The Cross-border Effects of Bank Capital Regulation

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Abstract

We propose a model for studying the international collaboration of bank capital regulation under the principle of reciprocity. We show that such a regime makes countries strategically compete for scarce bank equity capital. Raising capital requirements in a country may generate bank capital outflows as well as inflows. We pin down the condition for the sign of the capital flow and the associated externality, and highlight the implications for macroprudential regulation. Compared to full collaboration, individual countries are likely to set Basel III's Counter-Cyclical Capital Buffer too high in normal times, and too low in bad times.

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1 Introduction

International standards for bank capital regulation have evolved over the past three decades from a simple, common 8% minimum capital requirement to a broad, complex toolkit. Macroprudential considerations have been a driving force behind this evolution. Prudential risks and macroeconomic cycles differ across countries, which suggests the need for heterogeneous and time-varying capital requirements. The current regime (Basel III) promotes such flexibility and provides discretion in implementation to national regulators. However, in order to maintain a level playing field, the Basel Committee has introduced the principle of reciprocity: the capital requirement set by a regulator applies to all bank loans made in its jurisdiction, irrespective of which jurisdiction the bank belongs to. This principle fundamentally alters strategic incentives among regulators.

The existing literature has studied non-reciprocal regimes. In these circumstances, international competition for market shares within a country is a key driver of strategic incentives: Regulators can give an advantage to banks in their jurisdiction by cutting capital requirements as this allows them to operate at a cheaper cost than banks from other jurisdictions. So, to the extent that regulators care about the profits of the banks they regulate, they have an incentive to undercut one another in order for their banks to steal market share (Dell'Ariccia and Marquez (2006)). The introduction of reciprocity is designed to eliminate this market-share externality. Still, even in the current regime, regulators and policymakers regularly express concerns about the international spillovers of capital requirements. Yet there is little agreement on what the relevant externalities are and a formal framework for assessing them is needed.[1]

We find that, under reciprocity, what matters is not competition for market shares but competition for bank capital. Key is how much equity capital is allocated to lending in different jurisdictions. Changes in capital requirements alter this allocation and effectively generate bank equity capital flows. We propose a

[1]Typical concerns associated with higher capital requirements in a given country go from impairing the competitiveness of the domestic financial system (Osborne (2015)) to a reduction in domestic banks' foreign exposures, therefore impairing the functioning of foreign financial systems (de Guindos (2019)), and to cross-border relocation of risk-shifting activities (ESRB (2018), page 90).

model to study such capital flows and their implications for strategic interactions between regulators. The model has two dates and two countries (*Home* and *Foreign*), in which banks finance loans with a mix of insured deposits and equity capital. These banks face capital requirements in a reciprocity regime. Bank equity capital is mobile and there is global competition for it.

Our contribution is threefold: First, we show that, perhaps against conventional wisdom, an increase in the capital requirement in a country does not necessarily generate outflows of bank equity capital – *inflows* are possible too. We pin down the conditions under which either case occurs. Second, as bank equity capital is scarce, changes to capital requirements in a country impose, through capital flows, an externality onto the other country. We show that this *capital flow externality* is central to the incentive for national regulators to deviate from a collaborative optimum. If this externality is positive, it is associated with an incentive to deviate downwards, i.e., undercut the other country (and vice versa if the externality is negative). Third, we point out the implications for the coordination of macroprudential capital regulation. In particular, under reciprocity and absent coordination, macroprudential capital requirements are likely to be set too high in normal and good times, but too low in troubled times, when bank equity capital is particularly scarce.

To understand these results, let us expose the main mechanism of the model, starting from the perspective of a single country. The banking sector is competitive but, at the banking sector level, the returns to lending are diminishing. Consider the revenue banks receive from loans, net of repayments to depositors. This constitutes the resources available to pay the bank's shareholders, so we refer to it as banker revenue. This revenue is hump-shaped in aggregate lending in much the same fashion as a monopolist's profit is hump-shaped in quantities. Now, holding aggregate bank equity fixed, an increase in capital requirements contracts credit. Given the hump shape, this can either increase or decrease banker revenues. This means that, ceteris paribus, there is a revenue maximising capital requirement. Moreover, hump-shaped revenues imply that the return on bank equity is hump-shaped in lending too. It turns out the revenue and return maximising requirements are the same. The bottom line is that if the capital requirement is initially below the return maximising level, then an increase will raise returns (and, vice versa, it would lower returns if we start above the return maximising level).

Now take the case of two countries with equity capital that can be freely allocated by banks to lending in either one. Consider a competitive equilibrium for a given pair of capital requirements. A basic no-arbitrage argument implies that return on equity be equalised across countries. If, for some reason, the return on equity increases in a country, capital will be reallocated to it to restore equilibrium. So if, ceteris paribus, a higher Home capital requirement increases Home returns, this will trigger a capital inflows (and outflows if returns decrease). This implies that the sign of the capital flow induced by a capital requirement change hinges on whether the initial requirement is greater or less than the revenue maximising requirement.

In our model, banks issue equity competitively and the global aggregate supply of bank equity is upward sloping. Hence, in equilibrium, the return on bank equity is also equated to the marginal cost of raising it. Consider a change in the capital requirement in Home that attracts capital. There are two potential sources for this adjustment: the quantity of bank equity supplied globally can increase, or capital can flow into Home from Foreign, therefore generating a spillover. The extent to which these two margins are used depends on the relative elasticity of their associated supply curves. At one extreme, if the global supply for *new* equity capital is perfectly inelastic (i.e., there's a fix supply of global capital), all capital flowing into Home must flow out from Foreign; we have a 100% spillover. Conversely, if the global supply is perfectly elastic, all capital flowing to Home will be newly raised capital; and there is no spillover.

Having characterised the market equilibrium (for a given set of capital requirements), we then turn to a policy game (i.e., we endogenise the requirements). We compare the collaborative outcome with the Nash equilibrium when regulators seek to maximise net output subject to deadweight losses from financial instability. Bank equity capital alleviates these deadweight losses and hence is socially valuable. This gives rise to the competition across countries for bank equity capital: whether national regulators have an incentive to deviate upwards or downwards from the collaborative outcome depends on the sign of the capital flows the deviation would generate.

As we have explained, the sign of the capital flow depends on whether the initial requirement is higher or lower than the return maximising requirement. As an important aside: maximising returns is different from maximising welfare or even bank profits (as the return does not account for the banks' cost of funds). Still, the return maximising requirement is a useful threshold for the direction of capital flows.

We assess incentives for regulators to deviate from the collaborative outcome. We provide a closed form solution for the optimal collaborative capital requirement, and an associated closed form condition for whether this requirement is tighter or not than the return maximising requirement. Together with a numerical solution for the Nash equilibrium, this allows us to formulate empirical predictions. In particular, the following factors make it more likely, ceteris paribus, that capital requirements will be set too low by competing regulators: i) the supply of bank equity capital is particularly tight; ii) bank risk-shifting incentives are acute; iii) the aggregate loan demand is relatively elastic, or iv) deadweight losses are severe. And vice versa: competitive regulators will tend to set capital requirements too high under the opposite conditions, e.g., if equity capital is relatively abundant and bank risk-shifting incentives are mild.

Within the current regulatory framework, these predictions apply best to the setting of the counter-cyclical capital buffer (CCyB). The CCyB is the headline macroprudential capital requirement and consists of an add-on to the common minimum capital ratio. The buffers are set in each jurisdiction and must be reciprocated (within limits). Our analysis suggests that competing regulators will set the CCyB too high when bank capital is not too scarce, which we interpret as normal and good times.[2] In this case, there are gains from coordinating on more modest raises. However, if bank capital is very scarce (think of bad or

[2]As we discuss in Section 5, the empirical literature is generally consistent with the notion that banks respond, most of the time, to increases in capital requirements by partially raising more equity (as well as adjusting assets). In our model, this happens if and only if higher capital requirements generate capital inflows. This supports our interpretation of normal times being those where capital is not too scarce and where the level of lending is larger than the level that maximises banker revenue.

troubled times, for instance after a big negative shock to bank equity capital), then competing regulators will have an incentive to cut the CCyB by more than what collaboration requires. By design, the CCyB cannot be negative, which could mitigate this second discrepancy.

Overall, the reciprocity framework is incomplete and varies across geographies and for specific types of buffer. Nonetheless, national discretion with reciprocity appears to be the direction of travel in international accords. So, understanding how strategic interactions play out in this regime is important, and we see our paper as a first step in that direction.

The reference paper for the non-reciprocal regime is Dell'Ariccia and Marquez (2006). To illustrate how, in such a regime, regulators' strategic interactions fundamentally differ from those in the one we study, it is best to highlight a key mechanism in their paper. They have a representative bank based in each of two countries, but both banks operate in both countries. Each bank has a fixed amount of equity capital and faces the capital requirement imposed by their country of origin regulator. A key point is that a decrease in capital requirement by the Home regulator decreases the cost of capital for the Home-based bank, which gives it a competitive advantage in both markets and allows it to grab market share from its Foreign competitor. This is an externality that naturally gives incentives for countries to undercut one another. Adopting a reciprocity regime kills such a market-share externality.[3]

A related paper is Acharya (2003), which looks at how discretion in resolution regimes can undermine the benefit of coordination in capital regulation. In a similar international context, a series of papers study the interaction between capital regulation and other policy levers. Morrison and White (2009) examine the link between banking regulation and supervisory quality. In their set up, capital requirements are a substitute to the regulator's ability to distinguish sound banks from weak ones. Competition among regulators creates a selection effect: high quality banks prefer to be chartered by high ability regulator, which also sets lower capital requirements than low ability regulators. Other examples include,

[3]The Dell'Ariccia and Marquez (2006) model is more involved and also embed a financial stability externality, which reinforces the market share externality. We also allow for additional externalities but only discuss them in Section 5 since they are not the focus of our analysis.

Buck and Schliephake (2013) and Gersbach et al. (2020), which respectively focus on capital regulation interactions with supervision intensity and fiscal policy.

In addition, a sequence of papers have focused on international coordination of bank supervision, without focusing on capital requirements. Carletti et al. (2016) and Colliard (2019) both consider the role of central and local supervision when local supervisors have informational advantages but neglect cross-border externalities. Similarly, Calzolari and Loranth (2011) and Calzolari et al. (2018) consider how the presence of multinational banks alter supervisory incentives. And finally, Faia and Weder (2016), Bolton and Oehmke (2018) and Segura and Vicente (2019) study how the resolution of banks should be coordinated across borders.

2 The model

The model has two dates: 0 and 1. Decisions are made at date 0. At date 1, all stochastic variables are realised, and production and consumption take place.

We consider two sovereign countries: Home and Foreign. There is a single, tradeable good which can be consumed or used as physical capital in production, in which case the goods depreciate fully.

In each country, there is a mass of banks, a representative firm, a representative household, a representative banker, and a regulator. We describe here the details of the environment in the Home country. The Foreign country has the same environment (although we do not necessarily impose symmetry in parameter values); foreign variables are marked with a $'$.

Preferences Private agents only value date 1 consumption, are risk neutral, and act competitively. We define and discuss regulator preferences in Section 4 and 5.

Firms and technology The representative firm operates a Cobb-Douglas production technology: $Ak^{\alpha}l^{\alpha}$, where k is capital, l is labour, $0 < \alpha < 1$, and $A \geq 0$ is a random variable that captures TFP. The firm invests in capital at date 0. At date 1, A realises and the firm hires labour and produces.

We normalise $\mathbb{E}_0[A] = 1$ and assume that A is distributed over $[A^L, A^H]$, with a corresponding density function $g(A)$, which we assume is smooth, unless otherwise specified. Our analysis does not require to impose any specific structure on the dependence between A and A', but for simplicity we assume that the joint distribution has full support over $[A^L, A^H] \times \left[A'^L, A'^H\right]$.

The firm is penniless and borrows from the bank to invest in capital. In equilibrium, it makes zero profits in all states. We therefore abstract from firm ownership.

All agents have access to a riskless storage technology with a zero rate of return.

Banks Banks issue equity (protected by limited liability), take deposits, and lend to firms subject to a capital requirement constraint that we define below. The banks can potentially lend in both countries. There is free entry and equity can be raised globally (i.e. from bankers in either country). For our main results, it is not necessary to specify or restrict where deposits are raised.

Bankers Only bankers can invest in bank equity. Bank equity is costly in the sense that its global supply curve is, generally, upward sloping. However, it will also be useful to consider two extreme cases where the supply curve is flat (perfectly elastic) or vertical (perfectly inelastic). This is why we formalise the curve as follows.

The representative banker in Home is endowed with some initial wealth. However, bankers can generate additional date-0 wealth, at a disutility cost. To fix ideas, we use the following metaphor: bankers can obtain new goods in a virtual mine, some goods are easy to mine, some are not. The more people that are mining, globally, the harder the goods are to mine. The disutility of 1 hour of banker labour is 1. But the number of hours needed to mine 1 good is linearly increasing in global mining activity .

Formally, we define banker marginal disutility from mining $(1 + z)$ as:

$$1+z(N,N') \equiv \begin{cases} 1 & N+N' \leq \omega \\ 1+\kappa \times \underbrace{(N+N'-\omega)}_{\textbf{global mining}} & N+N' > \omega \end{cases}. \tag{1}$$

where $N+N'$ is the equity raised by banks globally, ω is the initial global endowment of banker wealth, and $\kappa \geq 0$ is a cost parameter.

Bankers supply equity competitively. Therefore, their marginal disutility function (1), which corresponds to the marginal cost of mining, constitutes the bank equity inverse supply curve. Most of our analysis focuses on the case where $\kappa > 0$ and $N+N' > \omega$. However, having the option to set $\kappa = 0$ or $\kappa \to \infty$ allows us to study the extreme cases where bank equity is perfectly elastic or perfectly inelastic.

Households The representative household is endowed with one unit of labour, which it supplies inelastically (and, for simplicity, without disutility) at date 1. It also has a large endowment of goods at date 0, which it initially allocates between insured bank deposits and the storage technology. We assume that this endowment is sufficiently large that the storage technology is always used in equilibrium. This pins down a households' opportunity cost of funds of unity.

Capital requirements: the reciprocity regime We follow the *principle of reciprocity* in bank capital regulation: capital requirements are in effect set by the regulator of the country where the lending takes place.

In our model, there is only one type of regulatory capital for banks: equity. To avoid confusion with physical capital, we henceforth refer to it as bank *equity capital*. Consider a given bank i, with equity capital n_i, and denote x_i and x'_i the quantity it lends in Home and Foreign respectively. Irrespective of its country of incorporation, this bank faces a capital requirement that takes the form:

$$n_i \geq \gamma x_i + \gamma' x'_i, \tag{2}$$

where $\gamma \in [0,1]$ and $\gamma' \in [0,1]$ are parameters set by the Home and Foreign regulators, respectively.

In practice banks are under the regulatory jurisdiction of the country where they are incorporated, and banks have three ways to lend across borders: directly, through branches, or through subsidiaries incorporated in the country of the borrower. Direct lending and lending made through branches fall, therefore, under the jurisdiction of the country where the bank is incorporated and, *de jure*, is not subject to the capital requirement imposed in the jurisdiction where the lending takes place. So, in principle, banks in different countries may face different capital requirements when lending to the same firm.

However, reciprocity levels the playing field. Concretely, any capital requirement set by the Home regulator is also imposed, by the Foreign regulator, on lending in Home of banks that fall under the Foreign regulator's jurisdiction (and vice versa). So, *de facto*, branches, subsidiaries, and direct cross-border loans all face the same capital requirement set by the country where the lending takes place.

Equilibrium In an equilibrium: banks and firms maximise expected pure profits; households and bankers maximise utility (i.e. date-1 consumption less any disutility of labour in date 0); and markets clear.

The problems of the firm and the household are trivial. Price taking behaviour implies that, in equilibrium:

- Deposits promise a zero rate of return.
- The wage, or household labour income, is given by: $w = A(1-\alpha)K^{\alpha}$, where K is aggregate capital. Given labour supply is normalised to 1, the aggregate wage bill is $W = w$.
- The loan gross interest rate is $A\alpha X^{\alpha-1}$, where X is aggregate lending.[4]
- Bankers break even in expectation. Their required return on bank equity should equal the marginal disutility of mining $(1+z)$.

[4] For simplicity, we consider an interest rate contingent on the realisation of TFP. Since firm defaults are costless, the realised repayment is exactly identical to that what it would be in an equilibrium under a standard debt contract with face value $A^{H}\alpha X^{\alpha-1}$ per unit of debt.

Market clearing requires:

- $K = X$

The banks' problem is more involved and is the focus of the next section.

3 Positive analysis: equity capital flows for given capital requirements

We are interested here in the equilibrium behaviour of banks for a given pair of capital requirements. That is, we treat γ and γ' as parameters, and study how a small change in γ affects the equilibrium allocation. We endogenise capital requirements in Section 4. To ease notation we sometimes drop function dependencies and denote functions evaluated at the market equilibrium with a star. We represent partial derivatives with a subscript, and we write total derivatives in full.

3.1 Preliminaries

The following lemma holds:

Lemma 1. *When banks default with strictly positive probability in equilibrium: (i) capital requirements are binding; and (ii) each individual bank perfectly specialises as either a lender in Home or Foreign.*

Proof. All proofs are in Appendix A. □

To fix attention on equilibria where capital requirements have interesting effects, unless otherwise stated, we make the following assumption to ensure banks default with positive probability:

ASSUMPTION 1: (risky banks) $A^L, A'^L = 0$, with $g(A^L), g'(A'^L) > 0$ and $\gamma, \gamma' < 1$.

Assumption 1 means that the results in Lemma 1 always hold. To interpret them: government guarantees generate an implicit subsidy for banks, which is maximised when the bank operates with maximum leverage. This is a well known result. Additionally, guarantees induce banks, ceteris paribus, to minimise diversification. Under the reciprocity regime this means that banks prefer to operate as subsidiaries in each country (i.e. with a separate balance sheets) rather than branches (or direct cross-border lending) to maximise the option value given by limited liability. Accordingly, one interpretation of our setup is that banks set up holding companies that operate across borders through separate specialised entities in each country. Alternatively, we can think of individual banks as stand-alone specialised lenders in each country.

For ease of exposition, and to save on notation, we will present our analysis using the latter interpretation. Either way, a bank's country of incorporation has no bearing on its behaviour in our model. Hence, we do not distinguish banks along that dimension.

The key object: Equity capital *allocated* to lending The key object in our analysis will be N, the aggregate quantity of bank equity capital *allocated* to banks specialising in lending at Home. A change in γ causes a reallocation of equity capital between Home, N, and Foreign, N': effectively a bank equity capital flow. It turns out that the direction of this capital flow will be key in governing the strategic interactions between regulators. We will come on to this. In this section we first look at the economic mechanisms that pin down the direction of the capital flow in the market equilibrium.

The bank's problem We consider a representative bank specialising in lending to Home firms. We denote this bank's equity n. Since capital requirements bind, the bank's lending is $x = \frac{n}{\gamma}$, the proceeds from lending are given by $\frac{n}{\gamma}\left(A\alpha X^{\alpha-1}\right)$ where the bank takes X as given. To lend an amount $\frac{n}{\gamma}$, the bank raises a total of $(1-\gamma)\frac{n}{\gamma}$ of deposits on which it pays zero interest. Aggregating across banks we have $X = \frac{N}{\gamma}$.

Define A^0 as the realisation of A such that the bank just has sufficient proceeds from its loans to make depositors whole. That is:

$$A^0(N,\gamma) = \frac{(1-\gamma)}{\alpha\left(\frac{N}{\gamma}\right)^{\alpha-1}}.$$

The revenue available for shareholder payouts is in expectation:

$$\frac{n}{\gamma}\int_{A^0(N,\gamma)}^{A^H}\left(\alpha A\left(\frac{N}{\gamma}\right)^{\alpha-1}-(1-\gamma)\right)g(A)dA.$$

Shareholders receive zero in the event of default. $A\alpha\left(\frac{N}{\gamma}\right)^{\alpha-1}$ is the unit proceeds from lending and $(1-\gamma)$ is the unit the cost of deposits. Shareholders have collectively invested equity capital n, hence, their expected return on equity is:

$$R(N,\gamma) \equiv \frac{\int_{A^0(N,\gamma)}^{A_H}\left(\alpha A\left(\frac{N}{\gamma}\right)^{\alpha-1}-(1-\gamma)\right)g(A)dA}{\gamma}. \tag{3}$$

The shareholders are the bankers and since their required return is $1+z$, the bank's optimisation problem can be written as:

$$\max_{n\geq 0} nR(N,\gamma) - n(1+z). \tag{4}$$

Market equilibrium Let N^* denote an equilibrium level of Home capital. We have the following result:

Proposition 1. *For all $\gamma, \gamma' \in (0,1)$, there exists a unique pair $\{N^*, N'^*\}$ such that returns on equity in both countries are equal to the bankers' required return. This pair is implicitly defined by:*

$$R(N^*,\gamma) = R'(N'^*,\gamma') = (1+z(N^*,N'^*)), \tag{5}$$

For intuition, note that, for a given γ, an increase in N implies more lending and therefore more aggregate physical capital. From diminishing returns, it directly follows that both $R(N,\gamma)$ and $R'(N',\gamma')$ are decreasing in N and N', respectively. This leads to the result since $z(N,N')$ is weakly increasing, and strictly from $N + N' = \omega$.[5]

3.2 International spillovers

We now turn to how capital requirements alter banks' allocation of capital to either Home or Foreign.

Starting from equilibrium, we now consider the effect of marginal changes in the Home capital requirement γ. The no-arbitrage condition in Proposition 1 implicitly defines a function $N^*(\gamma)$. There are two possible cases. Either $z^* = 0$, in which case capital is not costly in equilibrium and changes in the capital requirement in one country will not affect the equilibrium in the other country. The interesting case is $z^* > 0$. Then, we have:

$$\frac{dN^*}{d\gamma} = -\left(R^*_\gamma\right) \underbrace{\left(\frac{\kappa - R'^*_{N'}}{\kappa\left(R'^*_{N'} + R^*_N\right) - R^*_N R'^*_{N'}}\right)}_{\equiv \xi^*}. \tag{6}$$

The response of N^* to γ depends on κ and on the sensitivity of returns to the aggregate equity capital invested in the respective country. We will elaborate further on these objects below. However, to proceed, a useful starting point is the following lemma:

Lemma 2. *We have $\xi^* < 0$, therefore in the case where $z^* > 0$, we have $\frac{dN^*}{d\gamma} \gtreqless 0 \Leftrightarrow R^*_\gamma \gtreqless 0$.*

The intuition is simple: a change in capital requirements will trigger an increase (decrease) in bank equity capital in Home if, and only if, this change increases (decreases), ceteris paribus, the return on Home bank equity in equilibrium.[6]

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[5]While the pair $\{N^*, N'^*\}$ is unique, the proportion of bank equity capital sourced from bankers in Home or Foreign is indeterminate as bankers are indifferent in equilibrium. This is inconsequential for our results. See the end of Section 5.1 for further considerations on the ownership of equity capital.

[6]If $z^* = 0$, then $\frac{dN^*}{d\gamma} = -\frac{R^*_\gamma}{R^*_N}$ and Lemma 2 still directly applies, except at the point where $N^* + N'^* = \omega$, where $z(N^*, N'^*)$ is at a kink, which implies that $\frac{dN^*}{d\gamma}$ is not defined. However, Lemma 2 readily extends in terms of the subgradient.

By construction, new capital can only be raised by Home banks from two different sources: (i) bankers can mine more capital so that the global stock of bank capital will increase; (ii) there can be a flow of capital from Foreign banks to Home banks. In general, we get a combination of the two.

It is useful to think about $z(N, N')$ and R' as two (individual) inverse supply curves for N, with respective slopes κ and $R^*_{N'}$. With this in mind, the proposition that expresses the capital flow naturally follows:

Proposition 2. *If $N^* + N'^* < \omega$, then $z^* = 0$ and $\frac{dN^{*\prime}}{d\gamma} = 0$. In the case of interest where $N^* + N'^* > \omega$, then*

$$\frac{dN^{*\prime}}{d\gamma} = -\underbrace{\left(\frac{\kappa}{\kappa + (-R'^*_{N'})}\right)}_{\equiv SP^* \in [0,1)} \frac{dN^*}{d\gamma}.$$

The minus sign indicates that any change in capital in Home is met by a change of opposite sign in Foreign. The expression in the bracket pins down the proportion of the change in Home capital that translates to a change in Foreign. Intuitively, this expression, which we denote SP (for spillover), is pinned down by the relative slopes of the implicit supply curves we mentioned above.

Now, there are two special cases. If $\kappa = 0$, a change in capital requirements at Home does not affect the equilibrium cost of bank capital, supply adjusts entirely through mining, and there is no spillover. Second, $\kappa \to \infty$ implies a spillover of 100%. This corresponds to the extreme case where bank equity capital is in fixed supply. In this case, so long as $R^* > 1$, it is obvious that any change in equity capital at Home should be met by an exactly opposite change in Foreign.

Remark. Strictly speaking, non-zero spillovers require that $\kappa > 0$ not necessarily that $z^* > 0$: What is needed to generate a spillover is not that capital is costly per se but that the equilibrium cost of capital is affected by a change in capital requirements. In a modified version of the model where $z^* > 0$ (say, due to a tax advantage of debt) but $\kappa = 0$, there would be no spillover as a change in the capital requirement at Home would have no affect on the equilibrium cost of capital in Foreign.

3.3 The direction of international bank equity flows

Lemma 2 and Proposition 2 show that the direction of equity capital flows hinges on the sign of R_γ. We now discuss the economics of this sign and establish a general result: for any γ' there is a threshold value for γ that pins down the direction of the capital flow.

3.3.1 An increase in γ raise can return on equity ($R_\gamma > 0$)

Given that R embeds a subsidy from government guarantee, which is decreasing in the capital requirement, it is perhaps natural to assume that R_γ is negative and higher capital requirements always generate capital outflows. However, there is an alternative force working through competition that means that increases in γ can raise bank return on equity. Broadly speaking, raising γ restricts competition and increases total profits in the home banking sector. Keeping aggregate bank equity constant, more profits means a higher return on equity. In effect, tighter capital requirements act as a collusion device for otherwise competitive banks. However, this intuition needs to be qualified and this is best done by first outlining a simplified problem with fixed N.

Consider our environment, but without uncertainty (set $A = 1$) and consider a monopolist bank with predetermined capital N that maximises pure profits. The monopolist's optimal level of lending, $\mathring{X}$, is given by:

$$\mathring{X} \equiv \arg\max_{X} \alpha X^{\alpha} - X - zN.$$

The objective is hump-shaped in X, which reflects monopoly rents: starting from low levels, it increases up to $\mathring{X}$ where it peaks and then decreases. If pure profits are hump shaped in X so is the return on equity (given N):

$$R = \frac{1}{N}\left(\alpha X^{\alpha} - (X - N)\right),$$

and they are both maximised at the same level of lending, $\mathring{X}$. Now, substituting $N = \gamma X$, the return on equity can be written as:

$$R(N,\gamma) = N^{\alpha-1}\alpha\left(\frac{1}{\gamma}\right)^{\alpha} - \frac{1}{\gamma} + 1,$$

which is U-shaped in $1/\gamma$ and hence hump-shaped in γ. So, R_γ can take either sign and will be positive if $N/\gamma > \mathring{X}$.

Finally, adding back uncertainty, we return to Equation (3):

$$R(N,\gamma) = \int_{A^0(N,\gamma)}^{A^H}\left(AN^{\alpha-1}\alpha\left(\frac{1}{\gamma}\right)^{\alpha} - \frac{1}{\gamma} + 1\right)g(A)dA$$

By definition the integrand is nil at A_0, and the function is also hump-shaped in γ. So, unless the level of lending is already lower than what a monopolist would chose (taking N as given), an increase in γ contracts credit ($X = N/\gamma$). This increases pure profits $(RN - (1+z)N)$ and, therefore, the return on equity (R). For fixed N, therefore, this hump shape means that there is a threshold level for γ that pins down the sign of R_γ. However, in our model N is an equilibrium object that depends on γ itself. What we show next is that a threshold capital requirement still exists if one endogenises N and, as a result, $\frac{dN^*}{d\gamma}$ can have either sign in equilibrium.

3.3.2 There is a threshold for γ that pins down the sign of the capital flows

Theorem 1. $\forall\ \gamma' \in (0,1)$, *there exists a* $\widehat{\gamma}(\gamma') > 0$ *such that,* $\forall \gamma \in (0,1)$

$$\begin{cases} \frac{dN^*}{d\gamma} > 0 & ; \gamma < \widehat{\gamma}(\gamma') \\ \frac{dN^*}{d\gamma} = 0 & ; \gamma = \widehat{\gamma}(\gamma') \\ \frac{dN^*}{d\gamma} < 0 & ; \gamma > \widehat{\gamma}(\gamma') \end{cases}$$

Note that the theorem does not restrict $\widehat{\gamma}$ to be smaller than 1. If $\widehat{\gamma}(\gamma') > 1$, then it is simply the case that $\frac{dN^*}{d\gamma} > 0$ for all admissible γ. Hence, there always exists values for γ that are low enough (i.e., in between 0 and $\widehat{\gamma}$) for a marginal increase in γ to raise the Home bank's return on equity and, therefore, trigger capital inflows to Home.

Interpreting the theorem Endogenising N does not the change the fundamental intuition that returns are hump-shaped in γ. To see this, imagine that $R_\gamma^* > 0$: an increase in γ increases N^*. This inflow increases lending, which then reduces returns but this never fully offsets the partial effect represented by R_γ^* (otherwise we would have an outflow). This means R_γ^* and $\frac{dR^*}{d\gamma}$ share the same sign. However, the latter, total derivative is smaller in absolute magnitude due to the offset from the capital flow. Formally, using Equation (6), we have

$$\frac{dR^*}{d\gamma} = R_\gamma^* + \frac{dN^*}{d\gamma} R_N^* = R_\gamma^*(1 - \xi^* R_N^*), \tag{7}$$

where $(1 - \xi^* R_N^*) \in (0, 1)$ captures the offsetting affect of the equity capital flows.

This means $\hat{\gamma}(\gamma')$ is the capital requirement that maximises R^* given γ'; that is, it is the requirement at which $\frac{dR^*}{d\gamma} = 0$. From the definition of ξ^*, this can only be true when R_γ^* and, hence, $\frac{dN^*}{d\gamma}$ are equal to zero.

In equilibrium banks cannot benefit from an increase in γ So $\frac{dR^*}{d\gamma}$, $\frac{dN^*}{d\gamma}$ and R_γ^* are all zero at the same value, $\widehat{\gamma}(\gamma')$. An implication is that $\widehat{\gamma}(\gamma')$ is the value of the capital requirement that maximises not only R^*, but also banker revenues, $R^* N^*$, and the shareholder payout net of the capital invested $(R^* - 1)N^*$. However, what $\widehat{\gamma}(\gamma')$ does not maximise is bank pure-profits, simply because the latter includes the cost of bank equity. In our model, perfect competition implies that pure profits are always zero in equilibrium, so the cost increase just offsets the increase in revenue. More generally, when banks have market power, like in reality, the increase in cost always more than offsets an increase in revenue. This is why banks do not actually benefit from an increase in γ even if revenue is increasing in γ (which is reassuring given real world banks' notorious dislike of capital requirements).

4 Welfare analysis: strategic interactions among regulators

We now treat γ and γ' as the choice variables of strategic national regulators who maximise their objective taking each other's behaviour as given (for completeness, we now also allow for $\gamma, \gamma' = 1$). To maintain tractability, we assume the environments in both countries are initially identical.

4.1 Preliminaries

Financial stability Government guarantees generate a moral hazard problem as banks fail to internalise the expected shortfall of their assets in the event of default. Formally, this expected shortfall, which comes at a cost for the taxpayer, is defined as

$$S \equiv \int_{A^L}^{A^0} \left(\underbrace{(1-\gamma)\frac{N}{\gamma}}_{\text{tot. deposits}} - \underbrace{\alpha A \left(\frac{N}{\gamma}\right)^{\alpha}}_{\text{lending proceeds}} \right) f(A)dA.$$

Furthermore, banking sector defaults can entail deadweight losses in addition to the cost of reimbursing depositors. For instance, business disruptions may add a fixed cost to any default, funding a bailout may involve distortionary taxes, and these costs may spillover across borders. We capture such deadweight losses using a generic, reduced-form function:

$$L(N, \gamma; \Theta),$$

where we explicitly account for N and γ and use Θ to summarise all other variables (from Home or Foreign) that could affect deadweight losses in Home. The loss function in Foreign is symmetric.

The regulators' objective Consider now the following national accounting identity (in expectation):

$$\underbrace{K^\alpha - K}_{\equiv\text{NDP}} = X^\alpha - X = \underbrace{\underbrace{W}_{\text{labour income}} + \underbrace{(R-1)\,N}_{\text{shareholder income}} - \underbrace{S}_{\text{Shortfall}}}_{\equiv\text{NDI}}. \tag{8}$$

Shareholder income, $(R-1)N$, is banker revenue minus capital invested. It includes the expected transfer from the taxpayer, and this is why S is deducted in NDI.

Neither mining costs nor deadweight losses appear in equation (8). Deadweight losses must, by definition, enter any sensible welfare function. Mining costs, however, are more ambiguous. We modelled them as a disutility from labour for bankers, but this is of course a metaphor. We could equally have modelled them as a pure rent. The substantive question is whether, in the real world, bank capital is socially costly or not. Even though it is a fundamental question in banking, it turns out that it does not substantially affect our results (we come back to this in Section 5). For tractability and comparability with the literature, we do not include mining costs in our regulator's objective. It reads:

$$\Pi = \text{NDP} - L.$$

Capital market behaviour In what follows we consider regulators who choose their policy (γ and γ') facing optimising private agents. So, regulators take as given that, in equilibrium: (i) capital requirement are binding;[7] and that (ii) market forces will equate bank return on equity across countries (per Proposition 1).

Definition 1. Denote $\Pi^*(\gamma, \gamma')$ the value of the Home regulator's objective where the economy is at the market equilibrium consistent with γ,γ'.

From now on, we will distinguish between the *collaborative optimum*, the *Nash equilibrium of the policy game* (both defined below), and the *market equilibrium* which holds for any given combination of capital requirements.

[7]Strictly speaking, the capital requirements may be only *weakly* binding. In cases where the capital structure is undetermined we break the tie assuming it is pinned down by the requirement.

4.2 Collaborative outcome and incentives to deviate

The collaborative optimum To start, consider a collaborative game where regulators choose a common capital requirement to jointly maximise the sum of the objectives.

Definition 2. The collaborative optimum is given by

$$\gamma^c \equiv \arg\max_{\gamma} \Pi^*(\gamma, \gamma') + \Pi'^*(\gamma', \gamma)$$

$$\text{s.t.} \begin{cases} \gamma = \gamma' \\ R^* = R^{*\prime} = 1 + z^* \end{cases} .$$

The first constraint captures that collaborative regulators pick a common capital requirement, and the second is the capital market equilibrium conditions.[8]

The key object we will consider is an externality which we denote Π'^c_γ. It captures the marginal effect, for Foreign, of an increase in the Home capital requirement (given γ'), *at the collaborative optimum.* Formally:

Definition 3. $\Pi'^c_\gamma \equiv \Pi'^*_\gamma(\gamma, \gamma') \mid_{\gamma=\gamma'=\gamma^c}$

If $\Pi'^c_\gamma > 0$, this means that the Home regulator has an incentive to locally decrease γ at the collaborative optimum. This suggests a race to the bottom i.e. a non-collaborative equilibrium with a lower capital requirement than in the collaborative outcome. And, vice versa, $\Pi'^c_\gamma < 0$ suggests a race to the top.

For notational convenience, we now drop function dependencies for the remainder of this section. Henceforth, like in Π'^c_γ, functions with a c superscript are evaluated at the collaborative optimum: i.e., in the market equilibrium where both capital requirements are set at γ^c. Functions with a star superscript are evaluated at a market equilibrium for arbitrary capital requirements.

[8] Since we are assuming that the regulators set a common capital requirement for both countries, we do not consider the possibility for regulators to pick asymmetric capital requirements, even if this could increases joint surplus. However, as argued by Dell'Ariccia and Marquez (2006), coordinated regulation that imposes asymmetric capital requirements is likely to be politically challenging.

The loss function To go further, we need to impose some structure on the L function. For now, we assume $L_\gamma < 0$, $L_N \geq 0$, and $\Theta_{\gamma'} = \Theta_{N'} = 0$. A straightforward example is the one where the deadweight losses are proportional to the amount needed for bailing out the banks. That is $L = \lambda S$, with $\lambda > 0$, which is the specification we will use for our concrete examples below.

The first condition ($L_\gamma < 0$) is natural as it, for instance, applies if deadweight costs arise from bank losses in default states (which are reduced both in probability and size by tighter capital regulation). The second ($L_N \geq 0$) is natural too, as keeping γ constant, an increase in N scales up the banking sector, which is hardly consistent with a decrease in deadweight losses. The third restriction ($\Theta_{\gamma'} = \Theta_{N'} = 0$) implies that capital requirement in Foreign does not affect L in other ways than either directly, or indirectly through an effect on N^*. We are therefore ruling out some potentially relevant channels. However, these assumptions allow us to focus the analysis in two beneficial ways. First, as the loss function is symmetric, we can write

$$\frac{dL'^*}{d\gamma} = \frac{dN'^*}{d\gamma} L'_{N'},$$

that is, changes in capital requirements in Home only affect deadweight losses in Foreign through international capital flows of bank equity, which is precisely the focus of our analysis. So, we have:

$$\Pi'^c_\gamma = \frac{dN'^c}{d\gamma} \left(\underbrace{\text{NDP}'^c_{N'} - L'^c_{N'}}_{\text{shadow value of } N'} \right), \tag{9}$$

where $\frac{dN'^c}{d\gamma}$ is the change in the market equilibrium value of N following a marginal increase in γ evaluated at $\gamma, \gamma' = \gamma^c$.

The sign of the externality Under our set of assumptions, the sign of the externality is pinned down by the following proposition:

Proposition 3. *Assume $L_\gamma < 0$, $L_N \geq 0$, and $\Theta_{\gamma'} = \Theta_{N'} = 0$. Then, NDP$'^c_{N'} - L'^c_{N'} > 0$. Hence, Π'^c_γ has the same sign as $\frac{dN'^c}{d\gamma}$ and, therefore, $\Pi'^c_\gamma \gtreqless 0 \Leftrightarrow \gamma^c \gtreqless \hat{\gamma}(\gamma^c)$.*

In words, what Proposition 3 states is simply that the social shadow value of bank equity capital is positive at the collaborative optimum.[9] Therefore, any change in policy by a national regulator that siphons off bank capital from the other country constitutes a negative externality (and vice versa if the deviation generates outflows to the other country). From Theorem 1, we know that $\frac{dN'^c}{d\gamma} \Leftrightarrow \gamma \gtreqless \hat{\gamma}(\gamma')$, and the final result follows: The sign of the externality at the collaborative optimum depends on whether $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$. This is a condition in terms of endogenous objects, so it does not tells us which cases can actually occur and, if so, in which circumstances. We now study an example in which what outcome occurs can be expressed in terms of primitives of the model.

Example in closed form Consider a special case in which i) $\kappa \to \infty$ (so that the supply of bank equity capital is, in effect, perfectly inelastic);[10] ii) $L = \lambda S$, and iii) A follows a binary distribution.[11] This case allows us to obtain closed form solutions and show how deadweight losses, market power, and moral hazard affect the properties of the collaborative optimum, in particular how they affect whether $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$.

Proposition 4. *Assume $\kappa \to \infty$; $L = \lambda S$ with $\lambda > 0$, and $A \in \left\{0, \frac{1}{q}\right\}$, with $0 < q < 1$ and $\Pr(A = \frac{1}{q}) = q$. If $\gamma^c < 1$, then*

$$\lambda \gtreqless \frac{q - \alpha}{(1-q)\,\alpha} \Leftrightarrow \gamma^c \gtreqless \hat{\gamma}(\gamma^c) \Leftrightarrow \Pi'^c_{\gamma} \lesseqgtr 0.$$

To understand, proposition 4, first note that the shadow value of bank capital

[9]Even though mining is not a social cost, bankers need to be compensated for it. So, the positive shadow value simply reflects scarcity. To formally see why $\text{NDP}'^c_{N'} - L'^c_{N'} > 0$, suppose instead that the contrary holds. If, $\text{NDP}'^c_{N'} - L'^c_{N'} < 0$, then it must be the case that an increase in X', given γ^c, decreases welfare (since $X' = \gamma^c N'$). Now, keeping N' constant, an increase in γ^c has two effects. First, it mechanically decreases X', which would improve welfare as we have just established. Second, it reduces L', which is also beneficial to welfare. Hence, $\text{NDP}'^c_{N'} - L'^c_{N'} < 0$ cannot be true at the optimum as the policymaker could do better by increasing γ^c. Instead, at the collaborative optimum, $\text{NDP}'_{N'} - L'_{N'} > 0$. So raising the capital requirement still has two effects, but of opposite sign: there is still a reduction in L', which is good for welfare, but now the increase in bank capital scarcity, which reduces X', implies a welfare decreasing contraction of economic activity.

[10]This implies that $\xi^* \to (R'^*_{N'} + R^*_N)^{-1}$ and $\text{SP}^* \to 1$.

[11]We depart here from the smooth distribution assumption imposed elsewhere.

at the collaborative optimum ($\text{NDP}'^{c}_{N'} - L'^{c}_{N'}$) is equal to $(1-q)\,\lambda$. In line with Proposition 3, this is strictly positive. Hence, the sign of the externality is that of $\frac{dN'^{c}}{d\gamma}$. The proposition shows that there is a threshold that bisects the parameter space and pins down this sign. This threshold depends on the values for parameters λ, q, and α. We now consider the role of each of these parameters.

Deadweight losses: the role of λ The first order condition to the collaborative problem boils down to:[12]

$$\alpha\left(\frac{\omega/2}{\gamma^{c}}\right)^{\alpha-1} = \underbrace{\alpha\left(X^{c}\right)^{\alpha-1}}_{\text{MPK}} = 1 + \underbrace{(1-q)\,\lambda}_{\text{Marg. deadweight loss}}. \tag{10}$$

Collaborative regulators equate the social benefit lending (the marginal product of capital) to the social cost (one plus the marginal increase in deadweight losses the lending generates). The higher λ, the higher are deadweight losses at the margin and hence collaborative regulators desire a more constrained level of equilibrium lending, which translates to a higher γ^{c}. And if λ is high enough, we can have $\gamma^{c} > \hat{\gamma}(\gamma^{c})$, i.e. it is optimal to tighten regulation further than at which returns are maximised.

Now, as γ^{c} is monotonically increasing in λ, we can define a $\bar{\lambda}$ such that γ^{c} coincides with what the return maximising requirement: i.e $\gamma^{c}(\bar{\lambda}) = \hat{\gamma}(\gamma^{c}(\bar{\lambda}))$. We have $\bar{\lambda} = \frac{q-\alpha}{(1-q)\alpha}$ which corresponds to the threshold in the proposition.

Moral hazard and market power: the role of α and q We know that $\hat{\gamma}(\gamma^{c})$ is the capital requirement that maximises return on equity, given $\gamma' = \gamma^{c}$. Moreover, in this example, the return on bank equity is:

$$R = \frac{q}{N}\left(\frac{1}{q}\alpha\left(\frac{N}{\gamma}\right)^{\alpha} - (1-\gamma)\frac{N}{\gamma}\right)$$

[12] To see this, note that as $\kappa \to \infty$, the collaborative problem corresponds to that of a social planner in a closed economy with a banker's endowment of $\omega/2$:

$$\gamma^{c} \equiv \arg\max_{\gamma}\left(\frac{\omega/2}{\gamma}\right)^{\alpha} - \left(\frac{\omega/2}{\gamma}\right) - (1-q)\,\lambda\left((1-\gamma)\,\frac{\omega/2}{\gamma}\right).$$

From Lemma 2 and Theorem 1, we also know that $\hat{\gamma}(\gamma^c)$ is such that $R^c_\gamma = 0$. Hence, it is the case that:

$$\frac{\alpha}{q} \times \ \alpha \left(\frac{N^*}{\hat{\gamma}(\gamma^c)} \right)^{\alpha-1} = 1.$$

We can then define $\hat{X}(\gamma^c) \equiv \frac{N^*(\hat{\gamma}(\gamma^c),\gamma^c)}{\hat{\gamma}(\gamma^c)}$, which is the level of lending that maximises the Home return on equity given $\gamma' = \gamma^c$. This change of variable is useful because $\hat{X}$ actually does not depend on γ^c. Indeed, it is pinned down in closed form by α and q:

$$\frac{\alpha}{q} \times \ \alpha \hat{X}^{\alpha-1} = 1. \tag{11}$$

The upshot of this is that, to assess the role of α and q on whether $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$, we can directly assess how they affect whether $X^c \lesseqgtr \hat{X}$.

Let us start with q. In Equation (11), its presence can be linked to moral hazard induced by deposit insurance. Shareholders get nothing when the bank goes bust. The realised MPK in the good state is $\frac{\alpha X^{\alpha-1}}{q}$, and this is what matters for their revenue. So, the higher the q, the lower $\hat{X}$. On the other hand, the regulators also care about the downside, and even more so that it generates deadweight losses. So, the relevant MPK in Equation 10 is $\alpha \left(X^c\right)^{1-\alpha}$ and, instead, q appears on the marginal cost side in the term $(1-q)\,\lambda$. The higher the q, the less probable deadweight losses are, and the lower γ^c regulators will choose. Together, it is clear that the higher q the more likely $\hat{X} < X^c$, and vice versa.

Parameter α captures decreasing returns to lending, which both matters for the regulators and is relevant for return maximisation. However, return maximisation exploits market power. This is why we have a factor α^2 in equation (11) compared to a factor α for the collaborative regulators (10). So, the lower the alpha, the stronger the market power, the lower $\hat{X}$, and the more likely $\hat{X} < X^c$.

The role of ω The parameter ω does not appear in the condition of Proposition 4. Its absence is due to the assumed discrete distribution function (and Proposition 4 focusing on $\gamma^c < 1$), so that the probability of default is fixed at $1 - q$. In the general case, ceteris paribus, a higher ω allows the planner to achieve a lower probability of default (a higher q). This indicates a lower γ^c, which decreases the externality and can make it negative. We will confirm this in a numerical exercise in the next subsection.

4.3 The non-cooperative policy game

So far, we have focused on incentives to deviate from the collaborative optimum. If $\Pi_{\gamma}^{\prime c} > 0$, the incentive is to deviate downward, that is to undercut the other regulator, which suggests a race to the bottom (and similarly, a race to the top if $\Pi_{\gamma}^{\prime c} < 0$). To confirm such intuition, we now consider a policy game, where each regulator sets its capital requirement taking the other regulator's behaviour and the market no arbitrage condition as given.

Concretely, we look at the same environment as in the closed form example above (i.e., $\kappa \to \infty$, $L = \lambda S$), except that we go back to a continuous distribution function, so that we can study the role of ω and show that lower ω increases the externality and can make it positive. The Nash equilibrium of such a game cannot be solved for in closed form, so we do so numerically.

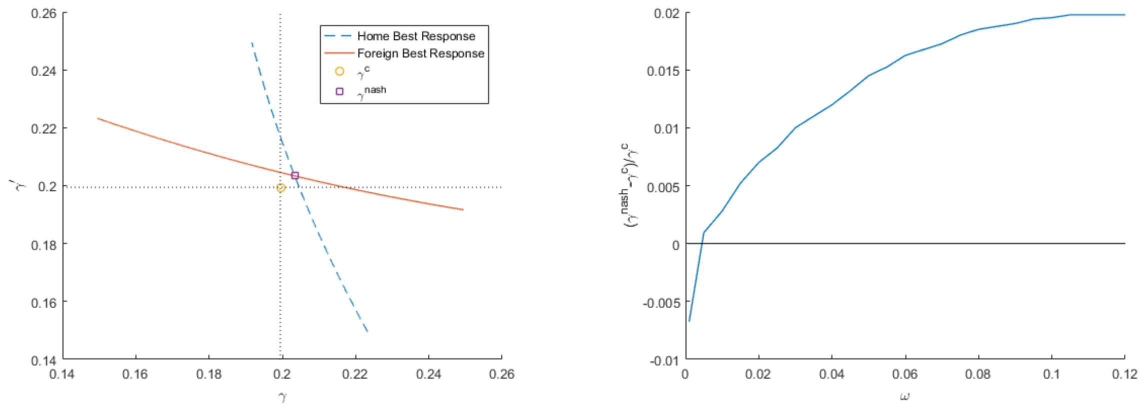
Numerical examples: collaborative optimum versus Nash equilibrium Figure 1 presents the numerical example.[13] The Nash equilibrium of the policy game is denoted γ^{nash}. The left panel presents a first example for a given value of ω. It displays γ^c and γ^{nash}, which is at the intersection of the best response curves. As we can see, for each regulator, the best response to γ^c would be to pick a higher capital requirement (which reflects the negative externality associated with siphoning off equity capital). As expected, we get $\gamma^{\text{nash}} > \gamma^c$; regulation is set at an overly tight level compared to collaboration.

The right panel illustrates that the reverse can happen if ω is low enough. The graph shows how the sign and magnitude of the difference between γ^{nash} and γ^c varies with ω. For high values of ω this difference is positive but when ω is relatively low it switches sign. As before, collaborative regulators set γ^c trading off

[13]Note that we use a log-normal distributions for A and A'. This is technically in violation of $A^L, A'^L = 0$ in Assumption 1; however, in the parameter space (including the range of capital requirements we look at) we consider banks always default with positive probability in equilibrium, which is what matters.

Figure 1: Policy choices under different levels of ω

Panel A: Best Response Curves ($\omega = 0.1$) Panel B: $\frac{\gamma^{\text{nash}} - \gamma^c}{\gamma^c}$ against ω



Notes - Parameter choices: $log(A) \sim N(0, \sigma^2)$, $\sigma^2 = 0.02$, $\lambda = 5$, $\alpha = 0.75$. Panel A: Best response curves for the Home (blue dashed line) and Foreign (red solid line) policymakers, γ^{nash} denoted by the purple square, γ^c denoted by the green circle ($\omega = 0.1$). Panel B: Plots γ^{nash} relative to γ^c as a function of ω.

that more lending boosts NDP but also generates deadweight losses. When ω is large, the regulator can achieve, with a relatively high γ^c, a combination of low marginal deadweight losses and high levels of lending, so that $\gamma^c < \hat{\gamma}(\gamma^c)$. This is what happens in the left panel, where raising the requirement imposes a negative externality and $\gamma^{\text{nash}} > \gamma^c$. However, when ω is relatively low, reflecting that bank equity capital is intrinsically quite scarce, high levels of lending are too costly in terms of deadweight losses. Accordingly, if ω is low enough, collaborative regulators pick a γ^c high enough that $\gamma^c > \hat{\gamma}(\gamma^c)$. In that case, competing regulators have an incentive to undercut one another and $\gamma^{\text{nash}} < \gamma^c$.

Remark. Here, κ is infinite, however, if it was finite, decreases in κ would generate similar effects to increases in ω, in the sense that these parameters have opposite effects on the intrinsic scarcity of bank equity capital.

4.4 Empirical predictions and policy implications

The above makes clear that the direction of the capital flows depends on the structure of the lending market and the state of the economy. The key condition is $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$. Both these objects are affected in non-trivial ways by the model parameters. So our empirical predictions are based on how parameters affect them on a relative basis. In particular, we can state that the following factors factors will raise γ^c relative to $\hat{\gamma}(\gamma^c)$, and hence increase the equity capital outflow from Home following a unilateral capital requirement increase: (i) Substantial market power in the banking sector or, more generally, high elasticity of loan demand (i.e. a high α in our closed-form example); (ii) Strong regulatory preferences for financial stability mandate and the avoidance of deadweight losses (i.e. a high λ, or more generally a high intensity of deadweight losses); (iii) Strong incentives to shift-risks or make one sided bets (i.e. a low q); (iv) Scarce bank capital (i.e. low ω or high κ), as would occur, for instance, following a crisis. The combination of these factors can be such that Home regulators have an incentive to deviate towards lower regulation than under collaboration.

This reasoning can be used to formulate implications for the setting of the countercyclical capital buffer (CCyB). This buffer is the headline macroprudential capital requirement and the Basel III accords mandate that signatories reciprocate changes in the CCyB in other jurisdictions. In normal times, when the banking sector is healthy (bank capital is relatively abundant, risk-shifting incentives are contained, etc.) we are more likely to be in a situation where national regulators have an incentive to deviate upward. Doing so would in fact attract capital from abroad an impose a negative externality onto other countries. So national regulators have an incentive to tighten requirements too much in good times. Hence, there may be gains on coordinating on looser regulation. Conversely, after a negative shock, when ω is low and risk-shifting incentives are more important, national regulators have incentives to cut the CCyB too aggressively.[14]

The Basel III accords also specify limits to the CCyB. In particular: (i) reciproc-

[14]It is not obvious that one may want to reduce capital requirements after a negative shock as risks and risk-shifting incentives are high. However, reductions may be optimal if the hit to bank equity capital more than offsets the increase in risk (Malherbe (2020)).

ation is only mandatory up to a 2.5% buffer, and (ii) the buffer cannot be set at a negative level. Interestingly, through the lens of our model, these limits could mitigate strategic behaviour. In bad times, the effective lower bound on the CCyB limits the scope for regulators to undercut one another. Whereas in good times, if a national regulator set the buffer above 2.5%, other regulators will not be obliged to reciprocate. Of course, whether 0% and 2.5% are appropriate bounds is an involved question; considerations range from what is the collaborative outcome in practice to how much discretion individual countries need to respond to asymmetric shocks. A full analysis of these is beyond the scope of this paper.

5 Discussion

5.1 Generalising the bank capital flow externality

We now put our results in a broader perspective by considering how regulatory incentives change in our model when the objective function is more general. A more general objective could be written as

$$\Pi(\gamma, \gamma') = \xi^W W + \xi^R (R-1) N - \xi^S S - L(\gamma, N, \Theta),$$

where ξ^W, ξ^R, and ξ^S are weakly positive Pareto weights. Without restrictions on the loss function, the generalised externality would then be:

$$\Pi_{\gamma}'^{c} = \frac{dN'^{c}}{d\gamma}\left(\xi^W W_{N'}'^{c} + \xi^R (R'^{c} + R_{N'}'^{c} N'^{c} - 1) - \xi^S S_{N'}'^{c}\right) - \left(L_{N'}'^{c}\frac{dN'^{c}}{d\gamma} + L_{\Theta'}'^{c}\Theta_{N}'^{c}\frac{dN^{c}}{d\gamma} + L_{\Theta'}'^{c}\Theta_{\gamma}'\right). \tag{12}$$

Using the definition of SP, we can rearrange Equation (12) as follows:

$$\Pi_{\gamma}'^{c} = \underbrace{\frac{dN'^{c}}{d\gamma}\left(\underbrace{\xi^W W_{N'}'^{c} + \xi^R (R'^{c} + R_{N'}'^{c} N'^{c} - 1) - \xi^S S_{N'}'^{c} - L_{N'}'^{c} + \mathrm{SP}^{c} L_{\Theta'}'^{c}\Theta_{N}'^{c}}_{\text{shadow value of bank capital}}\right)}_{\text{capital flow externality}} - L_{\Theta'}'^{c}\Theta_{\gamma}'^{c}. \tag{13}$$

Comparing the above to Equation (9) from Section 4,[15] three points are in order. First, generalising the objective function does not affect the existence of the capital flow externality: $\frac{dN'^c}{d\gamma}$ still multiplies a term that can be interpreted as the shadow value of bank equity capital.[16] Note also, that the sign of $\frac{dN'^c}{d\gamma}$ still hinges on whether $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$.

Second, as long as the shadow value of bank capital is positive at the collaborative optimum, the main logic of our analysis goes through: if R^c_γ is positive, an increase in γ generates a negative externality on Foreign because it siphons off capital from it. In the previous section, this shadow value is always positive at the collaborative optimum. This is an intuitive result and seems natural to the debate we are interested in. With arbitrary Pareto weights and loss functions, one cannot in principle rule out cases where the shadow value is negative at the collaborative optimum. In that case, bank equity capital becomes a sort of hot potato that regulators would prefer to pass onto their neighbours. The empirical relevance of such a case seems, however, limited.

Third, a more general loss function can also generate additional externalities. The term $L'^c_{\Theta'}\Theta'^c_\gamma$ in Equation 13 captures the direct affect of capital requirements at Home on deadweight losses in Foreign. Such additional externalities may either reinforce, mitigate, or even offset the bank equity capital flow externality (see below for examples of such externalities in previous literature).

Different objectives and alternative loss functions also imply different values of γ^c. So, this affects whether $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$. To fix ideas, it is useful to think of the stark example where the regulators just trade off shareholder income with financial stability (say with an objective function $\Pi = (R-1)N - \lambda S$). In that case, we will have $\gamma^c > \hat{\gamma}(\gamma^c)$. This is because, without financial stability costs, regulators would, by construction, pick $\gamma^c = \hat{\gamma}(\gamma^c)$. Hence, financial stability concerns make them choose a higher γ^c. Therefore, in such a case, raising γ always repels bank

[15]Our model corresponds to the case where $\xi^W, \xi^V, \xi^S = 1$ and $\Theta'_\gamma = \Theta'_N = 0$ in this generalised setup.

[16]In Equation (13) the shadow value has a slightly different interpretation as it includes the term $\mathrm{SP}^c L'^c_{\Theta'}\Theta'^c_N$. This captures how an outflow of capital from *Home* affects deadweight losses in Foreign (this is proportional to the inflow into Foreign). Hence, here the shadow value captures more than just how a marginal effect of an extra unit capital on the Foreign regulator's objective *ceteris paribus*; it also accounts for the fact the capital must come from somewhere.

equity capital, which is a positive externality onto Foreign, and results in a race to the bottom. Other specifications for the loss function, which could be a way to capture different sorts of additional externalities will also affect γ^c, with similar implications.

The main takeaway from this discussion is that the coincidence of the sign of the capital flow and its associated externality is a quite robust result. And so is the fact that the sign of the externality hinges on $\gamma^c \gtreqless \hat{\gamma}(\gamma^c)$. However, additional ingredients will affect which case is more likely, and can add other externalities that will affect Π'^c_γ independently (e.g., through $L'^c_{\Theta'}\Theta'^c_\gamma$ as explained above).

Remark. While our welfare function in Section 4 has equal Pareto weights, it does not, strictly speaking capture the case of a utilitarian regulator. First, our regulators do not internalise mining costs. This is justified if bank capital is not *per se* socially costly but since we modelled mining costs as a disutility of labour a utilitarian regulator would internalise them. Second, and more generally, our welfare function is defined in terms of net *domestic* income rather than net *national* income. Quantifying the latter requires specifying which taxpayer is liable for cross border bailouts. If the Home taxpayer is responsible for paying bailouts required by banks lending in Home, the shadow value of bank capital for a utilitarian regulator is given by $W'^c_{N'} + R'^c_{N'}\frac{\omega}{2} - S'^c_{N'} - L'^c_{N'} + \mathrm{SP}^c L'^c_{\Theta'}\Theta'^c_N$. The externality will still conform to the structure of equation (13), and the takeaways above also apply.

5.2 Is there empirical evidence for the sign of the externality?

When setting capital requirements, do regulators have incentives to undercut one another or, to the contrary, to engage in a race to the top? The mere fact that international standards are formulated in terms of minima suggests the former. However, such standards were initially set when requirements were not reciprocated. A formal empirical analysis of the new regime is beyond the scope of this paper (and, to our knowledge, such a study does not exist in the literature). The short time frame and relatively infrequent changes in the CCyB are significant hurdles.

However, one can still draw some inference on the sign of the externality arising from capital flows from the empirical literature on bank behaviour following capital requirement changes.

There is a substantial literature identifying the effect of capital requirements at the bank level by exploiting the heterogeneous impact of regulatory reforms, stress tests, or supervisory interventions (Francis and Osborne (2012); Bahaj et al. (2016); Gropp et al. (2019); Imbierowicz et al. (2018); Juelsrud and Wold (2020); De Jonghe et al. (2020)). The message from these papers is that banks facing an increase in capital requirement raise more equity. In some cases, the response is not significantly different than zero, but there is no evidence that banks reduce their levels of capital in response to higher requirements.

In our model, this bank-level relationship corresponds to $\frac{dn}{d\gamma} \geq 0$. Extrapolating at the aggregate level, this suggests $\frac{dN}{d\gamma} \geq 0$ and therefore conditions under which regulators have an incentive to deviate upwards.[17] Now, if $\frac{dN}{d\gamma} > 0$, unless bank equity capital is supplied perfectly elastically (which would corresponds to $\kappa = 0$ in our model), it must be that $\frac{dN'}{d\gamma} < 0$.

Consistent with this prediction, there is also bank-level evidence that tighter capital requirements at home causes domestic banks to cut lending abroad (see Aiyar et al. (2014); Forbes et al. (2017)), which would also correspond to $\frac{dn'}{d\gamma} < 0$ in our model. From a broader perspective, Buch and Goldberg (2017) provide a meta analysis showing that, in general, the tightening of prudential policies (including capital requirements) spillover to generate less lending abroad.

More empirical research is required to determine the direction of aggregate equity capital flows following sector-wide changes in requirements. However, the existing empirical evidence does suggest that the direction of equity capital flows is such that higher capital requirements at Home do generate on average a negative externality on Foreign.

[17] Interpreting bank-level estimates at the aggregate level may draw a biased picture. However, one would typically expect competitive pressures to result in a single banks raising proportionally less capital following an idiosyncratic requirement increase than the entire banking sector would do following a sector-wide increase.

5.3 Other externalities and other regulatory tools

The market share externality and other externalities As we mentioned in the introduction, a key mechanism in Dell'Ariccia and Marquez (2006) illustrates how regulators' strategic interactions fundamentally differ in a non reciprocal regime. In their model there is a representative bank based in each country, but both banks operate in both countries. Each bank has a fixed amount of equity capital and face the capital requirement imposed by their country of origin regulator. A key point is that a decrease in capital requirement by the Home regulator decreases the cost of capital for the Home-based bank, which gives it a competitive advantage in both markets and allows it to grab market shares from its Foreign competitor. This is a negative externality that naturally leads to a race to the bottom. Adopting a reciprocity regime kills such a market share externality.

Now, this is not the whole story in Dell'Ariccia and Marquez (2006). There is another subtle mechanism which reinforce the market share-externality. The probability that a bank defaults is affected by its monitoring activity. Increasing capital requirements effectively restricts competition and increases profitability, which improves banks' incentive to monitor. Because reduced competition raises profitability for all banks, an increase in capital requirements by one country improves monitoring incentives for banks in the other country. National regulators do not internalise such effect, which is what contributes to them setting requirements too low. This effect is not directly present in our model, but could be captured in reduced form by the term $L'^{c}_{\Theta'}\Theta'^{c}_{\gamma}$, in equation (13). For example if Θ' is the probability of default of Home banks with $\Theta'^{c}_{\gamma} < 0$, and $L'^{c}_{\Theta'} > 0$, the term $L'^{c}_{\Theta'}\Theta'^{c}_{\gamma}$ captures that a higher probability of default in Home increases expected deadweight losses in Foreign.

Firesale externalities (Kara (2016)) or contagion from cross-holdings (Niepmann and Schmidt-Eisenlohr (2013)) can also point to overly loose bank regulation compared to collaboration. These externalities can also be captured by the term $L'^{c}_{\Theta'}\Theta'^{c}_{\gamma}$. Finally, Haufler and Maier (2019) study another type of additional externality: if goods markets are integrated across countries regulators may have an incentive to deviate upwards. The logic follows from a form of terms-of-trade externality common in the international taxation literature (Devereux (1991)): policies that constrain a country's output, like capital requirements, impose a negative externality on trading partners leading to overly tight policy.

Other regulatory tools Our model focuses on capital requirements and abstracts from other regulatory tools. Previous literature has for instance looked at strategic interactions when regulators choose capital requirements as well as supervisory intensity (Buck and Schliephake (2013)) and at how unified capital requirements affects forbearance in resolution (Acharya (2003)).

In fact, there may already be strategic interactions among different types of capital requirements. In practice, the reciprocity framework is still incomplete. One could think of the CCyB as the marginal buffer and be tempted to conclude that if reciprocity applies to the marginal buffer, the whole system is essentially reciprocal. However, this is far from clear. National regulators have discretion over the definition of regulatory capital, over the calculation of riskweights, and bank-specific capital requirements. To give a specific example, Basel III includes buffers for systemically important banks. These buffers apply to specific banks, irrespective of where they lend, but national regulators have discretion with respect to the size of these buffers.

By combining the insights from Dell'Ariccia and Marquez (2006) and ours, one can speculate on how the average systemic bank buffer could interact with the CCyB. Imagine a regulator who wants to tighten capital requirements and considers a combination of these two tools. Assuming we are in normal times, the costs of raising the CCyB would be partially borne by foreign countries (our capital flow externality). In contrast, raising the systemic bank buffer would negatively affect the market shares of domestically incorporated systemic banks, and benefit other banks (including banks from different countries). A natural hypothesis is that such a regulator would rather raise the CCyB than raise the average systemic bank buffer. Such a prediction is of course speculative and a formal model is beyond the scope of the paper. However, this thought experiment serves to illustrate the complexity of the situation and stresses the need for further research in the area.

6 Conclusion

We have shown how the principle of reciprocity fundamentally affects strategic interactions between national regulators. In a non-reciprocal regime, regulators have an incentive to undercut one another's capital requirements to allow their own banks to steal market shares from international competitors. Reciprocity neutralises such incentives. The relevant strategic interaction becomes competition for scarce bank equity capital. Depending on economic conditions, a rise in a given country's capital requirement can generate capital flows of either sign. Outflows from that country are associated with a positive externality on other countries, inflows with a negative one. We argue that this capital flow externality is likely to make individual regulators set requirements too high (compared to full collaboration) in normal and good times and too low in bad times. Other forces can however mitigate or offset this externality.

Our results apply to reciprocal regimes in general. In the current regulatory environment, this corresponds best to the setting of Basel III's Counter-Cyclical Capital Buffer.

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A Proofs

Lemma. 1. *When banks default with strictly positive probability in equilibrium: (i) capital requirements are binding; and (ii) each individual bank perfectly specialises as either a lender in Home or Foreign.*

Proof. The bank defaults if

$$\alpha A X^{\alpha-1} x + \alpha A' \left(X'\right)^{\alpha-1} x' < x' + x - n - n',$$

where x and x' denotes lending and n and n' the capital invested in Home and Foreign respectively. This means we can define two functions that both represent the default boundary: points in the state space where the Bank just has sufficient resources to make depositors whole

$$\tilde{A}^0(A') = \frac{x' + x - n - n' - \alpha A' \left(X'\right)^{\alpha-1} x'}{\alpha X^{\alpha-1} x},$$

$$\tilde{A}'^0(A) = \frac{x' + x - n - n' - \alpha A X^{\alpha-1} x}{\alpha \left(X'\right)^{\alpha-1} x'}.$$

Given this, we can write the bank's problem as the following Lagrangian (ignoring non-negativity constraints, and noting that $\mathbb{E}[A] = \mathbb{E}[A'] = 1$)

$$\begin{aligned} &\max_{n,n',x,x'} \left(\alpha X^{\alpha-1} x + \alpha \left(X'\right)^{\alpha-1} x' - x' - x\right) + \\ \int_0^{\tilde{A}^0(A'^L)} \int_0^{\tilde{A}'^0(A)} &\left(x - x' - n - n' - \alpha A X^{\alpha-1} x - \alpha A' \left(X'\right)^{\alpha-1} x'\right) \tilde{g}(A, A') dA' dA + \\ &\psi\left(\frac{n}{\gamma} - x\right) + \psi'\left(\frac{n'}{\gamma'} - x'\right) - (n' + n)(z(N, N')), \end{aligned}$$

where $\tilde{g}(A, A')$ is the joint density of A and A'. The first row corresponds to the economic surplus generated by the bank's lending. The second row is strictly positive since the bank defaults with strictly positive probability (it would be zero otherwise) and captures the implicit subsidy to the bank arising the from the government's guarantee. The third row captures constraints arising from the capital requirements and the excess cost to the bank of the capital raised.

We first prove by contradiction that capital requirements must be binding in equilibrium. Imagine they are not. Then the bank can reduce n (or n') holding x and x' fixed. Since x and $x^{'}$ are fixed, revenue from lending is unaffected. However, the implicit subsidy is strictly decreasing in n (and n'): an increase in n both reduces the integrand and shifts the default boundary inwards. So, the deviation is profitable. Hence, capital requirements must bind in equilibrium.

We also prove the second part by contradiction. Consider a hypothetical equilibrium where banks do not perfectly specialise; that is, there is an interior solution at the bank level for both n and n'. Define the following function

$$\mathrm{rc}(A, \gamma) = \alpha A X^{\alpha-1} - (1-\gamma).$$

For a given A, the terms rc is the residual cash flows to the shareholders. It captures shareholder revenue, net of depositor repayment. We can write the return, for this bank, to one unit of equity being allocated to a loan in Home as:

$$r = \frac{\mathrm{rc}(1,\gamma)}{\gamma} - \int_0^{\tilde{A}^0(A'^L)} \int_0^{\tilde{A}'^0(A)} \left(\frac{\mathrm{rc}(A,\gamma)}{\gamma} \right) \tilde{g}(A, A') dA dA';$$

and similarly for a loan in Foreign.

If both n and n' are strictly positive in equilibrium, the bank must be indifferent at the margin. Then,

$$\begin{aligned} &\tfrac{\mathrm{rc}(1,\gamma)}{\gamma} - \textstyle\int_0^{\tilde{A}^0(A'^L)} \int_0^{\tilde{A}'^0(A)} \left(\tfrac{\mathrm{rc}(A,\gamma)}{\gamma} \right) \tilde{g}(A, A') dA' = \\ &\tfrac{\mathrm{rc}'(1,\gamma)}{\gamma} - \textstyle\int_0^{\tilde{A}^0(A'^L)} \int_0^{\tilde{A}'^0(A)} \left(\tfrac{\mathrm{rc}'(A,\gamma')}{\gamma'} \right) \tilde{g}(A, A') dA'. \end{aligned}$$

We now show that individual bank can profitably deviate by lending entirely in the Home country. To see this first note that the after-deviation revenues of such a bank is:

$$(n+n') \left(\frac{\mathrm{rc}(1,\gamma)}{\gamma} - \int_{A'}^{A^0} \left(\frac{\mathrm{rc}(A,\gamma)}{\gamma} \right) g(A) dA \right)$$

Making use of the fact that returns are equalised across countries allows us to compute the benefit from deviating as

$$(n+n')\left(\int_{A^L}^{\tilde{A}^0(A'^L)}\int_{A'^L}^{\tilde{A}'^0(A)}\left(\frac{\mathbf{rc}(A,\gamma)}{\gamma}\right)\tilde{g}(A,A')dA' - \int_{A^L}^{A^0}\left(\frac{\mathbf{rc}(A,\gamma)}{\gamma}\right)g(A)dA\right).$$

This benefit is strictly positive so long as there exists $A \in [A_L, A_0(A'_L)]$ such that $\mathrm{rc}(A,\gamma) > 0$, this is guaranteed by $g(A,A')$ having full support over $[A^L, A^H] \times [A'^L, A'^H]$. Since a profitable deviation exists an individual bank will never choose for both n and n' to be interior. Hence, the only possible equilibrium is one where individual banks perfectly specialise in each country and aggregate returns are equated by the aggregate number of banks participating in each country. □

Proposition. 1. *For all $\gamma, \gamma' \in (0,1)$, there exists a unique pair $\{N^*, N'^*\}$ such that returns on equity in both countries are equal to the bankers' required return. This pair is implicitly defined by:*

$$R(N^*,\gamma) = R'(N'^*,\gamma') = (1 + z(N^*, N'^*)).$$

Proof. To simplify notation, we omit function dependences on γ and γ'. First, let us establish that the double equality necessarily holds in equilibrium. If $R(N^*) > (1 + z(N^*, N'^*))$, the representative bank will scale up infinitely but, in equilibrium $n = N$ and $lim_{N\to\infty} R(N) = 0$. Likewise, If $R(N^*;\gamma) < (1 + z(N^*, N'^*))$ banks will choose $n = 0$, imposing $n = N$, $lim_{N\to 0} R(N^*;\gamma) = \infty$. Hence, in equilibrium we must have that $R(N^*) = (1 + z(N^*, N'^*)$. The same logic applies to $R'(N'^*)$

Remark. For what follows, when the steps are identical for Foreign, we only provide the proof for Home. Also, for the remainder of this proof, we omit stars to ease notation further.

It is straightforward that here is a single, interior N that solves $R(N) = 1 + z$. But z depends on the global demand for bank equity capital: $z = \kappa \times (N + N' - \omega)$. So, the question is whether there is a unique pair (N, N') that solves the system:

$$\begin{cases} R(N) = 1 + z(N, N') \\ R'(N') = 1 + z(N, N') \end{cases} \tag{14}$$

If the left-hand-side functions are invertible (we prove this later), we can write:

$$\begin{cases} N(z) = R^{-1}(1+z) \\ N'(z) = R'^{-1}(1+z) \end{cases}$$

From which, we can construct an implicit global demand function for bank capital: $N^D(z) \equiv N(z) + N'(z)$. Hence,

$$N^D(z) = R^{-1}(1+z) + R'^{-1}(1+z)$$

From $z = \kappa \times (N + N' - \omega)$, we have an explicit global supply function:

$$N^S(z) \equiv \frac{z}{\kappa} + \omega.$$

If $N^D(z)$ and $N^S(z)$ always cross in a single point, which pins down $N^* + N'^*$ and z^*. Then, unique corresponding values of N and N'^* arise from Equation (14). So, to complete the proof, we first show that (*i*) $R(N)$ and $R'(N')$ are invertible and, second, that (*ii*) $N^D(z)$ and $N^S(z)$ always cross only once.

(*i*) First note that from the smoothness of $g(A)$ we have that $R(N)$ is continuous in N. From the definition of R (Equation 3), we have:

$$R_N = \begin{cases} \frac{1}{\gamma^2}\alpha(\alpha-1)\left(\frac{N}{\gamma}\right)^{\alpha-2} & A^0(N) \leq A^L \\ \frac{1}{\gamma^2}\int_{A^0}^{A^H}\left(\alpha(\alpha-1)A\left(\frac{N}{\gamma}\right)^{\alpha-2}\right)g(A)dA & A^L < A^0(N) \leq A^H \\ 0 & A^H < A^0(N) \end{cases}.$$

The third case is irrelevant as it implies $R = 0$, which is ruled out in equilibrium (as shareholders would always lose 100% of their equity). Since $R_N < 0$ in the first two cases, $R(N)$ is invertible.

(*ii*) $N^S(z)$ is linearly increasing and $N^S(0) = \omega$. Given (*i*) we know that R^{-1} and R'^{-1} are decreasing in z. Hence, $N^D(z)$ is decreasing too. These ensure single crossing. $\square$

Lemma. 2. *We have* $\xi^* < 0$*, therefore in the case where* $z^* > 0$*, we have* $\frac{dN^*}{d\gamma} \gtreqless 0 \Leftrightarrow$

$R^*_\gamma \gtreqless 0$.

Proof. Start from the no-arbitrage condition:

$$R(N^*, \gamma) - R'(N'^*, \gamma') = 0.$$

Drop function dependencies and use stars to denote variables evaluated at the market equilibrium to get:

$$\frac{dN'^*}{d\gamma} = \frac{R^*_\gamma + R^*_N \frac{dN^*}{d\gamma}}{R'^*_{N'}}. \tag{15}$$

When $z^* > 0$, we also have:

$$R'\left(N'^*, \gamma'\right) - \kappa(N^* + N^{*\prime}) - 1 = 0.$$

Hence:

$$\frac{dN'^*}{d\gamma} = \frac{\kappa \frac{dN^*}{d\gamma}}{R'^*_{N'} - \kappa}. \tag{16}$$

Combining equations (15) and (16) yields

$$\frac{dN^*}{d\gamma} = -R^*_\gamma \left(\underbrace{\frac{\kappa - R'^*_{N'}}{\kappa \left(R'^*_{N'} + R^*_N\right) - R^*_N R'^*_{N'}}}_{\equiv \xi^*} \right).$$

We showed in the proof of Proposition 1 that $R'^*_{N'}, R^*_N < 0$. As, $\kappa \geq 0$, this means $\xi^* < 0$. Hence the sign of $\frac{dN^*}{d\gamma}$ is the same as the sign of R^*_γ. □

Proposition. 2. *If $N^* + N'^* < \omega$, then $z^* = 0$ and $\frac{dN^{*\prime}}{d\gamma} = 0$. In the case of interest where $N^* + N'^* > \omega$, then*

$$\frac{dN^{*\prime}}{d\gamma} = -\underbrace{\left(\frac{\kappa}{\kappa + (-R'^*_{N'})}\right)}_{\equiv SP^* \in [0,1)} \frac{dN^*}{d\gamma}.$$

Proof. The case $z^* = 0$ is straightforward. The case of interest directly follows from Equation (16) in the proof above, together with $R'^*_{N'} < 0$, which was shown in the proof of Proposition 1. □

Theorem. 1. $\forall\, \gamma' \in (0,1)$, *there exists a* $\widehat{\gamma}\,(\gamma') > 0$ *such that,* $\forall \gamma \in (0,1)$

$$\begin{cases} \frac{dN^*}{d\gamma} > 0 & ; \gamma < \widehat{\gamma}\,(\gamma') \\ \frac{dN^*}{d\gamma} = 0 & ; \gamma = \widehat{\gamma}\,(\gamma') \\ \frac{dN^*}{d\gamma} < 0 & ; \gamma > \widehat{\gamma}\,(\gamma') \end{cases}.$$

Proof. From Lemma 2 we know that the sign of $\frac{dN^*}{d\gamma}$ is the same as that of $R_\gamma\,(N^*, \gamma)$. Based on this, the proof proceeds in steps to show the following:

i) At low values of γ, $R_\gamma\,(N^*, \gamma)$ is positive.

ii) At low values of γ, $R_\gamma\,(N^*, \gamma)$ is strictly decreasing in γ.

iii) If $R_\gamma\,(N^*, \gamma)$ is nil, it is strictly decreasing in γ.

iv) If for some $\gamma < 1$, $R_\gamma\,(N^*, \gamma)$ is strictly negative, given iii) and the fact that $R_\gamma\,(N^*, \gamma)$ is continuous, it must be that $R_\gamma\,(N^*, \gamma) < 0$ for all value greater than such γ.

Points ii-iv together mean that there is at most one value for $\gamma \in (0,1)$ such that $R_\gamma\,(N^*, \gamma) = 0$. Given point i and Lemma 2, the theorem follows. To show these points formally, we start from Equation (3):

$$R\,(N;\gamma) = \frac{\left(\int_{A^0(N;\gamma)}^{A^H} \left(\alpha \left(\frac{N}{\gamma}\right)^{\alpha-1} - (1-\gamma)\right) g(A)dA\right)}{\gamma}.$$

Take the derivative with respect to γ to obtain

$$R_\gamma\,(N;\gamma) = \frac{-1}{\gamma^2} \int_{A^0(N;\gamma)}^{A^H} \left(\alpha^2 A \left(\frac{N}{\gamma}\right)^{\alpha-1} - 1\right) g\,(A)\,dA,$$

and note that $R_\gamma\,(N;\gamma)$ is continuous in γ for $\gamma \in (0,1)$.

Given N^*, the equilibrium level of lending X^* is pinned down by γ as follows: $X^*(\gamma) = N^*(\gamma)/\gamma$. We change variables accordingly to define:

$$H(X^*(\gamma), \gamma) \equiv R_\gamma\,(N;\gamma)\,|_{X^*(\gamma)}\,.$$

This object is the appropriate one to consider how the sign of R_γ depends on γ, *taking into account that N^* depends on γ.*

First, we have:

$$\lim_{\gamma \longrightarrow 0^+} H(X^*(\gamma), \gamma) = \frac{-1}{\gamma^2} \underbrace{\left(\lim_{\gamma \longrightarrow 0^+} \int_{A^0}^{A^H} \left(\alpha^2 A \left(X^* \right)^{\alpha - 1} - 1 \right) g\left(A \right) dA \right)}_{<0} > 0$$

To see why this limit is negative, consider X such that $\alpha \left(X \right)^{\alpha-1} = 1$. At such X, banks make strictly positive profits (lending just breaks even in expectation, but banks have no equity, so they only pick the upside, i.e., $A^0 > A^L$). This cannot be true in equilibrium. Instead, it must be the case that $\alpha \left(X^* \right)^{\alpha-1} < 1$. Since $\alpha < 1$, this is also true for $\alpha^2 \left(X^* \right)^{\alpha-1} < 1$. So, $H(X^*(\gamma), \gamma) > 0$ at low enough values of γ. This formalises step i) above.

Next, we can write the total derivative:

$$\frac{dH^*}{d\gamma} = H^*_\gamma + \frac{dX^*}{d\gamma} H^*_X$$

We have:

$$H^*_\gamma = -2\gamma^{-1} H^* + \underbrace{\left(-\gamma^{-2}\right) \underbrace{A^0_\gamma}_{<0} g(A_0) \underbrace{\left(\alpha^2 A^0 \left(X^*(\gamma) \right)^{\alpha-1} - 1 \right)}_{<0}}_{<0}. \tag{17}$$

That $\frac{\partial A_0}{\partial \gamma} < 0$ and $\left(\alpha^2 A^0 \left(X^*(\gamma) \right)^{\alpha-1} - 1 \right) < 0$ both follows from the definition of A^0.

$$\frac{dX^*}{d\gamma} H_{X^*} = \underbrace{\underbrace{\frac{dX^*}{d\gamma}}_{<0} \left(-\gamma^{-2}\right) \left[\underbrace{A^0_X}_{>0} g(A^0) \underbrace{\left(\alpha^2 A^0 \left(X^*(\gamma) \right)^{\alpha-1} - 1 \right)}_{<0} + \int_{A_0}^{A^H} \left(\underbrace{\alpha^2 A (\alpha - 1) \left(X^*(\gamma) \right)^{\alpha-2}}_{<0} \right) g\left(A \right) dA \right]}_{<0}. \tag{18}$$

That $A^0_X < 0$ follows from the definition of A^0, and $\frac{dX^*}{d\gamma} < 0$ simply reflects that an increase in γ raises banks cost of capital, which requires an increase in marginal return to lending in equilibrium (formally proving it requires a change in variable and additional notation; we omit it here but do it for the proof of Proposition 3 below, where we also deal with the kink that z^* exhibits at $N^* + N'^* = \omega$).

Now equations (17) and (18) mean that if $H^* > 0$, all the terms in $\frac{dH^*}{d\gamma}$ are negative. Hence, $R_\gamma(N^*;\gamma)$ is decreasing with γ when γ is sufficiently low that $H^* > 0$. This formalises step ii) above.

If $H^* = 0$, then $\frac{dH^*}{d\gamma} < 0$. Hence, if $R_\gamma(N^*;\gamma)$ is nil, it is still strictly decreasing in γ. This formalises step iii) above.

If $H^* < 0$ then $\frac{dH^*}{d\gamma}$ can have either sign. However, since the derivative is strictly negative at $H^* = 0$, increasing the capital requirement can never cause H^* to switch sign if $H^* < 0$. This completes step iv) above.

Steps i-iv) together imply the following: If there exists a $\gamma < 1$ such that $H(\gamma) = 0$, such γ is unique and, therefore, defines $\hat{\gamma}$. If $H > 0$ for all admissible $\gamma \in (0,1)$, then we can simply set $\hat{\gamma} = 1$. □

Proposition. 3. *Assume $L_\gamma < 0$, $L_N \geq 0$, and $\Theta_{\gamma'} = \Theta_{N'} = 0$. Then, $NDP'^c_{N'} - L'^c_{N'} > 0$. Hence, Π'^c_γ has the same sign as $\frac{dN'^c}{d\gamma}$ and, therefore, $\Pi'^c_\gamma \gtreqless 0 \Leftrightarrow \gamma^c \gtreqless \hat{\gamma}(\gamma^c)$.*

Proof. We start from Equation (9):

$$\Pi'^c_\gamma = \frac{dN'^c}{d\gamma}\left(\mathrm{NDP}'^c_{N'} - L'^c_{N'}\right).$$

Given Theorem 1, and the fact that countries are symmetric, what we need to show is $\mathrm{NDP}^c_N - L^c_N > 0$.

Step 1: Given that the two countries are symmetric and collaborative regulators are maximising with respect to a common capital requirement we can state their problem as

$$\gamma^c \equiv \arg\max_\gamma \Pi(\gamma) \equiv \mathrm{NDP}\left(N^*(\gamma,\gamma),\gamma\right) - L\left(N^*(\gamma,\gamma),\gamma\right),$$

where $N^*(\gamma,\gamma)$ denotes the capital market equilibrium value for N given capital requirements in both countries equate γ. Using a change of variable, we can rewrite the problem as

$$\gamma^c \equiv \arg\max_\gamma \Pi(\gamma) \equiv \widetilde{\mathrm{NDP}}\left(X^*(\gamma,\gamma),\gamma\right) - \widetilde{L}\left(X^*(\gamma,\gamma),\gamma\right), \tag{19}$$

where functions with tildes reflect the change of variable $X^* = \gamma N^*$.

We then have the partial derivatives:

$$\widetilde{\text{NDP}}_X \left(X^*(\gamma,\gamma),\gamma\right) = \gamma \text{NDP}_N \left(N^*(\gamma,\gamma),\gamma\right)$$

$$\widetilde{L}_X \left(X^*(\gamma,\gamma),\gamma\right) = \gamma L_N \left(N^*(\gamma,\gamma),\gamma\right)$$

and it is sufficient to show that $\widetilde{\text{NDP}}_X^c - \widetilde{L}_X^c > 0$.

Since, by definition, at a given X, NDP is not affected by γ, the first order condition associated with (19) is

$$\widetilde{\text{NDP}}_X^c \frac{dX^c}{d\gamma} - \widetilde{L}_X^c \frac{dX^c}{d\gamma} - \widetilde{L}_X = 0.$$

So

$$\frac{dX^c}{d\gamma}\left(\widetilde{\text{NDP}}_X^c - \widetilde{L}_X^c\right) = \underbrace{\widetilde{L}_\gamma}_{<0},$$

and, if $\frac{dX^c}{d\gamma} < 0$, it directly follows that $\widetilde{\text{NDP}}_X^c - \widetilde{L}_X^c > 0$.

Step 2 shows that $\frac{dX^*}{d\gamma} < 0$ (which implies $\frac{dX^c}{d\gamma} < 0$). In any equilibrium, we have

$$\tilde{R}(X^*,\gamma) - (1 + \widetilde{z}\,(X^*, X'^*, \gamma)) = 0,$$

where $\widetilde{z}\,(X^*, X'^*, \gamma)$ also denotes the z function with the relevant change of variables (including $X'^* = \gamma' N'^*$).

So if $z^* > 0$,

$$\frac{dX^*}{d\gamma} = -\frac{\tilde{R}_\gamma^* - \widetilde{z}_\gamma^*}{\tilde{R}_X^* - (1 - \text{SP}^*)\frac{\kappa}{\gamma}}.$$

By construction $\widetilde{z}_\gamma \geq 0$. Since $\text{SP}^* \in [0,1]$, we have $(1 - \text{SP}^*)\frac{\kappa}{\gamma} \geq 0$. Because of the diminishing marginal product of physical capital, we have $\tilde{R}_X < 0$. Finally, even though R_γ can be positive (as we show in Section 3), $\tilde{R}_\gamma$ cannot: an increase in γ keeping X constant can only decrease the return on bank capital, as this decreases the value of the implicit subsidy from the government guarantee without altering the gross revenues through changes in competition. Hence, when $z^* > 0$, we have $\frac{dX^*}{d\gamma} < 0$, and therefore $\frac{dX^c}{d\gamma} < 0$. If $z^* = 0$, then $\frac{dX^*}{d\gamma} = -\frac{\tilde{R}^*_\gamma}{\tilde{R}^*_X}$ and the same result follows directly, except at the point where $N^* + N'^* = \omega$, where $\widetilde{z}(X^*, \gamma)$ exhibit a kink in X^*, which implies that $\frac{dX^*}{d\gamma}$ is not defined. However, the result extends in terms of the sub-gradient, which concludes the proof. $\square$

Proposition. 4. *Assume* $\kappa \to \infty$*;* $L = \lambda S$ *with* $\lambda > 0$*, and* $A \in \left\{0, \frac{1}{q}\right\}$*, with* $0 < q < 1$ *and* $\Pr(A = \frac{1}{q}) = q$*. If* $\gamma^c < 1$*, then*

$$\lambda \gtreqless \frac{q - \alpha}{(1-q)\,\alpha} \Leftrightarrow \gamma^c \gtreqless \hat{\gamma}(\gamma^c) \Leftrightarrow \Pi'^c_\gamma \lesseqgtr 0.$$

Proof. With symmetric countries, the collaborative problem corresponds to that of a social planner in a closed economy with a banker's endowment of $\omega/2$. As $\kappa \to \infty$, the equity allocated to lending equals the endowment. Hence the problem boils down to:

$$\gamma^c \equiv \arg\max_{\gamma \in (0,1]} \left(\frac{\omega/2}{\gamma}\right)^\alpha - \left(\frac{\omega/2}{\gamma}\right) - (1-q)\,\lambda\,(1-\gamma)\,\frac{\omega/2}{\gamma}.$$

Focusing on an interior solution such that $\gamma^c < 1$, the first order condition is:

$$\alpha \left(\frac{\omega/2}{\gamma^c}\right)^{\alpha-1} = \alpha\,(X^c)^{\alpha-1} = 1 + (1-q)\,\lambda. \tag{20}$$

The return on equity under the assumptions in the example is given by:

$$R = \frac{q}{N}\left(\frac{1}{q}\alpha\left(\frac{N}{\gamma}\right)^\alpha - (1-\gamma)\frac{N}{\gamma}\right).$$

Since $\hat{\gamma}(\gamma^c)$ is such that $R^c_\gamma = 0$ (Lemma 2 and Theorem 1), it is given by:

$$\frac{\alpha}{q} \times \ \alpha\left(\frac{N}{\hat{\gamma}(\gamma^c)}\right)^{\alpha-1} = 1. \tag{21}$$

Combining (20) and (21) gives:

$$\lambda \gtreqless \frac{q - \alpha}{(1-q)\,\alpha} \Leftrightarrow \gamma^c \gtreqless \hat{\gamma}(\gamma^c)$$

and the second equivalence follows from Theorem 1. $\square$