g^c - 1$ and

$$J_{\varepsilon_g^c-1,g}^{int,o} = \frac{\varepsilon_m - \varepsilon_c^{lin}}{\varepsilon_m - \varepsilon_m^{-1}} J_{\varepsilon_g^c,g}^o,$$

where

$$\varepsilon_m^{-1} = \frac{\varepsilon_{g-1}^c + \varepsilon_{g-2}^c}{2}.$$

For the indicator function, we set $\mathbb{I}^{int}\left(J_{i,g}^{int,o} \geq 0\right) = \mathbb{I}\left(J_{i,g}^d \geq 0\right)$ for all $i \neq \varepsilon_g^c - 1$. When $i = \varepsilon_g^c - 1$, we set

$$\mathbb{I}^{int}\left(J_{i,g}^{int,o} \geq 0\right) = \frac{\varepsilon_m - \varepsilon_c^{lin}}{\varepsilon_m - \varepsilon_m^{-1}}.$$

A.4.2 Algorithm

The algorithm is as follows:

Define $J_{i,g} = \{J_{i,g}^d, J_{i,g}^{na}, J_{i,g}^e\}$

1. Outer loop: Guess labor market tightness for both markets: $\theta_H^{(k)}, \theta_L^{(k)}$ and $\tau^{(k)}$
2. Intermediate loop

(i) Guess $J_{i,g}^{(l)}$

3. Inner loop:

(i) Guess $J_{i,g}^{(l+1,j)} = J_{i,g}^{(l)}$

(ii) Compute $n_{i,g}^{d}$, using (16),

$$n_{i,g}^{d} = \frac{(1-p)\, f_g \mathbb{I}^{int}\left(J_{i,g}^{int,d,(l+1,j)} \geq 0\right)}{1-(1-p)(1-\delta_g - f_g)\,\mathbb{I}^{int}\left(J_{i,g}^{int,d,(l+1,j)} \geq 0\right)} l_{i,g}^{d}$$

and $n_{i,g}^{na}$, using (17) and (18),[24]

$$n_{i,g}^{na} = (1-p^m)\,\mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)} \geq 0\right) \frac{\pi\left((1-\delta_g)-f_g\right) n_{i-1,g}^{na} + f_g\left((1-\pi)\, l_{i,g}^{na} + \pi l_{i-1,g}^{na}\right)}{1-(1-\delta_g - f_g)(1-p^m)(1-\pi)\,\mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)} \geq 0\right)}$$

and

$$\begin{aligned} n_{i,g}^{e} &= \mathbb{I}^{int}\left(J_{i,g}^{int,e,(l+1,j)} \geq 0\right) \frac{(1-p^m)\, f_g l_{i,g}^{m,o}}{1-(1-\delta_g - f_g)(1-p^m)\,\mathbb{I}^{int}\left(J_{i,g}^{int,e,(l+1,j)} \geq 0\right)} \\ &+\mathbb{I}^{int}\left(J_{i,g}^{int,e,(l+1,j)} \geq 0\right) \phi \frac{(1-p^m)\left[(1-\delta_g - f_g)\left((1-\pi)\, n_{i,g}^{na} + \pi n_{i-1,g}^{na}\right) + f_g\left((1-\pi)\, l_{i,g}^{na} + \pi l_{i-1,g}^{na}\right)\right]}{1-(1-\delta_g - f_g)(1-p^m)\,\mathbb{I}^{int}\left(J_{i,g}^{int,e,(l+1,j)} \geq 0\right)} \end{aligned}$$

and then $n_{i,g} = n_{i,g}^{d} + n_{i,g}^{na} + n_{i,g}^{e}$, n_g and F, using (2) and (3).

(iii) Compute $n_{i,g}$ and interpolate. Then compute wages using (13), firm values (10), (11) and (12), worker values (5), (6), (7) and (8), interpolated employment and the solutions for wages

$$\begin{aligned} w_{i,g}^{d} &= \eta a \left(\frac{F}{n_g}\right)^{\frac{1}{\rho}} \varepsilon_i + (1-\eta)\frac{b_{i,g}}{1-\tau} + \beta\left(1-p^d\right) f_g \mathbb{I}^{int}\left(J_{i,g}^{int,d\prime} \geq 0\right) \eta J_{i,g}^{int,d\prime} \\ w_{i,g}^{e} &= \eta \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) + (1-\eta)\frac{b_{i,g}}{1-\tau} + \eta\beta(1-p^m) f_g \mathbb{I}^{int}\left(J_{i,g}^{e} \geq 0\right) J_{i,g}^{int,e} \\ w_{i,g}^{na} &= \eta \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) + (1-\eta)\frac{b_{i,g}}{1-\tau} + \eta\beta(1-p^m)(1-\phi) f_g \mathbb{I}^{int}\left(J_{i,g}^{int,na} \geq 0\right)(1-\pi) J_{i,g}^{int,na} \\ &+\eta\beta(1-p^m)(1-\phi) f_g \mathbb{I}^{int}\left(J_{i+1,g}^{int,na} \geq 0\right) \pi J_{i+1,g}^{int,na} + \eta\beta(1-p^m)\phi f_g \mathbb{I}^{int}\left(J_{i,g}^{int,e} \geq 0\right) J_{i,g}^{int,e}. \end{aligned} \quad (28)$$

[24] Note that labor market transition at grid point $i = 1$ is

$$n_{i,g}^{na\prime} = (1-\delta_g)(1-p^m)(1-\pi)\, n_{i,g}^{na} \mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)}(n') \geq 0\right) + (1-\pi)(1-p^m) f_g \left(l_{i,g}^{na} - n_{i,g}^{na}\right) \mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)}(n') \geq 0\right),$$

and, at grid point $i = I$,

$$\begin{aligned} n_{i,g}^{na\prime} &= (1-\delta_g)(1-p^m)\, n_{i,g}^{na} \mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)}(n') \geq 0\right) + (1-\delta_g)(1-p^m)\pi n_{i-1,g}^{na} \mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)}(n') \geq 0\right) \\ &+ f_g\left(l_{i,g}^{na} - n_{i,g}^{na}\right) \mathbb{I}^{int}\left(J_{i,g}^{int,na,(l+1,j)}(n') \geq 0\right)(1-p^m)\left(\left(l_{i,g}^{na} - n_{i,g}^{na}\right) + \pi\left(l_{i-1,g}^{na} - n_{i-1,g}^{na}\right)\right). \end{aligned}$$

This leads to slightly modified expressions when solving for steady state employment at these grid points. Similar modification applies to the transition rates for migrant population and labor force, as well as value functions.

(iv) Compute updated $J_{i,g}^{(l+1,j+1)}$ and $\delta_{J(j+1)} = \max_{i,g}\left(\left|J_{i,g}^{(l+1,j+1)} - J_{i,g}^{(l+1,j)}\right|\right)$. If $\delta_{J(j+1)} < c^r$ continue; otherwise go to step (ii).
(v) Compute $\delta_J = \max_{i,g}\left(\left|J_{i,g}^{(l+1)} - J_{i,g}^{(l)}\right|\right)$. If $\delta_J < c^r$ continue; otherwise go to step (i).

4. Use the solution for J and h_g (based on interpolated employment) to compute $\theta_H^{(k+1)}, \theta_L^{(k+1)}$ from the job creation condition (20), noting that $q_g = A(\theta_g)^{-\xi}$ and $f_g = A(\theta_g)^{1-\xi}$. Also, compute the updated tax from (22) using the solution for the wage $w_{i,g}^o$, employment $n_{i,g}^o$ and unemployment $u_{i,g}^o$ from (19). To do this we also need to solve for retirees from (26) and (15).

5. Compute $\delta_\theta = \max\{\theta_H^{(k+1)} - \theta_H^{(k)}, \theta_L^{(k+1)} - \theta_L^{(k)}\}$. If $\delta_\theta < c^r$ end; otherwise go to step (2).

A.5 Algorithm for solving dynamics

Note first that, using the CES properties of F, the firm value of an additional employed native worker in expression (10) can be written as

$$J_{i,H}^d(n) = a\left(\frac{Y}{n_H}\right)^{\frac{1}{\rho}} \varepsilon_i - w_{i,H}^d + \beta(1-\delta_H)(1-p) J_{i,H}^{int,d}(n'), \tag{29}$$

and

$$J_{i,L}^d(n) = (1-a)\left(\frac{Y}{n_L}\right)^{\frac{1}{\rho}} \varepsilon_i - w_{i,L}^d + \beta(1-\delta_L)(1-p) J_{i,L}^{int,d}(n'). \tag{30}$$

For established migrants we have, using (11),

$$J_{i,g}^e(n) = \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) - w_{i,g}^e + \beta(1-p^m)(1-\delta_g) J_{i,g}^{int,e}(n'), \tag{31}$$

and, for newly arrived immigrants, using (12),

$$\begin{aligned} J_{i,g}^{na}(n) &= \frac{\partial F}{\partial n_{i,g}}(n_H, n_L) - w_{i,g}^{na} + \beta(1-p^m)(1-\phi)(1-\delta_g)\left((1-\pi) J_{i,g}^{int,na}(n') + \pi J_{i+1,g}^{int,na}(n')\right) \\ &\quad + \beta(1-p^m)\phi(1-\delta_g) J_{i,g}^{int,e}(n'). \end{aligned} \tag{32}$$

Compute dynamics starting at some time period t. Assume that steady state is reached in period T.

1. Guess sequences of $\{n_{H,s}^{(k)}\}_{s=t}^T$, $\{n_{L,s}^{(k)}\}_{s=t}^T$, $\{\theta_{H,s}^{(k)}\}_{s=t}^T$, $\{\theta_{L,s}^{(k)}\}_{s=t}^T$ and $\{\tau_s^{(k)}\}_{s=t}^T$. Denote this vector of sequences by $\{\Psi_s^{(k)}\}_{s=t}^T$, i.e., $\Psi_s^{(k)} = \{n_{H,s}^{(k)}, n_{L,s}^{(k)}, \theta_{H,s}^{(k)}, \theta_{L,s}^{(k)}, \tau_s^{(k)}\}$

2. For T, compute $f_{H,T}$ and $f_{L,T}$ using the guess for $\theta_{H,T}$ and $\theta_{L,T}$. Then compute wages in period T using the dynamic version of (28) and interpolated employment. Then compute firm values using (29), (30), (31) and (32). Iterate backward from T to t.

3. Use labor market transition equations (16), (17) and (18) to compute $n_{H,s}^{(k+1)}$ and $n_{L,s}^{(k+1)}$ from $s = t$. Interpolate employment and compute unemployment using this. Then use resulting unemployment rates from (19) to compute $h_{g,s}$ in (21). Finally, use job creation (20) to compute labor market tightness $\theta_{H,s}$ and $\theta_{L,s}$ and use matching function to find sequence of job finding rates $f_{H,s}$ and $f_{L,s}$. Recursively proceed up to period T.

This gives updated sequences $\{n_{H,s}^{(k+1)}\}_{s=t}^{T}$, $\{n_{L,s}^{(k+1)}\}_{s=t}^{T}$, $\{\theta_{H,s}^{(k+1)}\}_{s=t}^{T}$ and $\{\theta_{L,s}^{(k+1)}\}_{s=t}^{T}$.

4. Finally, use the labor market transitions computed in step 3 in the expression for the tax rate

$$\tau = \frac{\sum_{g\in\{H,L\}}\sum_{i\in I} u_{i,g}b_{i,g} + \left(\sum_{o\in\{d,na,e\}}\sum_{g\in\{H,L\}}\sum_{i\in I} z_l\left(\omega_{i,g}^{o} - l_{i,g}^{o}\right) + z_{ret}\times ret\right)}{\sum_{o\in\{d,na,e\}}\sum_{g\in\{H,L\}}\sum_{i\in I} n_{i,g}^{o}w_{i,g}^{o}}, \tag{33}$$

to compute an updated sequence for $\tau_s^{(k+1)}$.

5. If the new sequence is close to the previous one, i.e. $\left\|\{\Psi_s^{(k+1)}\}_{s=t}^{T} - \{\Psi_s^{(k)}\}_{s=t}^{T}\right\| < M$, then quit. Otherwise, go to step 2.

A.6 Robustness checks

A.6.1 Implications of immigration with imperfect substitutability

In this section we document the implications of immigration in an alternative specification of our model where we assume that natives and migrants are imperfect substitutes in the production function. This alternative specification of the model is recalibrated to match the moments described in Table 3.

We still assume that the technology is given by (3) but where n_g now is given by

$$n_g = \left(\left(n_g^{d}\right)^{\frac{\rho_e-1}{\rho_e}} + \left(n_g^{m}\right)^{\frac{\rho_e-1}{\rho_e}}\right)^{\frac{\rho_e}{\rho_e-1}},$$

where ρ_e denotes the elasticity of substitution between natives and immigrants.

Finally, the effective (productivity-adjusted) employment of group $o \in \{d, m\}$, is given by

$$n_g^{o} = \sum_i \varepsilon_i n_{i,g}^{o}. \tag{34}$$

The marginal product is, for group $o \in \{d, m\}$,

$$\frac{\partial Y}{\partial n_{i,H}^{o}} = a\left(\frac{Y}{n_H}\right)^{\frac{1}{\rho}}\left(\frac{n_H}{n_H^{o}}\right)^{\frac{1}{\rho_e}}\varepsilon_i, \text{ and } \frac{\partial Y}{\partial n_{i,L}^{o}} = (1-a)\left(\frac{Y}{n_L}\right)^{\frac{1}{\rho}}\left(\frac{n_L}{n_L^{o}}\right)^{\frac{1}{\rho_e}}\varepsilon_i.$$

The firm values (10), (11) and (12) are now modified to be

$$J_{i,g}^{d}(n) = \frac{\partial F}{\partial n_{i,g}^{n}}(n_H, n_L) - w_{i,g}^{d} + \beta\left(1-p^{d}\right)(1-\delta_g)\,J_{i,g}^{d}(n'), \tag{35}$$

$$J_{i,g}^{e}(n) = \frac{\partial F}{\partial n_{i,g}^{m}}(n_H, n_L) - w_{i,g}^{e} + \beta(1-p^{m})(1-\delta_g)\left(J_{i,g}^{e}(n')\right) \tag{36}$$

and

$$\begin{aligned} J_{i,g}^{na}(n) &= \frac{\partial F}{\partial n_{i,g}^{m}}(n_H, n_L) - w_{i,g}^{na} + \beta(1-p^{m})(1-\phi)(1-\delta_g)\left((1-\pi)\,J_{i,g}^{na}(n') + \pi J_{i+1,g}^{na}(n')\right) \\ &\quad + \beta(1-p^{m})\,\phi\,(1-\delta_g)\left((1-\pi)\,J_{i,g}^{e}(n') + \pi J_{i+1,g}^{e}(n')\right). \end{aligned} \tag{37}$$

In line with the evidence in Busch et al. (2020), we calibrate this substitutability to 13 and then re-calibrate our model by matching the same moments as in the baseline specification.

Figure 11 reports the implications for GDP per capita and the employment-population ratio while Figure 12 documents the unemployment implications. Unsurprisingly, immigration is less detrimental for the economy when natives and immigrants are imperfectly substitutable in production. We note that the maximum decrease in GDP per capita in Figure 11 is roughly three quarters as large as in the baseline case. One contributing factor to this at both long and short horizons is the lower unemployment of natives that immigration yields in the imperfectly substitution case, as shown in Figure 12.[25] As can be seen from these two figures, all other quantities are affected very similarly in the two model specifications, i.e. for different degree of substitutability between natives and immigrants.

[25] The dynamics for immigrant unemployment is less affected by the degree of substitutability as the model is re-calibrated for each specification, and the calibration includes targets for the unemployment rate for immigrants as a function of the number of years in the country.

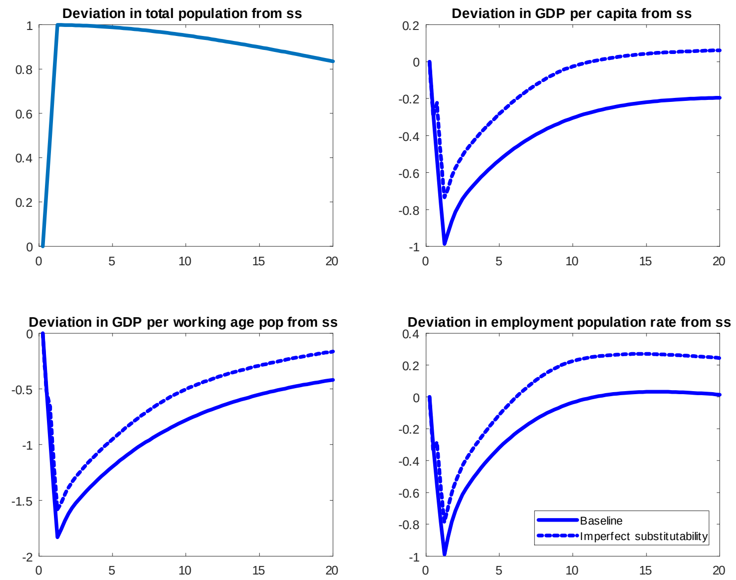

Figure 11: Model variant with imperfect substitutability between natives and migrants. The effect of a one percent migration shock on GDP and employment. Annual scale on x-axis.

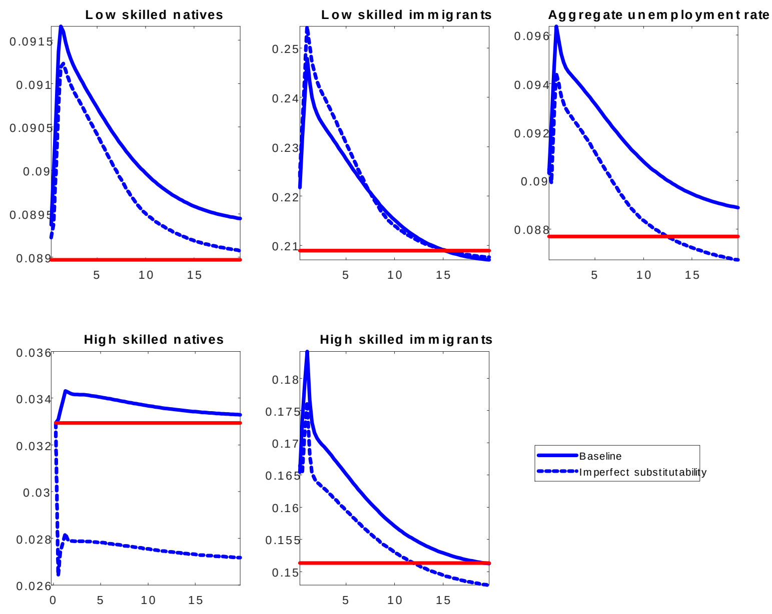

Figure 12: Model variant with imperfect substitutability between natives and immigrants. The effect of a one percent migration shock on various unemployment rates. The plot for the imperfect substitutability scenario has been adjusted to share the starting point with the baseline specification. Annual scale on x-axis.