Unilateral Practices, Antitrust Enforcement and Commitments

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January 31th 2022

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Abstract

This paper analyses the impact of commitments on antitrust enforcement. These tools, introduced in Europe by the Modernization reform of 2003, are now used intensively by the European Commission and by National Competition Agencies. We consider a setting where a firm can adopt a practice that is either pro- or anti-competitive; the firm knows the nature of the practice whereas the enforcer has only prior beliefs about it. If the firm adopts the practice, the enforcer then decides whether to open a case. When commitments are available, the firm can offer a commitment whenever a case is opened; the enforcer then decides whether to accept it or run a costly investigation that may or may not bring supporting evidence. We show that introducing commitments weakens enforcement when the practice is likely to be anti-competitive. The impact of commitments is however more nuanced when the practice is less likely to be anti-competitive.

Keywords: Antitrust enforcement, commitment, remedies, deterrence

1 Introduction

The modernization reform in 2003 has introduced new tools for the enforcement of European competition law. In particular, firms under investigation can now offer remedies intended to address the potential competitive concerns; if accepted by the European Commission, these remedies become binding commitments, but the case is closed and there is no fine or finding of infringement. [1] Since the adoption of regulation 1/2003,

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[1] Art. 9.1 of Regulation 1/2003 states: "Where the Commission intends to adopt a decision requiring that an infringement be brought to an end and the undertakings concerned offer commitments to meet

the European Commission – and national competition authorities, under similar rules – have increasingly relied on commitments, [2] raising a debate on the opportunity of such a widespread adoption. Critics fear that this tool may weaken deterrence and encourages firms to attempt dubious practices, by providing them with an escape way that, in case of an investigation, enables them to avoid paying any fine. Advocates claim instead that this instrument may shorten the duration of antitrust cases and deliver a fast and certain result, to be compared with the long, costly and uncertain process of running a full-scale investigation exposed to judicial review.

In this paper we develop a model where a firm must decide whether to undertake a profitable action which can be of either pro- or anti-competitive. The firm knows the nature of the action, whereas the enforcer only has prior beliefs about it. If the firm undertakes the action, the enforcer must then decide whether to open a case. Initially, the only instrument in the enforcer's toolbox is to run an in-depth investigation. Doing so is costly (for both the enforcer and the firm) but, when the action is anti-competitive, with positive probability the enforcer obtains evidence of infringement enabling it to adopt a decision forcing the firm to abandon the practice and to pay a fine.

We then introduce the possibility of commitments. We allows for any level of commitment, and assume they reduce the resulting impact of the practice on the enforcer's welfare criterion as well as on the firm's profit. In line with the procedure adopted by the European regulation, we let the firm have the initiative in offering the commitment. That is, when a case is opened, the firm can offer any commitment, in which case the enforcer then decides whether to accept the offered commitment and close the case, or run a full-scale investigation.[3]

When commitments are not available, the enforcer runs an investigation with certainty when it has very pessimistic priors, and with positive but decreasing probability when its beliefs become less pessimistic. When instead commitments are available, two classes of equilibria may arise. For sufficiently pessimistic priors, there exists a semi-separating equilibrium in which a commitment is offered (with some probability) only when the action undertaken is harmful, and the commitment is then accepted by the enforcer, who otherwise runs an investigation. For less optimistic priors, there exist pooling equilibria where the firm adopts the same strategy regardless of the type of action. One such equilibrium involves no commitments and simply replicates the outcome of the benchmark

the concerns expressed to them by the Commission in its preliminary assessment, the Commission may by decision make those commitments binding on the undertakings. Such a decision may be adopted for a specified period and shall conclude that there are no longer grounds for action by the Commission."

[2]According to Mariniello (2014), in the period 2004-2013 out of 47 decisions of the European Commission on Article 102 cases, 27 of them where closed with commitments. Japan has introduced a similar commitment procedure in 2018. See also Gautier and Petit (2018), p.213-6.

[3]This gives rise to a signalling game, where the informed party chooses the offered commitment. Choné *et al.* (2014) and Gautier and Petit (2018) consider instead a screening setting in which the enforcer has the initiative in proposing the commitment.

case. In the more interesting equilibrium, the firm offers the minimal commitment that the enforcer is ready to accept given its priors.

We show that the impact of commitments on the effectiveness of enforcement can be decomposed in terms of a *deterrence* effect reflecting the firm's incentive to adopt the action, regardless of its type, and a *screening* effect capturing the differential incentive to adopt a good or bad practice. The comparison of the two policy regimes, without and with commitments, shows that introducing commitments indeed undermines enforcement and reduces expected welfare when the practice is very likely to be socially harmful. The comparison is instead more nuanced for practices that are a priori less socially damaging.

Although commitments are nowadays a very common tool, there is surprisingly little theoretical analysis assessing their impact on antitrust enforcement. To the best of our knowledge, only two papers have provided a formal analysis. Choné *et al.* (2014) study a binary setting that restricts attention to all or nothing commitments. Compared with a full-fledged investigation, the use of commitments allows a faster termination of the infringement but weakens deterrence, as the firm avoids paying any fine. They study the solution to this trade-off when the enforcer can announce ex ante its policy, so as to influence firm behavior, and show that this optimal policy may not be credible if the agency can deviate ex post from the announced policy. Gautier and Petit (2018) focus on ex post enforcement in a setting in which the enforce can choose between arbitrary levels of commitments and full-fledged investigations. They show that commitments can be used to discriminate firms according to the social harm of their practices. They also note that this use of commitments can weaken deterrence.

We contribute to this literature by studying the implications of ex post enforcement on ex ante deterrence in a setting in which commitments, when available, are offered by the firms, as is the case in the formal procedure established in art. 9.1 of Regulation 1/2003. This gives rise to a signalling game, where the informed party chooses the offered commitment. Compared with the binary setting considered by Choné *et al.*, allowing for arbitrary levels of commitments tends to limit the enforcer's ability to discriminate different types of practices, from an ex post perspective, and further undermines deterrence from an ex ante perspective. As a result, introducing commitments may not be desirable. [4]

The paper is organized as follows: section 2 presents the model, section 3 considers the benchmark case without commitments, whereas Section 4 analyses the case where commitments are introduced. Section 5 compares the two policy regimes. Conclusions

[4] For informal policy discussions, see, e.g., Wils (2006,2008) and Mariniello (2014). The issues raised by commitments are also related to the literature on settlements, initiated by Shavell (1982) with a focus on litigation costs, and further developed by Bebchuk (1984) and Reinganum and Wilde (1986) by accounting for asymmetric information, and by Polinsky and Rubinfeld (1988) by studying the implications for ex-ante deterrence – for a comprehensive survey, see, e.g., Daughety and Reinganum (2011).

follow. All the proofs are in the Appendix.

2 The model

We present here the setting, which captures the key relevant ingredients of the antitrust enforcer's problem: the enforcer has initially limited evidence about a potential case, and must decide whether to open it. If it does so, then after a preliminary phase, the enforcer may either close the case, or proceed with an investigation, which is costly but may bring decisive evidence. Alternatively, the firm may offer a commitment if allowed; if accepted, it becomes compulsory and the case is closed with no sanctions nor guilty verdict.

Players, information and payoffs. The *enforcer* implements the antitrust policy and the *firm* can undertake an *action* (business practice) that has uncertain private and social impacts. Specifically, the action yields gross profits normalized to 1 and costs c, which is distributed according to a c.d.f. $F(c)$ over $[0, +\infty)$, with atomless density $f(c)$ and decreasing hazard rate $h(\cdot) = f(\cdot)/F(\cdot)$. With probability $\lambda \in (0, 1)$, the action is good[5] ($\theta = G$), in which case welfare increases by $W_G = W > 0$. Otherwise, the action is bad ($\theta = B$) and produces a social loss $W_L = -L < 0$. The welfare impact of the action, W_θ, is independent of the private cost c. The firm observes the action type, θ, and its cost, c, whereas the enforcer only knows their *ex ante* probability distributions, λ and $F(\cdot)$. In what follows, the type θ will refer interchangeably to the action or the firm.

Policy tools. According to the existing regulation, the enforcer can *open a case* based on a preliminary assessment that the action has been undertaken and of its likely effects. After a first evaluation, the enforcer can decide to either *close the case*, with no impact on profits and welfare, or proceed with *an investigation*. This costs $k > 0$, with sk to the firm and $(1 - s)\,k$ to the enforcer (where $s \in [0, 1]$), but enables the enforcer to obtain hard evidence with probability ρ when the action is bad (i.e., $\theta = B$), in which case it bans the practice and imposes a *sanction* S; if the action is good (i.e., $\theta = G$), the enforcer obtains no evidence and must therefore close the case.[6] We assume that $S \leq \bar{S} \equiv (1 - sk - \rho)/\rho$, implying that the firm always undertakes the action when it is costless. Alternatively, the firm can *offer a commitment* $C \in (0, 1]$, which, if accepted, limits profit and welfare proportionally and in the same manner: the profits of the firm is reduced to $1 - C$ and the welfare impact becomes $(1 - C)\,W_\theta$. We interpret the regulation as requiring the enforcer to accept the offered commitment whenever it yields an expected

[5] An alternative interpretation refers to a population of firms with heterogeneous costs and actions that may have a positive or negative impact on social welfare. $F(c)$ and λ then refer to the distribution of types.

[6] The enforcer may thus commit a type-II error (acquitting a firm despite a bad action), but no type-I error (convicting a firm for a good action).

welfare matching that of the best alternative (namely, opening an investigation or closing the case).[7]

Remark (constructive refusal). For ease of exposition, we will sometimes refer to offering no commitment as "offering $C = 0$". Note, however, that the enforcer then remains free to open an investigation or close the case, and/or to randomize between these two options.

Remark (discounting). We do not explicitly consider the time dimension of the enforcement process. Profits and welfare can be interpreted as the discounted stream of the corresponding payoffs. The sanction S, that occurs in a later stage of the process, should be interpreted as the pecuniary fine minus the profits gained in the meanwhile.

Timing. The timing of the game is as follows:

Stage 0: Nature draws the type $\theta \in \{G, B\}$ and the cost $c \in [0, 1]$, which are privately observed by the firm; the firm then decides whether to undertake the action. If the firm does not undertake the action, the game is over.

Stage 1: If an action is undertaken, the enforcer decides whether to open a case. If it does not, the game is over; otherwise, the firm then offers a commitment $C \in (0, 1]$ (with the convention that offering $C = 0$ is interpreted as offering no commitment).

Stage 2: Having observed the commitment C, the enforcer then chooses between the following policy options: accepting the commitment (if $C > 0$, see above remark), proceeding to an investigation, or closing the case.

Equilibrium. This setting corresponds to a *signalling game* between an informed player (the firm) and an uninformed player (the enforcer), in which the informed player moves first. We will look for the *Perfect Bayesian Equilibria* of this game with *pessimistic beliefs*: in case of an unexpected (i.e., out-of-equilibrium) move by the firm, we assume that the enforcer expects the action to be bad with probability 1.[8] If multiple equilibria exist that are Pareto ordered from the point of view of the two types of firm, we shall select the Pareto efficient one.

[7] That is, we do not allow the regulator to randomize over its decision when indifferent between accepting or not a commitment. Allowing for such randomization could generate additional semi-separating equilibria, which however are Pareto-dominated from the firms' standpoints; it could also generate additional pooling equilibria, which could not be Pareto-ranked without introducing additional structure on payoff functions.

[8] Pessimistic beliefs ensure the existence of an equilibrium.

3 Benchmark: no commitments

We start by studying the performance of "classic" policy intervention when commitments are not available. In this case, the last two stages of the above timing boil down to:

Stage 1 **(no commitments):** If an action is undertaken, the enforcer then chooses whether to proceed to an investigation or to close the case.[9]

If in stage 0 the firm undertakes the action $\theta = G, B$ with probability P_θ, then at the beginning of stage 1, upon observing that an action has indeed been undertaken, the enforcer updates its beliefs to:

$$\lambda_1 = \frac{\lambda P_G}{\lambda P_G + (1-\lambda) P_B}. \tag{1}$$

The enforcer then closes the case with some probability γ and opens an investigation with complementary probability $\iota = 1 - \gamma$. If the enforcer decides to close the case ($\gamma = 1$, superscript "c"), expected welfare is equal to:

$$W^c(\lambda_1) \equiv \lambda_1 W - (1-\lambda_1) L. \tag{2}$$

If instead the enforcer proceeds with a full investigation ($\iota = 1$, superscript "i") expected welfare is given by:

$$W^i(\lambda_1) \equiv \lambda_1 W - (1-\lambda_1)(1-\rho)L - k = W^c(\lambda_1) + (1-\lambda_1)\rho L - k. \tag{3}$$

Both functions are increasing in λ_1, with W^c being steeper than W^i, and satisfy $W^c(0) < W^i(0)$.[10] Compared to closing the case, running an investigation affects welfare in two ways. The term $(1-\lambda_1)\rho L$ corresponds to the improvement from terminating a bad action; it is the product of the posterior probability that the action is indeed socially harmful, the probability of obtaining hard evidence and the (avoided) social loss. The last term $(-k)$ denotes the total cost of an investigation. We maintain the following assumption:

Assumption 1: $k < \rho L$

[9] Formally, closing a case is then equivalent to not opening it; for the sake of exposition (in particular, for comparability with the case where commitments are allowed), we assume in what follows that the enforcer always opens a case; the relevant choice then boils down to closing the case or proceeding with an investigation.

[10] The relationships between the two curves is as follows. When $\lambda_1 = 0$, we have $0 > W^i = -(1-\rho)L - k > W^c = -L$ if $k < \rho L$ as assumed. Moreover, $0 < \frac{\partial W^i}{\partial \lambda_1} = W + (1-\rho)L < \frac{\partial W^c}{\partial \lambda_1} = W + L$. Indeed, when the enforcer becomes marginally more optimistic, i.e. $\lambda_1 \uparrow$, if he closes the case the expected welfare increases because the welfare gain W (welfare loss L) becomes more (less) likely. When, instead, the enforcer runs an investigation the welfare loss occurs only when the signal is uninformative, an event that realizes with probability $1-\rho$. Consequently, the avoided loss under more optimistic beliefs is $(1-\rho)L$.

This assumption ensures that the investigation costs are not so high to prevent any activity of the enforcer. Indeed the enforcer would not investigate if the action were known to be good ($\lambda_1 = 1$), but would do so if the action were known to be bad ($\lambda_1 = 0$). Hence, there exists a threshold probability, namely:

$$\lambda^{ci} \equiv 1 - \frac{k}{\rho L} \quad (\in (0,1)), \tag{4}$$

such that the enforcer is indifferent between closing the case and running an investigation when $\lambda_1 = \lambda^{ci}$, as the superscript suggests.

Two relevant cases can then be distinguished for the optimal enforcement policy. If $\lambda_1 \geq \lambda^{ci}$, then $W^c(\lambda_1) \geq W^i(\lambda_1)$ and the enforcer closes the case. The firm then always undertakes the action since by assumption $c \leq 1$,[11] regardless of the type of action $\theta = G, B$. It follows that $\lambda_1 = \lambda$. We conclude that when $\lambda \geq \lambda^{ci}$ the enforcer always closes the case and there is maximal and equal participation of both types, implying that the ex-ante and ex-post probabilities of the good action are the same.

If instead $\lambda_1 < \lambda^{ci}$, the action is sufficiently likely to be harmful to require to open an investigation. As we are going to analyze, in this case the participation is lower, the good action is undertaken with a higher probability than the bad actions and therefore the ex-post probability of a good action is larger than the ex-ante one. Indeed, the posterior λ_1 depends on the prior λ and on the probability that the firm undertakes the good and bad actions when it expects the enforcer to investigate. To disentangle this relationship, let us start by defining the cost of investigation for the good and bad type as

$$\overline{C}_G \equiv sk < \overline{C}_B \equiv sk + \rho(1+S). \tag{5}$$

This corresponds to the cost of standing up in a case and, for the bad type, also includes the expected lost profits and fine. Then, if the enforcer runs an investigation with probability $\iota \in [0,1]$, a firm of type $\theta = G, B$ undertakes the action if $c \leq 1 - \iota\overline{C}_\theta$; let

$$P_\theta(\iota) \equiv F(1 - \iota\overline{C}_\theta) \tag{6}$$

denote the resulting "participation" probability of that firm. Running an investigation discourages the firm from undertaking the action: $P_\theta(\iota)$ is decreasing in ι. As $S < \overline{S}$, this deterrence effect remains limited, but is larger for bad actions: $P_G(\iota) > P_B(\iota) > 0$. The enforcer's updated beliefs if the action is undertaken, given the probability of investigation $\iota \in [0,1]$, satisfy:

$$\lambda_1 = \lambda_1^e(\lambda, \iota) \equiv \frac{\lambda P_G(\iota)}{\lambda P_G(\iota) + (1-\lambda)P_B(\iota)} \geq \lambda \tag{7}$$

[11] As the cost c is distributed with smooth density, the decision of the firm in the particular case where $c = 1$ does not materially affect the analysis.

where the inequality stems from $P_G(\iota) \geq P_B(\iota)$. Indeed, if an investigation occurs with positive probability ($\iota > 0$), observing that the firm undertook the action makes the enforcer more optimistic, as the firm is more likely to do so when the action is good. Furthermore, as the deterrence effect is more pronounced for bad types, the posterior $\lambda_1^e(\lambda, \iota)$ increases with ι.[12]

As the posterior also increases with λ, it follows that, for $\lambda \leq \lambda^N$, where the superscript N stands for "No commitment", the enforcer always investigates (as $\lambda_1 = \lambda_1^e(\lambda, \iota) \leq \lambda_1^e(\lambda, 1) < \lambda^{ci}$), where $\lambda^N \in \left(0, \lambda^{ci}\right)$ is the unique solution to

$$\lambda^N \in \left(0, \lambda^{ci}\right) \quad \text{is the unique solution to} \quad \lambda_1^e(\lambda^N, 1) = \lambda^{ci}, \tag{8}$$

Conversely, for $\lambda \in (\lambda^N, \lambda^{ci})$, the enforcer must randomize: if it were to investigate with probability 1, the posterior $\lambda_1 = \lambda_1^e(\lambda, 1) > \lambda_1^e(\lambda^N, 1) = \lambda^{ci}$ would be too optimistic to warrant intervention; if instead it were to close the case with probability 1, the posterior $\lambda_1 = \lambda_1^e(\lambda, 0) = \lambda < \lambda^{ci}$ would be too pessimistic and trigger intervention. The equilibrium probability of investigation must therefore induce the enforcer to be indifferent between investigating or not; that is:

$$\iota = \tilde{\iota}(\lambda) \quad \text{is the unique solution to} \quad \lambda_1^e(\lambda, \iota) = \lambda^{ci}. \tag{9}$$

Building on this yields:

Proposition 1 (benchmark: no commitments) *Suppose that commitments are not allowed. There exists a unique equilibrium, $\mathcal{E}^N$, in which the enforcer opens an investigation with probability:*

$$\iota^N(\lambda) \equiv \begin{cases} 1 & \text{if } \lambda \in (0, \lambda^N] \\ \tilde{\iota}(\lambda) \in (0,1) & \text{if } \lambda \in (\lambda^N, \lambda^{ci}) \\ 0 & \text{if } \lambda \in [\lambda^{ci}, 1) \end{cases} \tag{10}$$

where $\tilde{\iota}(\lambda)$ decreases continuously from 1 for $\lambda = \lambda^N$ to 0 for $\lambda = \lambda^{ci}$, giving a firm of type θ an expected payoff equal to

$$\Pi_\theta^N(\lambda) = 1 - \iota^N(\lambda)\overline{C}_\theta. \tag{11}$$

Hence, the optimal policy $\iota^N(\lambda)$ requires maximal enforcement if $\lambda \leq \lambda^N$, limited enforcement if $\lambda \in (\lambda^N, \lambda^{ci})$ and no enforcement if $\lambda \geq \lambda^{ci}$. The enforcer is therefore less likely to intervene when it is more optimistic, which in turn encourages both types of firm to undertake the action.

[12] See the proof of Proposition 1 for a formal derivation.

Corollary 1 (participation in the absence of commitments) *The firm undertakes an action of type* θ *with probability* $P_\theta^N(\lambda) \equiv F(\Pi_\theta^N(\lambda))$*, which is continuous in* λ *and equal to* $F(1)$ *for* $\lambda \geq \lambda^{ci}$*. For* $\lambda < \lambda^{ci}$*, participation is strictly increasing in* λ *and higher for the good action:* $P_G^N(\lambda) > P_B^N(\lambda)$*. Expected participation, given by*

$$P^N(\lambda) \equiv \lambda P_G^N(\lambda) + (1-\lambda) P_B^N(\lambda), \tag{12}$$

is thus also increasing in λ *for* $\lambda \in (0, \lambda^{ci}]$.

Enforcement reduces the participation of both types of firm through the litigation costs sk, but has a positive effect on the composition of active firms, as bad types are also deterred through the expected sanction and lost profits $\rho(1+S)$. This composition effect increases with the probability of investigation, which in turn depends on the enforcer's prior beliefs. As a result, it is maximal for $\lambda \leq \lambda^N$, where the enforcer investigates for sure, decreases in λ in the range $(\lambda^N, \lambda^{ci})$, and vanishes for $\lambda \geq \lambda^{ci}$, where the enforcer stops investigating.

The expected welfare in the different regions according to the equilibrium probability of investigation $\iota^N(\lambda)$ is:

$$EW^N(\lambda) = \begin{cases} \lambda P_G^N(\lambda) \left[W - \iota^N(\lambda)k\right], & \\ -(1-\lambda)P_B^N(\lambda) \left[(1-\iota^N(\lambda)\rho)L + \iota^N(\lambda)k\right] & \text{if } \lambda \in (0, \lambda^{ci}), \\ \lambda W - (1-\lambda)L & \text{if } \lambda \in \left[\lambda^{ci}, 1\right). \end{cases}$$

The following corollary provides a useful expression of expected welfare for $\lambda \in (0, \lambda^{ci})$.

Corollary 2 (expected welfare) *The expected welfare for* $\lambda \in (0, \lambda^{ci})$ *can be expressed as*

$$EW^N(\lambda) = P^N(\lambda) W^i(\lambda_1^N(\lambda)) \tag{13}$$

where

$$\lambda_1^N(\lambda) = \lambda_1^e(\lambda, \iota^N(\lambda)) = \begin{cases} \lambda \frac{P_G^N(\lambda)}{P^N(\lambda)} \in (\lambda, \lambda^{ci}) & \text{if } \lambda \in (0, \lambda^N), \\ \lambda^{ci} & \text{if } \lambda \in \left[\lambda^N, \lambda^{ci}\right). \end{cases} \tag{14}$$

In particular, expected welfare is decreasing in λ *for* $\lambda \in \left[\lambda^N, \lambda^{ci}\right)$.

As welfare is normalized to zero when the action is not undertaken, expected welfare can be expressed as the probability of observing an action, measured by the participation $P^N(\lambda)$, multiplied by the resulting expected welfare, given the posterior belief $\lambda_1^N(\lambda)$. From Proposition 1, for $\lambda \leq \lambda^{ci}$ the enforcer either investigates for sure (if $\lambda \leq \lambda^N$), or it is indifferent between investigating or not (if $\lambda \in (\lambda^N, \lambda^{ci})$). In both cases, the resulting expected welfare is given by $W^i(\lambda_1^N(\lambda))$.

Corollary 2 points out that, in the range $\lambda \in (\lambda^N, \lambda^{ci})$, expected welfare *decreases* as the enforcer becomes more optimistic. In this range, the enforcer is too optimistic for investigating with certainty; its posterior belief thus remains equal to λ^{ci} (to ensure indifference between investigating or not), and the probability of investigation $\iota^N(\lambda)$ moreover decreases as the enforcer becomes more optimistic (to maintain the posterior $\lambda_1^e(\lambda, \iota^N(\lambda))$ to λ^{ci}). It follows that participation increases, and that each action yields an updated expected welfare equal to $W^i(\lambda^{ci}) < 0$.

Remark: too much or little enforcement? As just noted, enforcement deters good actions, as well as bad ones, from being undertaken. Depending on which force prevails, the level of enforcement may be excessive or insufficient. Indeed, taking as given the investigation probability ι, and taking into account the firm's response, the expected welfare as a function of ι can be expressed as:

$$EW(\iota) \equiv \lambda P_G(\iota)(W - \iota k) - (1-\lambda) P_B(\iota)\left[(1-\iota\rho)L + \iota k\right],$$

and so its derivative with respect to ι is given by:

$$\begin{aligned} \frac{dEW(\iota)}{d\iota} &= \lambda P_G'(\iota)(W - \iota k) - (1-\lambda) P_B'(\iota)\left[(1-\iota\rho)L + \iota k\right] \\ &\quad + (\rho L - k)(1-\lambda) P_B(\iota) - \lambda P_G(\iota) k, \end{aligned}$$

When $\lambda \in (\lambda^N, \lambda^{ci})$, the equilibrium probability $\iota = \tilde{\iota}(\lambda)$ is such that the posterior $\lambda_1^e(\lambda, P_B, P_G)$ coincides with λ^{ci}, which in turn implies that the terms in the second line cancel out. Using $P_\theta'(\iota) = -\overline{C}_\theta f_\theta(1 - \iota\overline{C}_\theta)$, it follows that slightly intensifying the enforcement activity (i.e., increasing ι slightly above $\tilde{\iota}(\lambda)$) would increase expected welfare whenever:

$$\frac{\lambda}{1-\lambda} < \frac{\overline{C}_B}{\overline{C}_G} \frac{f(1-\iota\overline{C}_B)}{f(1-\iota\overline{C}_G)} \frac{L - \tilde{\iota}(\rho L - k)}{W - \tilde{\iota}k}.$$

When instead this condition is not satisfied, there is over-enforcement: expected welfare would increase if the enforcer could commit itself to slightly reduce the frequency of investigations.

4 Commitments

We now revert to the general setting, where commitments are available, moving from stage 2 backwards.

4.1 The enforcer's response.

Let denote by $\lambda_2(C)$ the enforcer's updated belief at the beginning of stage 2, given the observed commitment C, and by $\nu_\theta(C)$, the equilibrium probability that, in stage 1, a firm of type θ offers a commitment C.[13] Upon observing a commitment C that is indeed offered with positive probability, the enforcer updates its beliefs to:

$$\lambda_2(C) = \frac{\lambda_1 \nu_G(C)}{\lambda_1 \nu_G(C) + (1-\lambda_1)\nu_B(C)}, \tag{15}$$

where, as in the previous Section, λ_1 is the interim belief at the beginning of stage 1 once the enforcer observes the action. If instead the enforcer observes a commitment C that should not be offered in equilibrium (i.e., $\nu_G(C) = \nu_B(C) = 0$), under pessimistic beliefs the posterior is $\lambda_2(C) = 0$.

Once observed the offered commitment C the enforcer can close the case with probability γ, investigate with probability ι or accept the commitment with probability $\alpha = 1 - \gamma - \iota$.

The expression of expected welfare at stage 2 is given by (2) if the enforcer closes the case, and by (3) if the enforcer proceeds with a full investigation, with the caveat that these must be evaluated with the posterior $\lambda_2 = \lambda_2(C)$. These two policy options yield a positive expected welfare only when the enforcer is sufficiently optimistic:

$$W^c(\lambda_2) > 0 \Longleftrightarrow \lambda_2 > \lambda^c \equiv \frac{L}{W+L}, \tag{16}$$

$$W^i(\lambda_2) > 0 \Longleftrightarrow \lambda_2 > \lambda^i \equiv \frac{(1-\rho)L + k}{(1-\rho)L + W}. \tag{17}$$

Finally, if the enforcer accepts the commitment ($\alpha = 1$), expected welfare[14] is given by:

$$W^a(\lambda_2, C) \equiv (1-C)W^c(\lambda_2). \tag{18}$$

In what follows, we assume that investigations are sufficiently costly that the enforcer contemplates opening one only if expected welfare would otherwise be sufficiently negative; that is:[15]

Assumption 2: $k > \underline{k} \equiv (1-\lambda^c)\rho L$.

[13] Once it has been sunk in stage 0, the cost c reduces profit and welfare but has no incidence on subsequent decisions; in what follows, we will thus concentrate on the expressions of profit and welfare that are gross of the cost c, and will refer to θ as the type of the firm.

[14] Hence, $W^a(\lambda_2, C)$ rotates clockwise with respect to $W^c(\lambda_2)$ around $\lambda_2 = \lambda^c$ when C increases.

[15] Closing the case yields $W^c(\lambda)$, whereas running an in-depth investigation yields $W^c(\lambda) + (1-\lambda)\rho L - k$. Assumption 2 ensures that the former option strictly dominates for any $\lambda \geq \lambda^c$.

Under this assumption the ranking of the three thresholds is:

$$\lambda^{ci} < \lambda^c < \lambda^i.$$

The following Lemma characterizes the enforcer's optimal response, in stage 2, to the offered commitment $C \in (0, 1]$:

Lemma 1 (enforcer's response) *If the firm offers no commitment, thus inducing a posterior belief* $\lambda_2(0)$*, the enforcer closes the case if* $\lambda_2(0) > \lambda^{ci}$*, investigates if* $\lambda_2(0) < \lambda^{ci}$*, and is indifferent between the two policy options if* $\lambda_2(0) = \lambda^{ci}$*. If instead the firm offers a positive commitment* $C \in (0, 1]$*, inducing a posterior belief* $\lambda_2(C)$*, then:*

- *if* $\lambda_2(C) > \lambda^c$*, the enforcer closes the case;*
- *if* $\lambda_2(C) \in [\lambda^{ci}, \lambda^c]$*, the enforcer accepts the commitment;*
- *if* $\lambda_2(C) < \lambda^{ci}$*, the enforcer accepts the commitment if* $C \geq \underline{C}(\lambda_2)$*, where*

$$\underline{C}(\lambda_2) \equiv \frac{\rho\lambda^c(\lambda^{ci} - \lambda_2)}{\lambda^c - \lambda_2}, \tag{19}$$

and otherwise opens an investigation.

The intuition is straightforward. If $\lambda_2 > \lambda^c$, the enforcer closes the case, as this dominates opening an investigation (because $\lambda^c > \lambda^{ci}$) as well as accepting any commitment (because expected welfare is positive). If instead $\lambda_2 \leq \lambda^c$, the enforcer prefers accepting any commitment to closing the case (as expected welfare is non-positive). As long as $\lambda_2 \geq \lambda^{ci}$, however, opening an investigation is even less desirable than closing the case, and so the enforcer always accepts the offered commitment.

By contrast, if $\lambda_2 < \lambda^{ci}$, then the enforcer strictly prefers opening an investigation to closing the case, and will therefore accept a commitment only if it is large enough, namely if it satisfies $(1 - C)W^c(\lambda_2) \geq W^i(\lambda_2)$. This determines the minimal acceptable commitment $\underline{C}(\lambda_2)$ given by (19), which is decreasing and concave for $\lambda_2 < \lambda^{ci}$ and such that $\lim_{\lambda_2 \to \lambda^{ci}} \underline{C}(\lambda_2) = 0$.[16]

Figure 1 shows the enforcer's optimal response and the relevant loci for $\lambda_2 \in [0, 1]$.

Remark: on the efficiency of commitments. In a *complete information environment*, commitments would be an efficient way of dealing with bad actions: when the action is known to be bad (i.e., $\lambda = 0$), the minimal commitment that the enforcer is willing to

[16] We have $\frac{\partial \underline{C}}{\partial \lambda_2} = \frac{\rho\lambda^c(\lambda^{ci} - \lambda^c)}{(\lambda^c - \lambda_2)^2} < 0$ and $\frac{\partial^2 \underline{C}}{\partial \lambda_2} = \frac{\rho\lambda^c(\lambda^{ci} - \lambda^c)}{(\lambda^c - \lambda_2)^3} < 0$ for $\lambda_2 \leq \lambda^{ci}$.

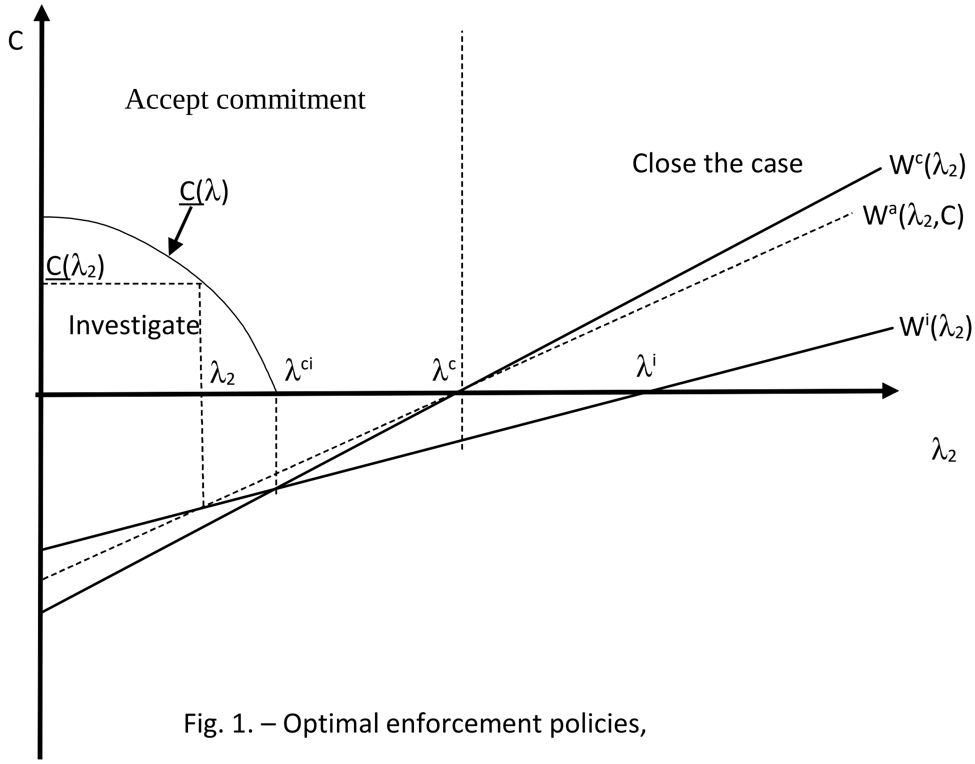

Fig. 1. – Optimal enforcement policies,

accept, $\underline{C}(0) = \rho - k/L$, is lower than the maximal commitment that the firm is willing to offer, $\overline{C}_B = \rho + sk + \rho S$. Hence, there is room for a mutually beneficial agreement. As we will see, *asymmetric information* substantially limits the efficiency of commitments.

Before proceeding, we note that, as in the absence of commitments (see Proposition 1), there is scope for enforcement if and only if $\lambda < \lambda^{ci}$. Specifically, letting Π_θ^C denote type θ's equilibrium profit when commitments are available, we have:

Proposition 2 (scope for enforcement)

(i) If $\lambda \geq \lambda^{ci}$, then there is a unique Pareto efficient equilibrium outcome, in which the enforcer never investigates nor accepts a positive commitment; as a result, the firm obtains its maximal payoff: $\Pi_B^C = \Pi_G^C = 1$.

(ii) If instead $\lambda < \lambda^{ci}$, then any equilibrium is such that, with positive probability the enforcer investigates or accepts a positive commitment; as a result, the firm does not obtain its maximal payoff, all the more so if the action is bad: $\Pi_B^C \leq \Pi_G^C < 1$.

In what follows we focus on the case $\lambda < \lambda^{ci}$, where there is indeed room for enforcement; we first characterize the Pareto-efficient equilibria (from firms' standpoint), before comparing them and drawing the policy implications.

4.2 Pareto efficient equilibria

We now move backward to the first two stages of the game. If in stage 0 the firm undertakes the action $\theta = G, B$ with probability P_θ, then at the beginning of stage 1, upon observing that an action has been undertaken, the enforcer updates its beliefs to λ_1, given by (1). The firm then chooses the set $\mathcal{C}_\theta$ of commitments to offer, and the probability $\nu_\theta(C)$ of offering any $C \in \mathcal{C}_\theta$, taking into account that the enforcer will then revise further its beliefs to $\lambda_2(C)$, given by (15) if $C \in \mathcal{C}_G \cup \mathcal{C}_B$ and by $\lambda_2(C) = 0$ otherwise (pessimistic beliefs), and will respond with the policy characterized by Lemma 1.

The following lemma restricts the set of candidate equilibria.

Lemma 2 (candidate equilibria) *If $\lambda < \lambda^{ci}$, in equilibrium either $\mathcal{C}_G = \mathcal{C}_B$ or $\mathcal{C}_G \subset \mathcal{C}_B$.*

Lemma 2 rules out the possibility that G reveals itself at stage 1 – in particular, it rules out fully separating equilibria. Indeed, if G were to reveal its type, the enforcer would then close the case, which would induce B to imitate. We are thus left with *pooling* equilibria, in which the two types use the same support, and *semi-separating* equilibria, in which G's support is strictly included in B's.[17]

4.2.1 Semi-separating equilibria

We first consider the semi-separating equilibria in which, in stage 1, the two types of firm may offer different commitments (i.e., $\mathcal{C}_G \neq \mathcal{C}_B$); from Lemma 2, it follows that, with positive probability B reveals itself at stage 1 (i.e., $\mathcal{C}_G \subset \mathcal{C}_B$). Furthermore, conditional on doing so, the best strategy for B is to offer the minimal commitment that the enforcer is then willing to accept, $\underline{C}(0)$. Hence, we can restrict attention to semi-separating equilibria in which $\mathcal{C}_B \setminus \mathcal{C}_G = \{\underline{C}(0)\}$ (i.e., B offers $\underline{C}(0)$ with positive probability and is the only one to do so) and at least one other commitment, C, is offered by both types. Note that, by construction, B's expected payoff is therefore equal to $1 - \underline{C}(0)$.

If in equilibrium the enforcer were offered another positive commitment $C > 0$, from Lemma 1 its response would be to close the case, investigate or accept C with probability one; but then, B could not be indifferent between offering C and $\underline{C}(0)$, a contradiction. Hence, $\mathcal{C}_B \cap \mathcal{C}_G = \{0\}$, that is, the good type offers no commitment, whereas the bad type randomizes between offering $\underline{C}(0)$ or no commitment. Furthermore, offering no commitment must induce the enforcer to randomize as well between closing the case or opening an investigation, otherwise B would not be indifferent between offering $\underline{C}(0)$

[17] Pooling and semi-separating equilibria usually refer to equilibria in which $\mathcal{C}_G \cap \mathcal{C}_B$ is a singleton; as we will see, the Pareto-efficient equilibria do have this property.

and no commitment. Hence, offering no commitment must induce $\lambda_2(0) = \lambda^{ci}$, so as to leave the enforcer indifferent between closing the case and opening an investigation; conversely, the enforcer must then investigate with probability (where the superscript S refers to semi-separating equilibria)

$$\iota^S = \frac{\underline{C}(0)}{\overline{C}_B} \tag{20}$$

so as to leave B indifferent between offering $\underline{C}(0)$ and no commitment. In this equilibrium, an action of type $\theta = B, G$ thus yields an expected payoff equal to $1 - \iota^S \overline{C}_\theta$. It follows that, in stage 0, the firm is less likely to undertake the action when it is bad: the participation rates are given by:

$$P_\theta^S \equiv P_\theta\left(\iota^S\right), \tag{21}$$

where $P_\theta(\cdot)$ is defined by (6), and $\overline{C}_B > \overline{C}_G$ implies $P_B^S < P_G^S$.

A necessary feature of this equilibrium is that offering no commitment induces a posterior $\lambda_2(0) = \lambda^{ci}$, so as to leave the enforcer indifferent between closing the case and opening an investigation. Given the above participation rates (which do not depend on the prior λ or on B's choice between offering $\underline{C}(0)$ or 0), the enforcer's posterior increases with the enforcer's prior, λ, and decreases with B's probability of offering no commitment, $\nu_B(0;\lambda)$. Hence, as the prior λ increases, $\nu_B(0)$ must increase as well. As the probability $\nu_B(0)$ cannot exceed 1, the condition $\lambda_2(0) = \lambda^{ci}$ imposes an upper bound on the prior λ. This upper bound, which we will denote by λ^S, is such that, in the limit case where $\nu_B(0) = 1$ – which corresponds to a pooling equilibrium where no commitment would be offered–, the posterior $\lambda_2(0) = \lambda_1$ given by (1) coincides with λ^{ci}; using (7) and (21), this amounts to:

$$\lambda_1^e(\lambda^S, \iota^S) = \lambda^{ci}, \tag{22}$$

which yields:

$$\lambda = \lambda^S \equiv \frac{\lambda^{ci} P_B^S}{\lambda^{ci} P_B^S + (1 - \lambda^{ci}) P_G^S} \quad (< \lambda^{ci}). \tag{23}$$

The following proposition shows that, conversely, as long as $\lambda < \lambda^S$ there exists a semi-separating equilibrium, which is moreover unique.

Proposition 3 (semi-separating equilibrium) *If $\lambda \in [\lambda^S, \lambda^{ci})$, there is no semi-separating equilibrium. If instead $\lambda < \lambda^S$, there exists a unique semi-separating equilibrium, $\mathcal{E}^S$, in which:*

- *G offers no commitment (i.e., $C = 0$), whereas B randomizes between offering $\underline{C}(0)$ and no commitment, in such a way that $\lambda_2(0) = \lambda^{ci}$;*

- *the enforcer accepts $\underline{C}(0)$ and, when offered no commitment, investigates with probability $\iota^S = \underline{C}(0)/\overline{C}_B$.*

In this equilibrium, for any $\lambda < \lambda^S$, the expected payoffs of the firm are given by

$$\Pi_G^S = 1 - \underline{C}(0)\overline{C}_G/\overline{C}_B > \Pi_B^S = 1 - \underline{C}(0) > 0. \tag{24}$$

The following corollary characterizes the expected welfare generated by the semi-separating equilibrium:

Corollary 3 (expected welfare) *If $\lambda < \lambda^S$, the unique semi-separating equilibrium yields participation rates given by (21) and an expected welfare given by:*

$$EW^S(\lambda) = P^S(\lambda)W^i(\lambda_1^S(\lambda)) \tag{25}$$

where

$$P^S(\lambda) = \lambda P_G^S + (1-\lambda)P_B^S \quad \textit{and} \quad \lambda_1^S(\lambda) = \frac{\lambda P_G^S}{P^S(\lambda)} \in (0, \lambda^{ci}). \tag{26}$$

As before, expected welfare can be expressed as the probability of observing an action, measured here by the participation $P^S(\lambda)$, multiplied by the resulting expected welfare, given the posterior belief $\lambda_1^S(\lambda)$. From Proposition 3, in the semi-separating equilibrium either B reveals itself and offers $\underline{C}(0)$, or both types offer no commitment, in such a way that the enforcer is always indifferent between investigating or not. Hence, the resulting expected welfare is given by $W^i(.)$.

4.2.2 Pooling equilibria

We now turn to the equilibria in which, in stage 1, both types of firm offer the same commitments (i.e., $\mathcal{C}_G = \mathcal{C}_B$). We first note that, if the enforcer is sufficiently optimistic, namely, for $\lambda \geq \lambda^S$, the equilibrium $\mathcal{E}^N$ characterized by Proposition 1 survives when commitments become available. To see why, recall first that, as λ increases to λ^S, the semi-separating equilibrium $\mathcal{E}^S$ from the previous section is such that B is indifferent between offering $\underline{C}(0)$ or no commitment, and chooses the latter with increasing probability $\nu_B(0,\lambda)$. B's indifference, is in turn ensured by the enforcer investigating with probability $\iota^S \equiv \underline{C}(0)/\overline{C}_B$ when no commitment is offered**.** As λ tends to λ^S, the equilibrium converges towards a pooling equilibrium where both types offer no commitment (i.e. $\nu_B(0,\lambda^S) = 1$) and the posterior $\lambda_2(0) = \lambda_1$ coincides with λ^{ci}. It follows that $\iota^S = \tilde{\iota}(\lambda^S)$, where $\tilde{\iota}(\cdot)$ is the enforcer's probability of investigation in the $\mathcal{E}^N$ equilibrium with no commitments, which is precisely designed to induce an interim belief λ_1 equal to

λ^{ci} by properly affecting the participation rates, as (9) shows. Conversely, firms' offering no commitment, together with the enforcer's investigating with probability $\tilde{\imath}(\lambda^S)$ and the participation rates given by (21), does constitute an equilibrium when commitments are available, as B is then indifferent between deviating, by offering the minimal acceptable commitment $\underline{C}(0)$, or not (because $\tilde{\imath}(\lambda^S) = \underline{C}(0)/\overline{C}_B$), and G thus strictly prefers to offer no commitment. In other words, for $\lambda = \lambda^S$, $\mathcal{E}^N$ remains an equilibrium (with appropriately expanded strategies) when commitments become available, as no type has an incentive to deviate by offering a positive commitment; as $\iota^N(\lambda) = \tilde{\imath}(\lambda)$ is decreasing in λ, $\mathcal{E}^N$ remains an equilibrium for $\lambda > \lambda^S$, where deviations become even less attractive.

However, another pooling equilibrium also exists, in which the firm offers the minimal acceptable commitment $\underline{C}(\lambda)$ and thus obtains $1 - \underline{C}(\lambda)$. This equilibrium exists as long as G (which has less to lose from an investigation) is not tempted to deviate and offer no commitment (or an unacceptable one) and be investigated; this is the case as long as $\underline{C}(\lambda) \leq \overline{C}_G$, or $\lambda \geq \underline{\lambda}(\overline{C}_G)$, where

$$\underline{\lambda}(C) \equiv \max\left\{0, \underline{C}^{-1}(C) = \frac{\rho\lambda^{ci} - C}{\rho\lambda^c - C}\lambda^c\right\} \in \left[0, \lambda^{ci}\right]. \tag{27}$$

denotes the lower value of the prior λ for which the enforcer is willing to accept a given commitment C.

The next proposition shows that these two pooling equilibria are the only (potentially) Pareto-efficient ones:

Proposition 4 (Pareto efficient pooling equilibrium)

(i) If $\lambda \in [\lambda^S, \lambda^{ci})$, the equilibrium $\mathcal{E}^N$ arising in the absence of commitment constitutes a pooling equilibrium.[18]

(ii) If $\lambda \in \left[\underline{\lambda}(\overline{C}_G), \lambda^{ci}\right)$ there is also a continuum of pooling equilibria in which the firm offers a commitment $C \in [\underline{C}(\lambda), \min\{\underline{C}(0), \overline{C}_G\}]$, which is accepted; among them, the Pareto-efficient equilibrium is $\mathcal{E}^P$, in which the firm offers the minimum acceptable commitment $\underline{C}(\lambda)$ and obtains $\Pi^P(\lambda) \equiv 1 - \underline{C}(\lambda)$.

By construction, the equilibrium $\mathcal{E}^N$ yields an expected welfare equal to $EW^N(\lambda)$, characterized by Corollary 2. The following corollary characterizes instead the expected welfare generated by the second pooling equilibrium $\mathcal{E}^P$:

[18] We slightly abuse notation here: the equilibrium strategies are indeed the same as in the absence of commitments, but they now survive a richer set of potential deviations.

Corollary 4 *The pooling equilibrium* $\mathcal{E}^P$ *yields an expected welfare given by:*

$$EW^P(\lambda) \equiv P^P(\lambda) W^i(\lambda), \tag{28}$$

where

$$P^P(\lambda) \equiv F(1 - \underline{C}(\lambda)). \tag{29}$$

The expected welfare can thus again be expressed as the probability of observing the action, multiplied by the expected welfare generated by an investigation. Furthermore, as both types of firm obtain the same profit in this pooling equilibrium, there is no updating and the expected welfare from an investigation is thus evaluated at the prior λ.

5 Comparison of policy regimes

We now study the desirability of adding commitments to the enforcer's toolkit. When a practice is likely to be welfare improving, namely, for $\lambda \geq \lambda^{ci}$, the enforcer never intervenes, regardless of whether commitments are allowed or not. When instead the practice is likely to be harmful (i.e., $\lambda < \lambda^{ci}$), there is scope for enforcement, with or without commitments. When they are not allowed, the equilibrium $\mathcal{E}^N$ entails an investigation with probability $\iota^N(\lambda)$ and yields expected welfare $EW^N(\lambda) = P^N(\lambda) W^i\left(\lambda_1^N(\lambda)\right)$. When instead commitments are allowed, three types of equilibria may emerge. First, a pooling equilibrium may replicate the same outcome as in the absence of commitments – in which case commitments thus have no impact. Alternatively, the semi-separating equilibrium $\mathcal{E}^S$, which exists for $\lambda < \lambda^S$, entails an investigation with probability ι^S and yields expected welfare $EW^S(\lambda) = P^S(\lambda) W^i\left(\lambda_1^S(\lambda)\right)$. Finally, the pooling equilibrium $\mathcal{E}^P$, which exists for $\lambda \geq \underline{\lambda}(\overline{C}_G)$, does not involve any investigation; it leads instead the enforcer to accept the commitment $\underline{C}(\lambda)$ and yields expected welfare $EW^P(\lambda) = P^P(\lambda) W^i(\lambda)$.

To identify the potential desirability of commitments, we consider here the case where there is indeed some scope for enforcement (i.e., $\lambda < \lambda^{ci}$), and focus on the new equilibria generated by commitments (i.e., $\mathcal{E}^S$ and $\mathcal{E}^P$). In principle, allowing for commitments may affect enforcement effectiveness in two ways. First, it may affect *deterrence*, as reflected by the participation term $P^\tau(\lambda)$, for $\tau = N, S, P$; indeed, as $W^i(\lambda_1) < 0$ in the relevant range $\lambda_1 \leq \lambda^{ci}$, deterrence is socially desirable: expected welfare increases when participation decreases. Second, it may affect the *screening* of bad actions by altering the probability of investigations; this is reflected by the term $W^i(\lambda_1^\tau)$, where $W^i(\cdot)$ is increasing and λ_1^τ increases with the probability $\iota^\tau(\lambda)$.[19] We show below that commit-

[19] In principle, commitments may also enhance welfare by reducing the cost of enforcement (e.g., by reducing the need for investigations). However, the above analysis shows that this is not the case as, in equilibrium, the offered commitments are never strictly preferable (from the enforcer's standpoint) to opening an investigation.

ments actually weaken (and possibly eliminate) screening, but can enhance deterrence when they give rise to the pooling equilibrium $\mathcal{E}^P$.

The following proposition shows that commitments are never desirable if they give rise to the semi-separating equilibrium to be selected for $\lambda \in (0, \lambda^S)$.

Proposition 5 *In the relevant range $\lambda \in (0, \lambda^S)$ in which the semi-separating equilibrium $\mathcal{E}^S$ exists, $EW^S(\lambda) < EW^N(\lambda)$.*

The probability of investigation ι is equal to $\iota^N(\lambda)$ for $\mathcal{E}^N$ and to $\iota^S = \underline{C}(0)/\overline{C}_B$ for $\mathcal{E}^S$. For $\lambda < \lambda^N$, $\iota^N(\lambda) = 1 > \iota^S$. For $\lambda > \lambda^N$, $\iota^N(\lambda) = \tilde{\iota}(\lambda)$, which, from (9) and (22), satisfies $\lambda_1^e\left(\lambda^S, \iota^N\left(\lambda^S\right)\right) = \lambda_1^e\left(\lambda^S, \iota^S\right) \left(= \lambda^{ci}\right)$, implying again $\iota^N(\lambda) > \iota^S$. Hence, in the regime without commitments the enforcer investigates more often. The reason is that, when commitments are available, the bad type can secure a profit $1 - \underline{C}(0)$ by offering a commitment $\underline{C}(0)$; this option limits in turn the enforcer's probability of investigation when the firm offers no commitment.

It follows that introducing commitments (i) encourages participation, as $P_\theta^S(\lambda) = F\left(1 - \iota^S\overline{C}_\theta\right) > F\left(1 - \iota^N(\lambda)\overline{C}_\theta\right) = P_\theta^N(\lambda)$, and (ii) deteriorates screening, as $\lambda_1^S(\lambda) = \lambda_1^e\left(\lambda, \iota^S\right) < \lambda_1^e\left(\lambda, \iota^N(\lambda)\right) = \lambda_1^N(\lambda)$. Commitments thus reduce enforcement effectiveness on both accounts, and are thus undesirable: $W^i\left(\lambda_1^S(\lambda)\right) < W^i\left(\lambda_1^N(\lambda)\right)$.

We now turn to the case where commitments give rise to the pooling equilibrium $\mathcal{E}^P$, which exists for $\lambda \in \left[\underline{\lambda}(\overline{C}_G), \lambda^{ci}\right)$ and in which the commitment $\underline{C}(\lambda)$ is offered and accepted with probability 1. By nature, this equilibrium performs particularly poorly in terms of screening, as it gives the same payoff to both types, and thus induces the same rate of participation: $P^P(\lambda) = P_B^P(\lambda) = P_G^P(\lambda) = F(1 - \underline{C}(\lambda))$. It follows that commitments can be desirable only if they enhance deterrence, and do so to an extent large enough to compensate the loss of screening. The next proposition identifies a number of situations in which this cannot occur. Specifically, define:

$$\begin{aligned}
\underline{h} &\equiv \frac{\lambda^c - \lambda^{ci}}{\rho\lambda^c\left(\lambda^i - \lambda^{ci}\right)} = \frac{(1-\rho)L + W}{\rho L}, \\
\overline{h} &\equiv \underline{h} \times \left[\frac{1-\lambda^i}{1-\lambda^{ci}} + \frac{\lambda^i - \lambda^{ci}}{\lambda^{ci}\left(1-\lambda^{ci}\right)}\frac{\overline{C}_B}{\overline{C}_B - \overline{C}_G}\right] (> \underline{h}).
\end{aligned}$$

We have:

This footnote should be expanded and moved to the main text, where it could be part of a broader discussion of the design of the commitments process: here, the firms are the proposers, and can therefore fully appropriate the cost savings; if instead the enforcer could pick the commitments (or pre-commit itself ahead of time to a given commitment policy), then it could appropriate part of these savings, in which case the commitments process may be more desirable.

Proposition 6 *In the relevant range* $\lambda \in \left[\underline{\lambda}(\overline{C}_G), \lambda^{ci}\right)$ *in which the pooling equilibrium* $\mathcal{E}^P$ *exists:*

(i) *if* $\lambda \leq \lambda^N$, *then* $EW^P(\lambda) < EW^N(\lambda)$;

(ii) *if instead* $\lambda > \lambda^N$, *then:*

a. *if* $h\left(1 - \underline{C}(\lambda^N)\right) < \underline{h}$, *then* $EW^P(\lambda) < EW^N(\lambda)$;

b. *if* $h(1) < \overline{h}$, *then* $EW^P(\lambda) < EW^N(\lambda)$ *for* λ *close enough to* λ^{ci}.

Recall that the pooling equilibrium $\mathcal{E}^P$ exists only for $\lambda \geq \underline{\lambda}(\overline{C}_G)$, implying that both types prefer to offer an acceptable commitment to facing an investigation with certainty, as $\underline{C}(\lambda) \leq \overline{C}_G \left(< \overline{C}_B\right)$. It follows that commitments cannot enhance deterrence when $\lambda \leq \lambda^N$, where the enforcer investigates with probability 1 in the absence of commitments: we then have $P^P(\lambda) = F(1 - \underline{C}(\lambda)) > F\left(1 - \overline{C}_\theta\right) = P_\theta^N(\lambda)$.

If $\lambda > \lambda^N$, in the absence of commitments the enforcer opens an investigation with probability $\tilde{\imath}(\lambda)$, leading to a participation equal to $P^N(\lambda) = \lambda P_G(\tilde{\imath}(\lambda)) + (1-\lambda) P_B(\tilde{\imath}(\lambda))$, where $P_\theta(\iota) = F\left(1 - \iota\overline{C}_\theta\right)$ and $\tilde{\imath}(\lambda)$ is decreasing in λ; in case of commitments, the equilibrium $\mathcal{E}^P$ generates a participation $P^P(\lambda) = P_\theta^P(\lambda) = F(1 - \underline{C}(\lambda))$, where $\underline{C}(\lambda)$ is also decreasing in λ. Hence, when λ increases above λ^N, deterrence is reduced in both regimes, all the more so as the enforcer becomes more optimistic. In particular, in both regimes deterrence vanishes for $\lambda = \lambda^{ci}$: $\underline{C}(\lambda^{ci}) = \tilde{\imath}\left(\lambda^{ci}\right) = 0$ (hence, screening also vanishes in the absence of commitments); as a result, expected welfare then coincides in both regimes.

Building on these insights, Proposition 6 provides conditions ensuring that expected welfare remains higher in the absence of commitments. Specifically, as $EW^N(\lambda) = P^N(\lambda) W^i\left(\lambda^{ci}\right)$ is decreasing in λ in the range $\lambda \in \left(\lambda^N, \lambda^{ci}\right)$, a "global" sufficient condition is for $EW^P(\lambda) = P^P(\lambda) W^i(\lambda)$ to be instead increasing in that range, that is:

$$\frac{dEW^P}{d\lambda}(\lambda) = \frac{dP^P}{d\lambda}(\lambda) W^i(\lambda) + P^P(\lambda) \frac{dW^i}{d\lambda}(\lambda) > 0,$$

where $P^P(\lambda) = F(1 - \underline{C}(\lambda))$ and $dP^P(\lambda)/d\lambda$ is therefore proportional to $f(1 - \underline{C}(\lambda))$; it follows that this condition amounts to imposing an upper bound on the hazard rate $h(1 - \underline{C}(\lambda)) = f(1 - \underline{C}(\lambda)) / F(1 - \underline{C}(\lambda))$, namely:[20]

$$h(1 - \underline{C}(\lambda)) \leq g(\lambda) \equiv \frac{(\lambda^c - \lambda)^2}{\rho\lambda^c(\lambda^c - \lambda^{ci})\left(\lambda^i - \lambda\right)}.$$

[20] See the proof of Proposition 6.

As both sides of this inequality are decreasing in λ, it holds throughout the range $\lambda \in (\lambda^N, \lambda^{ci})$ whenever $h\left(1 - \underline{C}\left(\lambda^N\right)\right) \leq \underline{h} \equiv g\left(\lambda^{ci}\right)$. Similarly, the local condition

$$\frac{dEW^P}{d\lambda}\left(\lambda^{ci}\right) > \frac{dEW^N}{d\lambda}\left(\lambda^{ci}\right) = \frac{dP^N}{d\lambda}\left(\lambda^{ci}\right) W^o\left(\lambda^{ci}\right)$$

ensures that expected welfare is higher in the absence of commitments for λ close to λ^{ci}. Using (9) and (9), $dP^N(\lambda)/d\lambda$ can be shown to be proportional to $F\left(1 - \underline{C}(\lambda)\right)$, implying that the above condition amounts again to imposing a (weaker) upper bound on the hazard rate, namely, $h(1) < \overline{h}$.

6 Conclusion

Since the reform in the enforcement of art. 101 and 102, known as "the modernization", the commitment procedure has become widely used in European antitrust cases. This instrument enables a firm under investigation to offer measures intended to limit the anticompetitive effects. If accepted by the enforcer, these remedies become binding commitments, but there is no fine or finding of infringement.

To study the impact of this reform, we consider a setting in which a firm has the opportunity to undertake a practice that may (exogenously) be pro- or anti-competitive. The firm knows the nature of the practice, whereas the enforcer only has prior beliefs about it. We first analyse the case where the enforcer can only rely on investigations, which are costly but bring evidence with some probability. We then compare the outcome of this policy regime with that where commitments are available which, if accepted, reduce both social effects and private profits.

As one would expect, when a practice is sufficiently likely to be pro-competitive, no enforcement takes place in either policy regime. The comparison of the policy regimes is therefore relevant for practices that are sufficiently likely to generate a social harm. When commitments are not available, the enforcer investigates with certainty when the practice is particularly likely to be harmful, and with positive but decreasing probability as prior beliefs become less pessimistic. Enforcement discourages some pro-competitive practices, since investigations are costly to the firm even in the absence of infringement, but deters to a larger extent the anti-competitive ones since a firm may then have to stop the practice and pay a fine. Thus, in this benchmark case, enforcement generates both *deterrence*, by discouraging both types of actions, and *screening*, by deterring bad actions to a larger extent than good ones. It is shown that, as a result, enforcement is socially beneficial. Interestingly, as the enforcer's prior beliefs become less pessimistic, expected welfare increases as long as the enforcer keeps investigating with certainty, but then decreases, as reducing the probability of investigation limits both deterrence and screening – obviously, once the enforcer becomes so optimistic that it stops investigating, expected welfare increases again as prior beliefs become further optimistic.

We then turn to the case where commitments are available. In line with practice, we assume that the firm can propose a remedy once an investigation is opened; this gives rise to a signalling game in which the firm proposes the minimal commitment that the enforcer is willing to accept rather than to proceed with an investigation.

Two types of equilibria may arise. When priors are sufficiently pessimistic, there exists a semi-separating equilibrium in which a firm that has undertaken a pro-competitive action offers no commitment, and is then investigated with positive probability, whereas a firm that has undertaken an anti-competitive action randomizes between offering no commitment - and being investigated - and offering the minimal commitment that the enforcer is ready to accept when facing such action. We show that giving a bad firm the opportunity of offering a commitment reduces the probability of investigations, even when no commitment is being offered. As a result, there is less screening as well as less deterrence than in the benchmark case without commitments. It follows that introducing commitments weakens enforcement and reduces expected welfare.

When instead prior beliefs are less pessimistic, there exists pooling equilibria, in which the firm adopts the same strategy regardless of the type of action. In one such equilibrium, no commitment is ever offered and the enforcer investigates with the same probability as in the benchmark case. Obviously, introducing commitments has then no impact on enforcement and welfare.

In another, more interesting equilibrium, both types offer the minimal acceptable commitment, given the enforcer's prior beliefs, and no investigation thus takes place. There is thus no screening, as the firm adopts the same strategy; the only impact of the policy is therefore in terms of deterrence. As long as prior beliefs are sufficiently pessimistic, introducing commitments reduces welfare by eliminating screening and softening deterrence. When prior beliefs are more optimistic, however, there may exist particular cases in which commitments enhance welfare, because in their absence the probability of investigation would be quite low anyway.

This analysis emphasizes the effectiveness of investigations in terms of deterrence and screening. This effectiveness confers an advantage to the benchmark regime when prior beliefs are pessimistic, as investigations are then likely to take place. This suggests that commitments should not be made available for practices that are likely to be socially harmful. This advantage becomes however less relevant when the prior is more optimistic, and little enforcement would be exerted anyway. As a result, more nuanced results are obtained for practices that are less likely to produce social harm.

We have assumed that, as is the case in practice, the firms have the initiative in proposing commitments. This has the implication that the firm appropriates the "gains from trade" when negotiating with the enforcer. It may be desirable to design instead the commitment procedure so as to confer greater bargaining power to the enforcer.

For given participation rates, this would induce the firm to offer greater commitments and thus improves welfare. This, in turn, would improve deterrence by reducing further the expected profit attached to both types of actions; in addition, it would improve screening in the semi-separating equilibrium described above, by reducing the value of the commitment option for a bad firm.

The policy discussion often emphasizes the benefits of commitments stemming from reduced enforcement costs. The idea is that avoiding an in-depth investigation enables the enforcer to save resources, which can be put to good use to pursue other cases, and possibly increase the number of cases being investigated. Our analysis however emphasizes that these cost savings tend also to reduce the minimal acceptable commitments. As a result, higher cost savings translate into lower commitments, which tend to reduce the effectiveness of the commitment procedure. Designing the procedure so as to ensure that commitments must exclusively remedy the competitive harm, thus ignoring any saving on investigation costs, may therefore also contribute to confer greater bargaining power to the enforcer and make the commitment procedure more effective.[21]

[21] Doing so would lead to greater commitments even if, as in the current setting, the firms have the initiative in proposing the commitments. Furthermore, while the cost savings would not be taken into consideration in the negotiations over the commitments, introducing the commitment procedure would still contribute to save on investigation costs whenever commitments would be accepted. In addition, greater commitments would improve deterrence, which would also contribute to reduce the number of cases and thus save on investigation costs.

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Appendix

A Proofs

Proof of Proposition 1. Suppose that, in equilibrium, undertaking the action induces the enforcer to investigate with probability ι. Anticipating this, the firm undertakes an action of type θ with probability $P_\theta(\iota)$ given by (6); the enforcer's updated beliefs, given by (7), can be written as

$$\lambda_1^e(\lambda,\iota) \equiv \frac{1}{1+\frac{1-\lambda}{\lambda}\Phi(\iota)}$$

where

$$\Phi(\iota) \equiv \frac{P_B(\iota)}{P_G(\iota)}(>0)$$

satisfies $\Phi(0) = 1$ and is strictly decreasing in ι:

$$\Phi'(\iota) = -\Phi(\iota)\left[\overline{C}_B h_B(\iota) - \overline{C}_G h_G(\iota)\right] < 0, \tag{30}$$

where

$$h_\theta(\iota) \equiv h(1-\iota\overline{C}_\theta),$$

and the inequality stems from the hazard rate $h(\cdot) = f(\cdot)/F(\cdot)$ being strictly decreasing and $\overline{C}_B > \overline{C}_G$. It follows that $\lambda_1^e(\lambda,\iota)$ strictly increases with ι as well as with λ, and moreover satisfies $\lambda_1^e(\lambda,0) = \lambda$ and $\lambda_1^e(\lambda,1) = \lambda^{ci}$ for $\lambda = \lambda^N(<\lambda^{ci})$, defined in (8). Therefore, there are three possible cases:

- If $\lambda \geq \lambda^{ci}$, then any $\iota > 0$ would induce $\lambda_1^e(\lambda,\iota) > \lambda^{ci}$, implying that the enforcer would be unwilling to investigate; hence, the enforcer closes the case (or never opens it): $\iota(\lambda) = 0$.
- If instead $\lambda \leq \lambda^N$, then any $\iota < 1$ would induce $\lambda_1^e(\lambda,\iota) < \lambda^{ci}$, implying that the enforcer would be unwilling to close the case; hence, the enforcer investigates whenever the firm undertakes the action: $\iota(\lambda) = 1$.
- Finally, if $\lambda^N < \lambda < \lambda^{ci}$, then the enforcer must be indifferent between closing the case or proceeding with an investigation: $\iota = 0$ would lead to $\lambda_1^e(\lambda,0) < \lambda^{ci}$, in which case the enforcer would rather investigate ($\iota = 1$), a contradiction; likewise, $\iota = 1$ would lead to $\lambda_1^e(\lambda,\iota) > \lambda^{ci}$, in which case the enforcer would rather close the case ($\iota = 0$), another contradiction. It follows that the enforcer's posterior must satisfy $\lambda_1^e(\lambda,\iota) = \lambda^{ci}$, which in turn requires the enforcer to investigate with probability $\tilde{\iota}(\lambda) \in (0,1)$, where $\tilde{\iota}(\lambda)$ is the unique solution in ι to $\lambda_1^e(\lambda,\iota) = \lambda^{ci}$.

and is implicitly defined by

$$\Phi(\tilde{\iota}(\lambda)) = \frac{\lambda}{1-\lambda}\frac{1-\lambda^{ci}}{\lambda^{ci}}. \tag{31}$$

The LHS of (31) is continuously differentiable in ι, whereas the RHS is continuously differentiable in λ. It follows that $\tilde{\iota}(\lambda)$ is continuously differentiable in λ. Using (30) and (31) yields:

$$\tilde{\iota}'(\lambda) = \frac{-1}{\lambda(1-\lambda)\left[\overline{C}_B h_B(\tilde{\iota}(\lambda)) - \overline{C}_G h_G(\tilde{\iota}(\lambda))\right]} < 0, \tag{32}$$

where the inequality follows from the monotonicity of $h(\cdot)$ and $\overline{C}_B > \overline{C}_G$. Hence, $\tilde{\iota}(\lambda) \in [0,1]$ is decreasing in λ for $\lambda \in \left[\lambda^N, \lambda^{ci}\right]$.

Finally, the participation of the good and bad types in the no-commitment regime (superscript "N") can be expressed as

$$P_\theta^N(\lambda) \equiv F(1 - \iota^N(\lambda)\overline{C}_\theta).$$

■

Proof of Corollary 1: The properties of $P_G^N(\lambda)$ and $P_B^N(\lambda)$ directly follow from $\overline{C}_G < \overline{C}_B$ and the properties of $\iota^N(\lambda)$. Moreover, for $\lambda \in (0, \lambda^N]$:

$$\frac{dP^N(\lambda)}{d\lambda} = P_G^N(\lambda) - P_B^N(\lambda) > 0,$$

and for $\lambda \in (\lambda^N, \lambda^{ci})$, using (32) and $p_\theta^N(\lambda) \equiv f\left(1 - \iota^N(\lambda)\,\overline{C}_\theta\right)$:

$$\begin{aligned} \frac{dP^N(\lambda)}{d\lambda} &= P_G^N(\lambda) - P_B^N(\lambda) + \lambda\frac{dP_G^N(\lambda)}{d\lambda} + (1-\lambda)\frac{dP_B^N(\lambda)}{d\lambda} \\ &= P_G^N(\lambda) - P_B^N(\lambda) + \frac{\lambda\overline{C}_G p_G^N(\lambda) + (1-\lambda)\overline{C}_B p_B^N(\lambda)}{\lambda(1-\lambda)\left[\overline{C}_B h_B^N(\lambda) - \overline{C}_G h_G^N(\lambda)\right]} > 0. \end{aligned} \tag{33}$$

■

Proof of Corollary 2. The posterior of λ is

$$\lambda_1^N(\lambda) = \lambda\frac{P_G^N(\lambda)}{P^N(\lambda)} > \lambda.$$

Expected welfare is given by:

$$EW^N(\lambda) = P^N(\lambda)\left\{[1-\iota(\lambda)]\,W^c(\lambda_1^N(\lambda)) + \iota(\lambda)W^i(\lambda_1^N(\lambda))\right\},$$

where from Proposition 1: for $\lambda \leq \lambda^N$, $\iota^N(\lambda) = 1$; and for $\lambda \in (\lambda^N, \lambda^{ci})$, $\lambda_1^N(\lambda) = \lambda^{ci}$, implying that the enforcer is indifferent between investigating or not, and so $W^c(\lambda_1^N(\lambda)) = W^i(\lambda_1^N(\lambda)) \left(= W^i(\lambda^{ci})\right)$. Hence, in both cases, $EW^N(\lambda) = P^N(\lambda)W^i(\lambda_1^N(\lambda))$.

Moreover, for $\lambda \in (\lambda^N, \lambda^{ci})$, where $EW^N(\lambda) = P^N(\lambda)W^i(\lambda^{ci})$, we have:

$$\frac{dEW^N(\lambda)}{d\lambda} = \frac{dP^N(\lambda)}{d\lambda}W^i(\lambda^{ci}) < 0,$$

as P^N is strictly increasing in λ from Corollary 1, and $W^i(\lambda^{ci}) < 0$. ■

Proof of Lemma 1. Suppose the firm offers no commitment ($C = 0$). In this case $W^a(\lambda_2, 0) = W^c(\lambda_2)$ and the relevant comparison is between closing the case and investigating. Then, for $\lambda_2 > (<)\lambda^{ci}$ closing the case welfare-dominates (is welfare-dominated by) investigating. When $\lambda_2 = \lambda^{ci}$ the enforcer is indifferent between the two options.

Suppose now the firm offers a (positive) commitment $C > 0$. Consider the first case: if the posterior belief is $\lambda_2 = \lambda_2(C) > \lambda^c$, then $W^c(\lambda_2) - W^a(\lambda_2) = CW^c(\lambda_2) > 0$; hence, the enforcer strictly prefers closing the case to accepting the commitment C. Since $\lambda_2 > \lambda^c > \lambda^{ci}$, then $W^c(\lambda_2) - W^i(\lambda_2) = k - (1-\lambda_2)\rho L > 0$ given Assumption 2. Hence closing the case dominates also proceeding to an investigation. In the second case, since $W^c(\lambda_2) < 0$ for $\lambda_2 \in \left[\lambda^{ci}, \lambda^c\right]$, $W^a(\lambda_2, C) \equiv (1-C)\, W^c(\lambda_2) \geq \max\left\{W^c(\lambda_2), W^i(\lambda_2)\right\}$ for $C > 0$. In the third case the same holds true if $C \geq \underline{C}(\lambda_2)$. Hence, in these latter two cases the policy rule of accepting the commitment when this option is not worse than the alternatives is optimal. Fourth, if $0 < C < \underline{C}(\lambda_2)$ it must be $\lambda_2 < \lambda^{ci}$ and from the definition of $\underline{C}(\lambda_2)$, $W^c(\lambda_2) < W^a(\lambda_2, C) < W^i(\lambda_2)$. Hence the enforcer opens an investigation. ■

Proof of Proposition 2. *i)* Suppose that $\lambda_1 \geq \lambda^{ci}$, and consider a candidate equilibrium in which both types offer no commitment ($C = 0$) and the enforcer closes the case. Both types thus obtain the maximal profit of 1; hence, they do not have an incentive to deviate in stage 1 and, in stage 0, they undertake the action with the same probability, $F(1)$. It follows that $\lambda_2(0) = \lambda_1 = \lambda \geq \lambda^{ci}$, and so in stage 2 the enforcer is indeed willing to accept the zero commitment (close the case). This establishes the existence of an equilibrium in which both types obtain a payoff of 1. As this is the maximal achievable payoff, it follows that any Pareto efficient equilibrium (from the two types' standpoints) yields the same outcome.

ii) Consider now the case $\lambda_1 < \lambda^{ci}$ and suppose there exists an equilibrium such that $\Pi_\theta = 1$, for some type $\theta \in \{B, G\}$, implying that the enforcer responds to that type's offered commitment (if any) by closing the case. As the other type could mimic it, it follows that both types obtain the maximal profit equal to 1. Hence, in stage 0 they undertake the action with probability $F(1)$, implying that $\lambda_1 = \lambda$. As $E[\lambda_2(C)]_{C \in \mathcal{C}_G \cup \mathcal{C}_B} =$

$\lambda_1 < \lambda^{ci}$, there exists $C \in \mathcal{C}_G \cup \mathcal{C}_B$ such that $\lambda_2 = \lambda_2(C) \leq \lambda_1 < \lambda^{ci}$. From Lemma 1, the enforcer then either investigates or accepts C, and the latter case arises only if $C \geq \underline{C}(\lambda_2) > 0$. It follows that the type offering C obtains a profit strictly lower than 1, a contradiction. Therefore, in any equilibrium, both types must obtain a profit strictly lower than 1. Furthermore, as G could mimic B, and B's ex post payoffs are always weakly lower than G's, in equilibrium B must obtain a weakly lower expected profit than G. ■

Proof of Lemma 2. Fix $\lambda < \lambda^{ci}$ and suppose that there exists a $C_G \in \mathcal{C}_G \backslash \mathcal{C}_B$. Upon observing C_G, the enforcer's belief becomes $\lambda_2(C_G) = 1$ and its optimal response is to close the case (or equivalently to accept the commitment if C_G; hence, $\Pi_G(C_G) = 1$. However, no such equilibrium can exist according to Proposition 2*(ii)*. Hence, in equilibrium either $\mathcal{C}_G = \mathcal{C}_B$ or $\mathcal{C}_G \subset \mathcal{C}_B$. ■

Proof of Proposition 3. The proof is structured in four steps. We first characterize a unique candidate equilibrium by establishing that $\mathcal{C}_B \backslash \mathcal{C}_G = \{\underline{C}(0)\}$ (step 1), $\mathcal{C}_G = \{0\}$ (step 2) and $\iota(0) = \underline{C}(0) / \overline{C}_B$ (step 3). We then establish existence for $\lambda < \lambda^S$ (step 4).

Step 1: $\mathcal{C}_B \backslash \mathcal{C}_G = \{\underline{C}(0)\}$. According to Lemma 2 the candidate semi-separating equilibrium satisfies $\mathcal{C}_G \subset \mathcal{C}_B$. Then, there exists $C_B \in \mathcal{C}_B \backslash \mathcal{C}_G$. We then have:

i) If $C_B > \underline{C}(0)$ then $\alpha(C_B) = 1$ and $\Pi_B(C_B) = 1 - C_B$. Any deviation to $\widetilde{C}_B \in [\underline{C}(0), C_0)$ would lead to $\alpha(\widetilde{C}_B) + \gamma(\widetilde{C}_B) = 1$; hence, B would obtain at least $1 - \widetilde{C}_B > 1 - C_0$ and thus benefit from the deviation, a contradiction.

ii) Likewise, if $C_B < \underline{C}(0)$ then $\iota(C_B) = 1$ and $\Pi_B(C_B) = 1 - \overline{C}_B$. Any deviation to $\widetilde{C}_B \in \left[\underline{C}(0), \overline{C}_B\right)$ would lead to $\alpha(\widetilde{C}_B) + \gamma(\widetilde{C}_B) = 1$; hence, B would obtain at least $1 - \widetilde{C}_B > 1 - \overline{C}_B$ and thus benefit from the deviation, a contradiction.

Hence, a semi-separating equilibrium (superscript "S") satisfies $\mathcal{C}_B \backslash \mathcal{C}_G = \{\underline{C}(0)\}$ and thus gives B a payoff equal to

$$\Pi_B^S = 1 - \underline{C}(0).$$

Step 2: $\mathcal{C}_G = \{0\}$. Offering any $C_G \in \mathcal{C}_G (= \mathcal{C}_G \cap \mathcal{C}_B)$ must give B the same payoff $\Pi_B^S = 1 - \underline{C}(0)$. Hence, doing so cannot induce the enforcer to accept it (B's indifference would require $C_G = \underline{C}(0)$), close the case with probability 1 (B would obtain $1 > \Pi_B^S$), or investigate with probability 1 (B would obtain $1 - \overline{C}_B < \Pi_B^S$). It follows that the enforcer must be randomizing; the policy rule then implies that it must be randomizing between closing the case and opening an investigation. We must therefore have $\lambda_2(C_G) = \lambda^{ci}$ (to induce the enforcer to randomize) and $\mathcal{C}_G = \{0\}$ (as the enforcer would accept any positive commitment).

Step 3: $\iota(0) = \underline{C}(0)/\overline{C}_B$**.** B's indifference condition then requires $\Pi_B(C_G) = 1 - \iota(C_G)\overline{C}_B = 1 - \underline{C}(0)$ or:

$$\iota(C_G) = \frac{\underline{C}(0)}{\overline{C}_B}.$$

Step 4: existence for $\lambda < \lambda^S$**.** Thus, in equilibrium, $\mathcal{C}_G = \{0\}$, $\mathcal{C} = \mathcal{C}_B = \{0, \underline{C}(0)\}$, $\iota(0) = \underline{C}(0)/\overline{C}_B$, $\alpha(\underline{C}(0)) = 1$, and the two types respectively obtain $\Pi_B^S = 1 - \underline{C}(0)$ and $\Pi_G^S = 1 - \frac{\overline{C}_G}{\overline{C}_B}\underline{C}(0)$. By construction, B is indifferent between offering 0 or $\underline{C}(0)$, whereas G strictly prefers offering 0 (as offering $\underline{C}(0)$ would yield a payoff of $\Pi_B^S < \Pi_G^S$). Furthermore, no type has an incentive to deviate to any $\widetilde{C} \notin \mathcal{C}$: as this would induce $\lambda_2(\widetilde{C}) = 0$, any $\widetilde{C} > \underline{C}(0)$ would be accepted but give both types $1 - \widetilde{C} < \Pi_B^S (< \Pi_G^S)$, whereas any $\widetilde{C} < \underline{C}(0)$ would trigger an investigation and give type θ a payoff $1 - \overline{C}_\theta < \Pi_\theta^S$.

Turning to stage 0, the probability of undertaking the action is $P_B^S = F(\Pi_B^S)$ for the bad type and $P_G^S = F(\Pi_G^S) (> P_B^S)$ for the good one. Hence:

$$\lambda_1(\lambda) = \frac{\lambda P_G^S}{\lambda P_G^S + (1-\lambda) P_B^S}. \tag{34}$$

To induce $\lambda_2(0) = \lambda^{ci}$, the bad type must offer zero commitment with sufficient probability. Specifically, using

$$\lambda_2(0) = \frac{\lambda_1(\lambda)}{\lambda_1(\lambda) + [1 - \lambda_1(\lambda)]\nu_B(0;\lambda)}, \tag{35}$$

we must have:

$$\nu_B(0;\lambda) = \frac{\lambda}{1-\lambda}\frac{1-\lambda^{ci}}{\lambda^{ci}}\frac{P_G^S}{P_B^S}, \tag{36}$$

where the right-hand side is always positive for $\lambda > 0$, but is lower than 1 only if $\lambda < \lambda^S$, where λ^S is defined in (23). Hence, the semi-separating equilibrium $\mathcal{E}^S$ exists only for $\lambda \in (0, \lambda^S)$. Conversely, for any $\lambda \in (0, \lambda^S)$, the equilibrium strategies described above do constitute a semi-separating equilibrium. ■

Proof of Corollary 3. For $\lambda \in [0, \lambda^S)$, the semi-separating equilibrium yields participation rates given by (21), which satisfy $P_G^S > P_B^S$ and do not depend on the prior λ. Furthermore, G offers no commitment with probability 1, whereas B does so with probability $\nu_B(0;\lambda)$, inducing a posterior equal to λ^{ci} that leaves the enforcer indifferent between investigating or not (either way, expected welfare is thus equal to $W^i(\lambda^{ci})$) and otherwise offers $\underline{C}(0)$, leaving again the enforcer indifferent between investigating or not (either way, it obtains $W^i(0)$).

Letting

$$P_0^S(\lambda) \equiv P_G^S + P_B^S \nu_B(0;\lambda)$$

denote the overall probability of no commitment being offered, expected welfare can therefore be expressed as:

$$\begin{aligned} EW^S(\lambda) &= P^S(\lambda)\left\{\frac{P_0^S(\lambda)}{P^S(\lambda)}W^i(\lambda^{ci}) + [1 - \frac{P_0^S(\lambda)}{P^S(\lambda)}]W^i(0)\right\} \\ &= P^S(\lambda)\,W^i\left(\frac{P_0^S(\lambda)}{P^S(\lambda)}\lambda^{ci}\right) \\ &= P^S(\lambda)\,W^i\left(\lambda_1^S(\lambda)\right), \end{aligned}$$

where the second equality relies on the linearity of $W^i(\lambda_1)$ in λ_1 (namely, $W^i(\lambda_1) = W^i(0) + \lambda_1[W + (1-\rho)L]$), whereas the last one uses $P_G^S/P_0^S(\lambda) = \lambda^{ci}$ and $P_G^S/P^S(\lambda) = \lambda_1^S(\lambda)$. ■

Proof of Proposition 4. Let $\lambda < \lambda^{ci}$ and suppose there exists a pooling equilibrium, i.e., satisfying $\mathcal{C}_G = \mathcal{C}_B = \mathcal{C}$. Let λ_1 denote the enforcer's interim belief, upon observing that the action has been undertaken. We consider three cases, depending on the value of this belief.

Case 1: $\lambda_1 > \lambda^{ci}$. By construction, there then exists $C \in \mathcal{C}$ such that $\lambda_2(C) \geq \lambda_1 > \lambda^{ci}$, implying from Lemma 1 that the enforcer either accepts C (if $\lambda_2(C) \leq \lambda^c$) or closes the case (if $\lambda_2(C) \geq \lambda^c$). In both instances, the two types obtain the same payoff (namely, either $1 - C$, or 1). As the firm must be indifferent between all commitments in $\mathcal{C}$, it follows that both types obtain the same expected payoff, and thus undertake the action for the same cost realization; hence, $\lambda_1 = \lambda < \lambda^{ci}$, a contradiction.

Case 2: $\lambda_1 = \lambda^{ci}$. By construction, there then exists $C \in \mathcal{C}$ such that $\lambda_2(C) \geq \lambda_1 (= \lambda^{ci})$. If $C > 0$, then from Lemma 1 the enforcer accepts it with probability 1 and both types thus obtain the same payoff, implying $\lambda = \lambda_1 = \lambda^{ci}$, which contradicts the working condition $\lambda < \lambda^{ci}$. Hence, $C = 0$ and the enforcer must either close the case, open an investigation, or randomize between these two options. Furthermore, if the enforcer closes the case with probability 1, then again both types obtain the same payoff, implying $\lambda = \lambda_1 = \lambda^{ci}$, a contradiction. Hence, the enforcer must open an investigation with positive probability: $\iota(0) > 0$, implying that B obtains $1 - \iota(0)\overline{C}_B$. As B could secure a payoff of $1 - \underline{C}(0)$ by offering a commitment $\underline{C}(0)$, the probability of investigation cannot be too large, namely:

$$\iota(0) \leq \frac{\underline{C}(0)}{\overline{C}_B} (< 1).$$

Suppose now that there exists another offered commitment, $C' \in \mathcal{C} \setminus \{C\}$. As offering C gives different payoffs to the two types, to ensure that both of them are indifferent between offering C or C', it must be the case that C' induces the enforcer to open an investigation with positive probability; furthermore, as just seen, to deter B from deviating this probability must be lower than 1. It follows that the enforcer must again be indifferent between opening an investigation or closing the case, which requires ($\lambda_2(C') = \lambda^{ci}$, and) $C' = 0$, a contradiction.

Hence, $\mathcal{C} = \{0\}$ and the enforcer opens an investigation with probability $\iota(0) \leq \underline{C}(0)/\overline{C}_B$, and closes the case otherwise. It follows that firms' participation is $P_\theta(\iota(0)$ as defined in (6) and $\lambda_1 = \lambda_1^e(\lambda, \iota(0))$, as defined by (7). The working condition $\lambda_1 = \lambda^{ci}$ therefore implies that $\iota(0) = \tilde{\iota}(\lambda)$ as defined in (9), which is decreasing in λ. That is, the candidate equilibrium coincides with the equilibrium that arises when commitments are not available. Notice that the equilibrium probability of investigation in the no-commitment case, $\iota^N(\lambda)$, is equal to 1 for $\lambda \leq \lambda^N$ and is equal $\tilde{\iota}(\lambda)$ and decreases from 1 to 0 as λ increases from λ^N to λ^{ci}. Then there exists $\hat{\lambda} \in (\lambda^N, \lambda^{ci})$ such that $\tilde{\iota}(\hat{\lambda}) = \underline{C}(0)/\overline{C}_B$, the participation rate is equal to the one in the semi-separating equilibrium ($P_\theta^N(\hat{\lambda}) = P_\theta^S$) and $\lambda_1 = \lambda_1^e(\hat{\lambda}, \tilde{\iota}(\hat{\lambda})) = \lambda^{ci}$. But then $\hat{\lambda} = \lambda^S$ as defined in (23). The constraint $\iota(0) = \tilde{\iota}(\lambda) \leq \underline{C}(0)/\overline{C}_B$ is therefore satisfied for $\lambda \geq \lambda^S$.

Conversely, for any $\lambda \in [\lambda^S, \lambda^{ci})$, there exists a pooling equilibrium that coincides with the equilibrium $\mathcal{E}^N$ arising when commitments are not available. Indeed, in this equilibrium a firm of type θ obtains a payoff $\Pi_\theta^N(\lambda) = 1 - \tilde{\iota}(\lambda)\overline{C}_\theta$, where $\tilde{\iota}(\lambda) \leq \underline{C}(0)/\overline{C}_B$, implying that B obtains

$$\Pi_B^N(\lambda) \geq 1 - \underline{C}(0),$$

whereas G obtains

$$\Pi_G^N(\lambda) \geq 1 - \overline{C}_G \frac{\underline{C}(0)}{\overline{C}_B} > \max\{1 - \overline{C}_G, 1 - \underline{C}(0)\},$$

where the inequality stems from $\underline{C}(0) < \overline{C}_B$ and $\overline{C}_G < \overline{C}_B$. As any deviant offer $\tilde{C}$ would induce $\lambda_2(\tilde{C}) = 0$, it would be accepted only if $\tilde{C} \geq \underline{C}(0)$, in which case each type θ would obtain $1 - \tilde{C} \leq 1 - \underline{C}(0) \leq \Pi_\theta^N(\lambda)$; if instead $\tilde{C} < \underline{C}(0)$, then the enforcer opens an investigation and each type θ obtains $1 - \overline{C}_\theta < \Pi_\theta^N(\lambda)$, where the inequality stems again from $\tilde{\iota}(\lambda) \leq \underline{C}(0)/\overline{C}_B < 1$. Hence, there is no profitable deviation.

Case 3: $\lambda_1 < \lambda^{ci}$. By construction, there then exists $C \in \mathcal{C}$ such that $\lambda_2(C) \leq \lambda_1$. If $C < \underline{C}(\lambda_2(C))$, then from Lemma 1 the enforcer opens an investigation and B thus obtains $\Pi_B = 1 - \overline{C}_B$. But then, B would be strictly better off offering $\tilde{C} \in (\underline{C}(0), \overline{C}_B)$: this would either lead the enforcer to accept it, giving B a payoff $1 - \tilde{C} > \Pi_B$, or to close the case (if $\tilde{C} \in \mathcal{C}$ and $\lambda_2(\tilde{C}) \geq \lambda^c$), giving B an even higher payoff. Hence, it must be $C \geq \underline{C}(\lambda_2(C))$ and, from Lemma 1, the enforcer accepts C whenever offered; furthermore, $C \geq \underline{C}(\lambda_2(C))$ and $\lambda_2(C) \leq \lambda_1 < \lambda^{ci}$ together imply $C > 0$.

Suppose now that there exists another offered commitment, $C' \in \mathcal{C} \setminus \{C\}$. By construction, the firm must be indifferent between offering C or C'. If the enforcer accepts C', it must do so with probability 1; indifference then requires $C' = C$, a contradiction. Likewise, if the enforcer closes the case with probability 1, then indifference requires

$C = 0$, another contradiction. In all other cases, the enforcer opens an investigation with positive probability, implying that B and G obtain different payoffs; hence, they cannot be both indifferent between C and C'. Hence, $\mathcal{C} = \{C\}$. It follows that both types offer C with probability 1, and so $\lambda_2(C) = \lambda_1$. Furthermore, from the previous reasoning, C satisfies $C \geq \underline{C}(\lambda_2(C))$ and is therefore accepted; hence, both types obtain the same payoff (namely, $1 - C$) and thus participate for the same cost realizations. Hence, $\lambda_2(C) = \lambda_1 = \lambda$.

From the above, $C \geq \underline{C}(\lambda)$ and both types obtain $1 - C$. Hence, if $C > \underline{C}(0)$, they would strictly benefit from slightly reducing the offered commitment, as the enforcer would still accept it even with a posterior $\lambda_2 = 0$. Furthermore, if $C > \overline{C}_G$, then G would therefore strictly benefit from deviating and offering no commitment, so as to induce the enforcer to open an investigation.

Hence, the candidate equilibria are such that $\mathcal{C} = \{C\}$, where $C \in [\underline{C}(\lambda), \min\{\underline{C}(0), \overline{C}_G\})$, and $\lambda_2(C) = \lambda_1 = \lambda$. Together, the conditions $\underline{C}(\lambda) \leq C \leq \overline{C}_G$ imply $\lambda \geq \underline{\lambda}(\overline{C}_G)$. Conversely, if $\lambda \geq \underline{\lambda}(\overline{C}_G)$, then any $C \in [\underline{C}(\lambda), \min\{\underline{C}(0), \overline{C}_G\}]$ can be supported in equilibrium. Indeed, in such an equilibrium both types obtain $1 - C$ and any deviant offer $\tilde{C}$ induces $\lambda_2(\tilde{C}) = 0$. Hence, $\tilde{C}$ would be accepted only if $\tilde{C} \geq \underline{C}(0)$, in which case both types would obtain $1 - \tilde{C} \leq 1 - \underline{C}(0) \leq 1 - C$; if instead $\tilde{C} < \underline{C}(0)$, then the enforcer opens an investigation and each type θ obtains $1 - \overline{C}_\theta \leq 1 - \overline{C}_G \leq 1 - C$. Hence, there is no profitable deviation.

Summing-up, these pooling equilibria exist only if $\lambda \in [\underline{\lambda}(\overline{C}_G), \lambda^{ci})$, in which case the Pareto-efficient one is for the lowest possible commitment, $\underline{C}(\lambda)$, which corresponds to $\mathcal{E}^P$, in which the firm offers the minimum acceptable commitment $\underline{C}(\lambda)$ and obtains $\Pi^P(\lambda) \equiv 1 - \underline{C}(\lambda)$. ■

Proof of Corollary 4. In the Pareto-efficient pooling equilibrium $\mathcal{E}^P$, the firm offers $\underline{C}(\lambda)$, which is accepted, and thus obtains $\Pi^P(\lambda) \equiv 1 - \underline{C}(\lambda)$. Hence, the expected participation is

$$P^P(\lambda) \equiv F(1 - \underline{C}(\lambda)).$$

As by construction $W^a(\lambda, \underline{C}(\lambda)) = W^i(\lambda)$, expected welfare is then given by:

$$EW^P(\lambda) = P^P(\lambda) [\lambda W - (1 - \lambda)L] (1 - \underline{C}(\lambda)) = P^P(\lambda) W^i(\lambda). \tag{37}$$

■

Proof of Proposition 5. The expected welfare generated by the equilibria $\mathcal{E}^S$ and $\mathcal{E}^N$, for $\lambda \in [0, \lambda^S)$, are respectively given by (25) and (13). Their difference can be expressed as:

$$\begin{aligned}\Delta EW(\lambda) &= EW^S(\lambda) - EW^N(\lambda)\\ &= \left[P^S(\lambda) - P^N(\lambda)\right] W^i(\lambda_1^S(\lambda)) + P^N(\lambda)\left[W^i(\lambda_1^S(\lambda)) - W^i(\lambda_1^N(\lambda))\right],\end{aligned}$$

where $EW^N(\lambda)$, $P^N(\lambda)$, $\lambda_1^N(\lambda)$ and $EW^S(\lambda)$, $P^S(\lambda)$, $\lambda_1^S(\lambda)$ are respectively defined by (13), (12), (14) and (25), (26).

In each equilibrium $\mathcal{E}^\tau$, for $\tau = N, S$, the participation is of the form $P^\tau(\lambda) = \lambda P_G^\tau(\lambda) + (1-\lambda) P_B^\tau(\lambda)$. In $\mathcal{E}^N$, the participation of type $\theta = G, B$ is given by

$$P_\theta^N(\lambda) = P_\theta\left(\iota^N(\lambda)\right),$$

where $P_\theta(\iota)$ is given by (6) and is strictly decreasing in ι, and $\iota^N(\lambda)$ is given by (10).

In $\mathcal{E}^S$, G offers no commitment, which triggers an investigation with probability $\iota^S = \underline{C}(0)/\overline{C}_B \in (0,1)$, whereas B is indifferent between doing the same or offering the commitment $\underline{C}(0)$, which is accepted by the enforcer. It follows that the participation of type $\theta = G, B$ is given by

$$P_\theta^S = P_\theta\left(\iota^S\right).$$

Furthermore, from (8) and (22), λ^N and λ^S are such that

$$\lambda_1^e\left(\lambda^N, 1\right) = \lambda_1^e\left(\lambda^S, \iota^S\right)\left(= \lambda^{ci}\right).$$

As $\lambda_1^e(\lambda, \iota)$ is increasing in both of its arguments, it follows from $\iota^S < 1$ that

$$\lambda^N < \lambda^S < \lambda^{ci},$$

which in turn implies that $\iota^N\left(\lambda^S\right) = \tilde{\iota}\left(\lambda^S\right)$; hence, from (9) and (22):

$$\lambda_1^e\left(\lambda^S, \iota^N\left(\lambda^S\right)\right) = \lambda_1^e\left(\lambda^S, \iota^S\right)\left(= \lambda^{ci}\right),$$

implying:

$$\tilde{\iota}\left(\lambda^S\right) = \iota^S.$$

As $\iota^N(\lambda)$ is (weakly) decreasing in λ, and strictly so for $\lambda \in \left(\lambda^N, \lambda^{ci}\right)$, it follows that, in the relevant range $\lambda < \lambda^S$, we have:

$$\iota^N(\lambda) > \iota^N\left(\lambda^S\right) = \tilde{\iota}\left(\lambda^S\right) = \iota^S.$$

Participation is therefore lower in $\mathcal{E}^N$ than in $\mathcal{E}^S$: $P_\theta^N(\lambda) < P_\theta^S$ for $\theta = G, B$, and so $P^N(\lambda) < P^S$. Furthermore, the enforcer's interim belief is more optimistic in $\mathcal{E}^N$ than in $\mathcal{E}^S$: given the participation rates, these beliefs are respectively given by $\lambda_1^N(\lambda) =$

$\lambda_1^e\left(\lambda, \iota^N(\lambda)\right)$ and $\lambda_1^S(\lambda) = \lambda_1^e\left(\lambda, \iota^S\right)$; hence, $\iota^N(\lambda) > \iota^S$ implies $\lambda_1^N(\lambda) > \lambda_1^S(\lambda)$. In addition, for any $\lambda < \lambda^S$, $\lambda_1^e\left(\lambda^S, \iota^S\right) = \lambda^{ci}$ implies $\lambda_1^S(\lambda) = \lambda_1^e\left(\lambda, \iota^S\right) < \left(\lambda^{ci} <\right)\lambda^i$; hence, $W^i\left(\lambda_1^S(\lambda)\right) < W^i\left(\lambda^i\right) = 0$.

Summing-up, in the relevant range $\lambda < \lambda^S$, we have $P^S(\lambda) > P^N(\lambda) > 0$ and $W^i(\lambda_1^S(\lambda)) < W^i(\lambda_1^N(\lambda)) < 0$. It then follows from the expression of $\Delta EW(\lambda)$ that $EW^S(\lambda) < EW^N(\lambda)$. ■

Proof of Proposition 6. The equilibria $\mathcal{E}^N$ and $\mathcal{E}^P$ co-exist in the interval $\lambda \in \left[\underline{\lambda}(\overline{C}_G), \lambda^{ci}\right)$, where we have $EW^N(\lambda) = P^N(\lambda)\, W^i\left(\lambda^{ci}\right)$ and $EW^P(\lambda) = P^P(\lambda)\, W^i(\lambda)$, where $W^i(\lambda) < W^i\left(\lambda^{ci}\right)$. Furthermore, for $\lambda \leq \lambda^N$, $\iota^N(\lambda) = 1$ and so $P^P(\lambda) = F\left(1 - \underline{C}(\lambda)\right) > F\left(1 - \overline{C}_\theta\right) = F\left(1 - \iota^N(\lambda)\,\overline{C}_\theta\right) = P_\theta^N(\lambda)$ for $\theta = G, B$, implying that $P^P(\lambda) > P^N(\lambda)$; it follows that:

$$
\begin{aligned}
\Delta EW(\lambda) &\equiv EW^P(\lambda) - EW^N(\lambda) \\
&= P^P(\lambda)\left[W^i(\lambda) - W^i(\lambda^{ci})\right] + \left[P^P(\lambda) - P^N(\lambda)\right] W^i(\lambda^{ci}) \\
&= < 0,
\end{aligned}
$$

where the inequality stems from both terms being negative, as (i) $P^P(\lambda) > 0$ and $W^i(\lambda) < W^i(\lambda^{ci})$ and (ii) $P^N(\lambda) > P^N(\lambda)$ and $W^i(\lambda^{ci}) < 0$.

We now focus on the case $\lambda > \lambda^N$, where $\lambda_1^N(\lambda) = \lambda^{ci}$ and, from Corollary 2, $EW^N(\lambda)$ is strictly decreasing in λ. As $\lim_{\lambda \to \lambda^{ci}} EW^N(\lambda) = \lim_{\lambda \to \lambda^{ci}} EW^P(\lambda) = F(1)\, W^i\left(\lambda^{ci}\right)$, it follows that commitments are never desirable if $EW^P(\lambda)$ is (weakly) increasing in λ. Using $W^i(\lambda) = [W + (1-\rho)L]\left(\lambda - \lambda^i\right)$ and:

$$
\begin{aligned}
P^P(\lambda) &= F\left(1 - \underline{C}(\lambda)\right) = F\left(1 - \frac{\rho\lambda^c(\lambda^{ci} - \lambda)}{\lambda^c - \lambda}\right), \\
\frac{dP^P}{d\lambda}(\lambda) &= f\left(1 - \underline{C}(\lambda)\right) \frac{\rho\lambda^c(\lambda^c - \lambda^{ci})}{(\lambda^c - \lambda)^2},
\end{aligned}
$$

the derivative of $EW^P(\lambda)$ can be expressed as:

$$
\begin{aligned}
\frac{dEW^P}{d\lambda}(\lambda) &= P^P(\lambda)\frac{dW^i}{d\lambda}(\lambda) - \frac{dP^P}{d\lambda}(\lambda) W^i(\lambda) \\
&= [W + (1-\rho)L]\, F\left(1 - \underline{C}(\lambda)\right)\left[1 - \frac{h\left(1 - \underline{C}(\lambda)\right)}{g(\lambda)}\right], \qquad (38)
\end{aligned}
$$

where $h\left(1 - \underline{C}(\lambda)\right)$ and

$$
g(\lambda) \equiv \frac{(\lambda^c - \lambda)^2}{\rho\lambda^c(\lambda^c - \lambda^{ci})\left(\lambda^i - \lambda\right)}
$$

are both strictly decreasing in λ:

$$\frac{d}{d\lambda}\left(h\left(1-\underline{C}\left(\lambda\right)\right)\right)=-h'\left(1-\underline{C}\left(\lambda\right)\right)\underline{C}'\left(\lambda\right),$$

where the inequality stems from $h(\cdot)$ and $\underline{C}(\cdot)$ being both decreasing functions, and:

$$g'\left(\lambda\right)=-\frac{\left(\lambda^{c}-\lambda\right)\left(2\lambda^{i}-\lambda^{c}-\lambda\right)}{\rho\lambda^{c}(\lambda^{c}-\lambda^{ci})\left(\lambda^{i}-\lambda\right)^{2}}<0,$$

where the inequality follows from $\lambda<\lambda^{ci}<\lambda^{c}<\lambda^{i}$. It follows that $EW^{P}(\lambda)$ is (weakly) increasing in λ in the range $[\lambda^{N},\lambda^{ci})$ if

$$h\left(1-\underline{C}\left(\lambda^{N}\right)\right)\leq\underline{h}\equiv g\left(\lambda^{ci}\right)=\frac{\lambda^{c}-\lambda^{ci}}{\rho\lambda^{c}\left(\lambda^{i}-\lambda^{ci}\right)}.$$

Likewise, for λ close to λ^{ci}, commitments are undesirable if:

$$\frac{dEW^{P}}{d\lambda}\left(\lambda^{ci}\right)>\frac{dEW^{N}}{d\lambda}\left(\lambda^{ci}\right).$$

which, using (38), (33) and $W^{i}(\lambda)=[W+(1-\rho)L]\left(\lambda-\lambda^{i}\right)$, amounts to:

$$[W+(1-\rho)L]F(1)\left[1-\frac{h(1)}{g\left(\lambda^{ci}\right)}\right]>-[W+(1-\rho)L]F(1)\frac{\lambda^{i}-\lambda^{ci}}{1-\lambda^{ci}}\left[\frac{\overline{C}_{B}}{\lambda^{ci}\left(\overline{C}_{B}-\overline{C}_{G}\right)}-1\right],$$

or $h(1)<\overline{h}$, where:

$$\begin{aligned}\overline{h}&\equiv\underline{h}\left\{1+\frac{\lambda^{i}-\lambda^{ci}}{1-\lambda^{ci}}\left[\frac{\overline{C}_{B}}{\lambda^{ci}\left(\overline{C}_{B}-\overline{C}_{G}\right)}-1\right]\right\}\\&=\underline{h}\left\{\frac{1-\lambda^{i}}{1-\lambda^{ci}}+\frac{\lambda^{i}-\lambda^{ci}}{\lambda^{ci}\left(1-\lambda^{ci}\right)}\frac{\overline{C}_{B}}{\left(\overline{C}_{B}-\overline{C}_{G}\right)}\right\}(>\underline{h}).\end{aligned}$$

■