Identification through the Forecast Error Variance Decomposition: an Application to Uncertainty*

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Abstract

We develop a novel identification scheme for Structural Vector Autoregression based on constraints on the Forecast Error Variance decomposition. We characterize the properties of this approach and provide Bayesian, but frequentist friendly, algorithms for estimation and inference. We use this strategy to investigate the effects of uncertainty and financial shocks on the economy by allowing for uncertainty endogeneity, disentangling real versus financial uncertainty, and separating uncertainty from pure financial shocks. Monte-Carlo exercises illustrate the effectiveness of this approach. Using US data, we find that macro uncertainty is mostly endogenous, and overlooking this channel can lead to distortions in the estimated effects of uncertainty on the economy. We find that financial uncertainty is more likely to be exogenous. Also, ignoring that uncertainty can originate from different, i.e., real or financial, sources biases the estimation. Finally, omitting the endogenous features of uncertainty leads to underestimate the effects of financial shocks on the economy.

Keywords: Causality, Financial Conditions, Identification, Structural Vector Autoregression, Uncertainty.

JEL: C11, C32, E32, E37, E44.

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1 Introduction and Related Literature

Since the influential paper of Bloom (2009), the business cycle relationship between uncertainty and macroeconomic variables and the underlying transmission mechanism have received extensive consideration.[1] Several measures of uncertainty have been proposed, and many scholars have analyzed the macroeconomic effects of uncertainty shocks.[2]

Three challenges come to the fore. First, most works usually employ structural vector autoregressions (SVARs) with some recursive identification scheme. The common assumption is that uncertainty is exogenous, i.e., it does not respond contemporaneously to economic variables, whereas economic variables react contemporaneously to uncertainty.[3] Recursive schemes are widespread due to the simplicity of implementation and interpretation, but for uncertainty it is extremely challenging to defend them as convincing identification strategies. In fact, the current evidence makes researchers unable to take up a position on the direction of the causality between uncertainty and economic variables: there is no uncontroversially accepted, theoretically grounded belief to assert that a specific recursive scheme is credible for identification of uncertainty shocks. On the contrary, both directions of causality are feasible and theory is also ambiguous about the sign of the effect of uncertainty on the economy. Uncertainty can affect the economy through firms' behavior, which is influenced by uncertainty because of (i) the real option argument (Bernanke, 1983; McDonald & Siegel, 1986); (ii) the delay of hiring and investment decisions (Bloom, 2009; Bloom et al., 2018; Leduc & Liu, 2016); (iii) the interaction with financial frictions that impact on firms' decisions (Arellano et al., 2018; Gilchrist et al., 2014; Alfaro et al., 2018). The uncertainty can influence the economy also through precautionary savings (Basu & Bundick, 2017; Fernández-Villaverde et al., 2011). On the other hand, some scholars have pointed out that bad economic and/or credit conditions are likely to rise uncertainty (Van Nieuwerburgh & Veldkamp, 2006; Bachmann & Moscarini, 2011; Fajgelbaum et al., 2017; Brunnermeier & Sannikov, 2014; Atkinson et al., 2021; Plante et al., 2018). Empirical contributions that have allowed for

[1] Bloom (2014) provides an excellent survey.

[2] A partial list of works consists of Bloom (2009), Bachmann et al. (2013), Caggiano et al. (2014), Jurado et al. (2015), Rossi and Sekhposyan (2015), Caldara et al. (2016), Baker et al. (2016), Basu and Bundick (2017), Cesa-Bianchi et al. (2018), Shin and Zhong (2020), Carriero et al. (2018b), Bloom et al. (2018), Angelini et al. (2019), Ludvigson et al. (2021), and Carriero et al. (2021).

[3] We use the terms exogenous (endogenous) as shorthand for predetermined (not predetermined) within the period.

both directions of causality include Carriero et al. (2021), Ludvigson et al. (2021), and Angelini et al. (2019). All these contributions have shown that the direction of causality might depend on the uncertainty typology and measure of choice. Additional literature pointed out that uncertainty can stimulate economic activity (growth options theory): a mean-preserving spread in risk originated from an unbounded upside combined with a limited downside can lead firms to invest and hire, since the rise in mean preserving risk raises expected profits.[4]

A separate challenge is about the origins of uncertainty. Standard theories claim that uncertainty originates from macroeconomic fundamentals, e.g., productivity, and that such real economic uncertainty, when interacted with market frictions, decreases real activity. However, it has been argued that uncertainty depresses the economy via its impact on financial markets (Gilchrist et al., 2014), or through sources of uncertainty specific to financial markets (Bollerslev et al., 2009). Furthermore, Ng and Wright (2013) discuss that financial uncertainty –as distinct from real economic or macro uncertainty– could have a pivotal role in recessions after 1982, both as a cause and as a propagation channel. The challenge also arises because the theoretical literature has focused on volatility coming from fundamentals, while empirical efforts have usually tested those frameworks employing uncertainty proxies that are strongly correlated with financial market variables. This naturally leads to wonder whether it is macro uncertainty or financial uncertainty (or both) to drive business cycle fluctuations. The current literature does not disentangle/identify the contributions of real versus financial uncertainty to business cycle fluctuations, nor it allows feedback between macro and financial uncertainty, e.g., in a typical SVAR one would find a proxy for -say- macroeconomic uncertainty, but not for financial uncertainty and vice versa. Exceptions are the small-scale models in Ludvigson et al. (2021) and Angelini et al. (2019) and the contribution in Shin and Zhong (2020).

Third, there is high degree of comovement between indicators of financial distress such as credit spreads and uncertainty proxies as both variables are "fast moving" (Caldara et al., 2016; Brianti, 2021; Caggiano et al., 2021). It is therefore difficult to impose plausible zero contemporaneous restrictions to identify these two disturbances. It is also difficult to impose sign restrictions as uncertainty and financial shocks could have theoretically

[4]For instance, see Oi (1961), Hartman (1972), Abel (1983), Bar-Ilan and Strange (1996), Pástor and Veronesi (2006), Kraft et al. (2018), Segal et al. (2015), and Fernández-Villaverde and Guerrón-Quintana (2020).

the same qualitative effects on both prices and quantities.

This paper proposes a new strategy to identify the real effects of uncertainty and financial shocks while addressing the empirical issues above, namely it allows for uncertainty endogeneity, identifies simultaneously different sources of uncertainty, and separates uncertainty from pure financial shocks. To our knowledge, this is the first paper to tackle those concerns in a single unified framework. Contrarily to most methodologies in the uncertainty literature, our identification scheme allows both a causal transmission channel going from uncertainty to the economic variables and the opposite causal mechanism going from the economic variables to uncertainty.

Specifically, we propose a novel and general multiple shocks identification scheme based on constraints on the Forecast Error Variance (FEV) decomposition of a Structural Vector Autoregression. While we focus on uncertainty and financial disturbances, the identification and estimation toolkit developed here can be applied in any SVAR where standard ordering and sign restrictions are not enough to identify competing shocks (see discussion in Section 2.3).

The foundation of our scheme consists of simultaneously discriminating between macro uncertainty, financial uncertainty, and financial shocks, where the latter is defined as a shock to credit supply and measured through the credit spreads. As an example, say we want to identify macro uncertainty shocks. We assume the latter need to explain the unexpected movements of macro uncertainty variable more than the fluctuations of financial uncertainty variable and credit spread. This restriction selects a set (multiplicity) of admissible structural models, that can be fairly wide. We therefore augment the identification scheme by adding the requirement that the identified innovation needs to maximize its contribution to a function depending on the FEV of macro uncertainty variable. In a similar fashion and within the same procedure, we simultaneously identify also financial uncertainty and credit supply shocks.

Our identification strategy consists of solving a constrained maximization problem, where the objective function is an equally weighted linear combination of the FEV of target variables and the constraints are inequality restrictions on the FEV. This corresponds to work out a quadratic optimization problem on the columns of the rotation matrix transforming reduced-form residuals into structural shocks. We provide a flexible toolkit and establish mild conditions under which the solution of the optimization problem exists and is unique. We present simple algorithms to run Bayesian estimation and inference, though the identification approach and properties hold in a frequentist setting. Our methodology differs from Uhlig (2004), who popularized optimization of the FEVD with sign restrictions as constraints for identification of a single shock. On the other hand, we employ inequality constraints on the FEVD, which are novel to the literature, as constraints of the maximization problem. Thus, it is worth stressing that our approach (i) identifies simultaneously a multiplicity of shocks, that is uncommon in the literature, and (ii) is well-suited (even) when sign restrictions are unavailable, or cannot help distinguish competing shocks: uncertainty disturbances are a natural example. Also, the fact that inequality restrictions on the FEVD correspond to quadratic constraints on the rotation matrix has some effects on estimation and inference with respect to the standard case of linear constraints, namely sign restrictions. The spirit of our approach also echoes Volpicella (2021) and Amir-Ahmadi and Drautzburg (2021), but significant differences arise. Volpicella (2021) put sign restrictions and bounds on the FEV to set-identify a single shock; on the contrary, we point-identify shocks, do not place bounds, and allow identification of a multiplicity of shocks. Amir-Ahmadi and Drautzburg (2021) employed set-identification through ranking restrictions on the impulse response functions, combined with standard sign restrictions.

We assess our identification showing that it is consistent with most of theoretical frameworks modeling uncertainty regardless of whether those models consider endogenous or exogenous uncertainty. Through a Monte-Carlo exercise we prove that our scheme recovers the impulse response functions in different Data Generating Processes (DGPs), with exogenous or endogenous uncertainty; our identification successfully captures the effect of uncertainty on the economy regardless the exogeneity extent of the uncertainty disturbances in the DGP.

We then estimate a SVAR with US data, and apply the proposed identification scheme. Firstly, we find that both macro and financial uncertainty shocks act as negative demand shocks, i.e., decrease the real activity and trigger a deflationary pressure. However, the responses to the two shocks are quantitatively substantially different: macro uncertainty has a stronger and more persistent effect on the real activity variables. Secondly, there is some evidence in favor of endogeneity of uncertainty. In particular, dismissing the feedback effect from the economy to macro uncertainty biases the estimation. These results are in line with Ludvigson et al. (2021), arguing that macro uncertainty is mostly endogenous, whereas financial uncertainty is mainly exogenous; they are compatible with Carriero et al. (2021), who found that some macro variables affect macro uncertainty (especially at quarterly frequency), and partially diverge from Angelini et al. (2019), who claimed that both macro and financial uncertainty are exogenous. Comparison with those papers will be fully discussed. Our findings suggest that, to study uncertainty and its effects, it is paramount to disregard naive schemes, such as sign restrictions and recursive ordering, and employ a more refined strategy to identification, such as the one we construct here. Thirdly, omitting in the same system that uncertainty has different sources (macro and financial uncertainty), as is the case in most of the literature, dramatically biases the impulse response functions. Also, recent theoretical literature has emphasized the pivotal role of financial conditions in amplifying and propagating to real economy the effects of uncertainty (Arellano et al., 2018; Christiano et al., 2014; Gilchrist et al., 2014; Brunnermeier & Sannikov, 2014; Alfaro et al., 2018). We find this channel is crucial for financial uncertainty, and ignoring it delivers biased estimation; however, it turns out to be negligible for the transmission of macro uncertainty. Finally, financial shocks are recessionary and ignoring the endogenous role of uncertainty leads to underestimating the effects of credit shocks on the economy. This is interesting as in the literature the role of uncertainty in the transmission of financial shocks is overlooked.

From a modeling standpoint, new Keynesian models with Rotemberg-type sticky prices[5] or real frictions in the labor market and nominal rigidities[6] replicate the recessionary co-movement of output and prices after a shock to uncertainty, but quantitative distortions arise from failing to model multiple sources and endogeneity of uncertainty.

In this paper, Section 2 introduces the identification strategy; Section 3 provides the estimation toolkit; Section 4 illustrates the effectiveness of our approach via Monte-Carlo experiments; Section 5 contains the application, its empirical contributions, and discusses comparison with alternative identification strategies; Section 6 concludes. Finally, Appendix A contains the proofs; Appendix B provides further evidence that our identified shocks are truly structural and presents additional robustness checks.

[5]See Oh (2020); Bonciani and Van Roye (2016); Leduc and Liu (2016); Basu and Bundick (2017); Cesa-Bianchi and Fernandez-Corugedo (2018); Katayama and Kim (2018).

[6]See Leduc and Liu (2016); Cacciatore and Ravenna (2020).

2 Model Specification and Identification

This section defines the SVAR and then introduces the identification strategy.

2.1 Model

Consider a SVAR(p) model

$$\boldsymbol{A}_0\boldsymbol{y}_t = \boldsymbol{a} + \sum_{j=1}^{p}\boldsymbol{A}_j\boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t \tag{2.1}$$

for $t = 1, \ldots, T$, where $\boldsymbol{y}_t$ is an $n\times1$ vector of endogenous variables, $\boldsymbol{\epsilon}_t$ an $n\times1$ vector white noise process, normally distributed with mean zero and variance-covariance matrix $\boldsymbol{I}_n$, $\boldsymbol{A}_j$ is an $n\times n$ matrix of structural coefficient for $j = 0, \ldots, p$. As is usual in the literature, structural disturbances are assumed to be uncorrelated. The initial conditions $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_p$ are given. Let $\boldsymbol{\theta} = (\boldsymbol{A}_0, \boldsymbol{A}_+)$ collect the structural parameters, where $\boldsymbol{A}_+ = (\boldsymbol{a}, \boldsymbol{A}_j)$ for $j = 1, \ldots, p$. The reduced-form VAR is as follows:

$$\boldsymbol{y}_t = \boldsymbol{b} + \sum_{j=1}^{p}\boldsymbol{B}_j\boldsymbol{y}_{t-j} + \boldsymbol{u}_t, \tag{2.2}$$

where $\boldsymbol{b} = \boldsymbol{A}_0^{-1}\boldsymbol{a}$ is an $n \times 1$ vector of constants, $\boldsymbol{B}_j = \boldsymbol{A}_0^{-1}\boldsymbol{A}_j$, $\boldsymbol{u}_t = \boldsymbol{A}_0^{-1}\boldsymbol{\epsilon}_t$ denotes the $n \times 1$ vector of reduced-form errors. $var(\boldsymbol{u}_t) = E(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma} = \boldsymbol{A}_0^{-1}(\boldsymbol{A}_0^{-1})'$ is the $n \times n$ variance-covariance matrix of reduced-form errors. Let $\boldsymbol{\phi} = (\boldsymbol{B}, \boldsymbol{\Sigma}) \in \boldsymbol{\Phi}$ collect the reduced-form parameters, where $\boldsymbol{B} \equiv [\boldsymbol{b}, \boldsymbol{B}_1, \ldots, \boldsymbol{B}_p]$, $\boldsymbol{\Phi} \subset \mathcal{R}^{n+n^2p} \times \boldsymbol{\Xi}$, and $\boldsymbol{\Xi}$ is the space of symmetric positive semidefinite matrices. Let the $n \times n$ matrix

$$\boldsymbol{IR}^h = \boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{A}_0^{-1} \tag{2.3}$$

be the impulse response at h-th horizon for $h = 0, 1, \ldots,$ where $\boldsymbol{C}_h(\boldsymbol{B})$ is the h-th coefficient matrix of $(\boldsymbol{I}_n - \sum_{h=1}^{p}\boldsymbol{B}_hL^h)^{-1}$. Its (i, j)-element denotes the effect on the i-th variable in $\boldsymbol{y}_{t+h}$ of a unit shock to the j-th element of $\boldsymbol{\epsilon}_t$.

As in Uhlig (2005), we define the set of all IRFs through an $n\times n$ orthonormal matrix $\boldsymbol{Q} \in \boldsymbol{\Theta}(n)$, where $\boldsymbol{\Theta}(n)$ characterizes the set of all orthonormal $n \times n$ matrices. Uhlig (2005) showed that $\{\boldsymbol{A}_0 = \boldsymbol{Q}'\boldsymbol{\Sigma}_{tr}^{-1} : \boldsymbol{Q} \in \boldsymbol{\Theta}(n)\}$ is the set of observationally equivalent

$\boldsymbol{A}_0$'s consistent with reduced-form parameters, where $\boldsymbol{\Sigma}$ relates to $\boldsymbol{A}_0$ by $\boldsymbol{\Sigma} = \boldsymbol{A}_0^{-1}(\boldsymbol{A}_0^{-1})'$, $\boldsymbol{\Sigma}_{tr}$ denotes the lower triangular Cholesky matrix with non-negative diagonal coefficients of $\boldsymbol{\Sigma}$. The likelihood function depends on $\boldsymbol{\phi}$ and does not contain any information about $\boldsymbol{Q}$, leading to ambiguity in decomposing $\boldsymbol{\Sigma}$. Thus, there is a multiplicity of $\boldsymbol{Q}$'s which deliver $\boldsymbol{A}_0$ given $\boldsymbol{\phi}$. Specifically, the impulse response of variable i to shock j at horizon h, i.e., (i,j)-element of $\boldsymbol{IR}^h$, can be expressed as $\boldsymbol{e}_i'\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}\boldsymbol{Q}\boldsymbol{e}_j \equiv \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{q}_j$, where $\boldsymbol{e}_i$ is the i-th column vector of $\boldsymbol{I}_n$, $\boldsymbol{q}_j$ is the j-th column of $\boldsymbol{Q}$ and $\boldsymbol{c}_{ih}'(\boldsymbol{\phi})$ represents the i-th row vector of $\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}$. Identification therefore requires to placing a set of restrictions on $\boldsymbol{Q}$. For example, imposing $\boldsymbol{Q} = \boldsymbol{I}_n$ implies a recursive ordering identification, i.e., the Cholesky decomposition, whereas sign restrictions specify a set of admissible $\boldsymbol{Q}$'s.

2.2 Identification Strategy

Our identification scheme identifies $k \leq n$ shocks $j \in 1, \dots, k$, denoted by $\boldsymbol{q}_j = \boldsymbol{Q}\boldsymbol{e}_j$, where $\boldsymbol{q}_j'\boldsymbol{q}_{\tilde{j}} = 0$ for $j \neq \tilde{j}$ is the standard orthogonality condition. Specifically, in our application $k = 3$ and shocks of interest are macro uncertainty, financial uncertainty, and credit supply disturbances. For clarity, we will breakdown identification into 2 steps, but in practice that corresponds to solve a unique maximization problem.

Say we want to identify macro uncertainty shocks: we assume that macro uncertainty disturbances explain the unexpected movements of macro uncertainty variable more than the fluctuations of financial uncertainty and credit spreads upon impact (Step 1), and that is the key assumption. Among these candidates, the innovation that maximizes its contribution to an objective function consisting of the movements of macro uncertainty variable is identified as the macro uncertainty shock (Step 2, instrumental to reduce the identification uncertainty[7]). The same methodology applies to identification of financial uncertainty and credit supply shocks. However, we point out that this approach does not take any stand about the exogeneity/endogeneity feature of uncertainty: uncertainty can impact on macro variables, and the latter can hugely affect uncertainty (in principle, even more than uncertainty disturbances). In fact, Section 4 shows that our identification

[7]Step 2 stresses the idea that movements in uncertainty are substantially driven by uncertainty disturbances (after controlling for other sources of uncertainty and financial conditions), while unanticipated deterioration in credit conditions are largely caused by an adverse financial shock (after controlling for uncertainty). Among others, this is consistent with some literature (Caldara et al., 2016; Brianti, 2021) claiming that the shocks should have the maximum effect on their endogenous counterpart (after the appropriate controls).

assumptions are consistent with structural models regardless whether those frameworks consider endogenous or exogenous uncertainty, and successfully recover the impulse response functions of different DGPs.

We now formalize the strategy above. Let $CFEV_j^i(\tilde{h})$ denote the FEV at horizon $\tilde{h}$ of variable i explained by the j-th structural shock:

$$CFEV_j^i(\tilde{h}) = \boldsymbol{q}_j' \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}\boldsymbol{q}_j, \tag{2.4}$$

$$= \boldsymbol{q}_j' \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{i}}(\boldsymbol{\phi})\boldsymbol{q}_j, \tag{2.5}$$

where $\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{i}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}$ is a $n \times n$ positive semidefinite matrix. Equation (2.5) describes the contribution between 0 and 1 of the shock j to the unexpected fluctuations of variable i at horizon $\tilde{h}$.

Without loss of generality, suppose that (i) $j = 1$ is the first shock, $j = 2$ is the second shock, $j = 3$ is the third shock and so on; (ii) the n endogenous variables are ordered such that $i = 1$ is the macro uncertainty variable, $i = 2$ is the financial uncertainty variable, and $i = 3$ is the credit spreads. Define the following $I_{-j} = \{1, \dots, k\}/\{j\}$ as a subset of the shocks of interest. Thus, the identification of $\boldsymbol{Q}_{1:k} = [\boldsymbol{q}_1, \boldsymbol{q}_2, \dots, \boldsymbol{q}_k]$, with $k = 3$ and $j \in 1, 2, 3$, requires to solve the following constrained optimization problem:[8]

$$\boldsymbol{Q}_{1:k}^* = \arg\max_{\boldsymbol{Q}_{1:k}} \sum_{i=1}^{k} \boldsymbol{q}_i' \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{i}}(\boldsymbol{\phi})\boldsymbol{q}_i \tag{2.6}$$

subject to

$$\boldsymbol{q}_j' \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{j}}(\boldsymbol{\phi})\boldsymbol{q}_j \geq \boldsymbol{q}_j' \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{i}}(\boldsymbol{\phi})\boldsymbol{q}_j \ for\ j = 1, \dots, k, \quad \forall i \in I_{-j} \tag{2.7}$$

and

$$\boldsymbol{Q}_{1:k}' \boldsymbol{Q}_{1:k} = \boldsymbol{I}_n. \tag{2.8}$$

In our application, we set $\tilde{h} = 0$. For $j = 1$, we are identifying the macro uncertainty shock as the innovation that maximizes its contribution to the FEV of the macro uncertainty variable subject to the following constraints. Restrictions (2.7) establish that (for $j = 1$) the contribution of the macro uncertainty shock to the FEV of the macro uncertainty variable must be higher than the contribution to the FEV of financial uncertainty

[8]Once columns 1 to k are identified, we can always construct orthogonal columns $k + 1$ to n.

variables and credit spreads (upon impact in the application). Those restrictions are instrumental to separate macro uncertainty shocks from financial uncertainty and credit supply disturbances. For $j = 2, 3$, problem (2.6)-(2.8) identifies financial uncertainty and credit supply shock, respectively. Restrictions (2.8) makes sure that the identified shocks are orthogonal to each other.[9] As a shorthand notation, let $\boldsymbol{\Gamma}(\boldsymbol{\phi}, \boldsymbol{Q}) \geq \mathbf{0}$ be the whole set of inequality constraints on the FEVD represented by (2.7).[10]

Our strategy simultaneously identifies three shocks and forces them to be orthogonal. A well-known caveat of single identification is that one-at-a-time identified shocks can be still correlated to each other; this seems particularly relevant in uncertainty literature. For instance, Cascaldi-Garcia and Galvão (2020) showed that news and uncertainty shocks tend to be correlated, and as such are not truly structural, if identified separately; Caldara et al. (2016) separated uncertainty and financial shocks by imposing ordering restrictions, finding that the order hugely affects the results. The cost of allowing simultaneous identification of a multiplicity of shocks is that the optimization problem is in principle non-convex. However, in Section 3 we establish mild conditions under which the problem is easy to solve and computationally (extremely) feasible.

2.3 Discussion

The machinery of this paper, which has a natural and interesting application to uncertainty and financial shocks, can be applied in any empirical setting where sign restrictions are not enough/not available to identify and separate competing disturbances. However, nothing prevents scholars from combining the methodology here with sign restrictions as constraints in equation 2.7, and this should be evaluated depending on the specific application. Researchers often have qualitative prior information beyond the sign of impulse-responses that can be used for identification of multiple shocks. De Graeve and Karas (2014) argued bank runs (adverse deposit market supply shocks) hit uninsured banks harder than insured institutions. Kilian and Murphy (2012) claimed that qualitative information beyond sign restrictions is necessary to distinguish demand and supply shocks

[9] The orthogonality restriction matters only if we restrict multiple shocks simultaneously. For individual identification, we can always construct vectors in the Nullspace of the restricted shocks.

[10] In a completely different setup, i.e., identification of monetary policy via sign restrictions, Wolf (2020) stresses that in principle inequality constraints are necessary, but not sufficient, to successfully separate shocks because linear combinations of structural shocks can still satisfy the constraints. We find this is not the case in our Monte-Carlo experiment and empirical application.

in the oil market. Similarly, separation between news and surprise shocks requires to rank the relative effect of those disturbances over target variables (an application to productivity surprise and new shocks is provided by Amir-Ahmadi and Drautzburg (2021)). For example, we might argue that (lump-sum) fiscal transfers increase the spending of leveraged households more than those with low leverage. Also, conditional on how the payments are funded, some households might reduce spending and private consumption, e.g., if their taxation goes up. Thus, we might want to relax sign restrictions and rely on constraining the ability of the shock to explain fluctuations in private consumption. In order to separate credit and housing shocks, Furlanetto et al. (2017) argued that the former explain variation of total credits to households and firms more than the contributions to the fluctuations in the real estate value, and the other way around. We view these typologies of restrictions as underutilized, especially for identification of multiple shocks, and formally provide a machinery to systematically use them in a context of point-identification, avoiding the drawbacks of set-identification that affect most of the aforementioned studies (Baumeister & Hamilton, 2015; Giacomini & Kitagawa, 2020).

3 Estimation and Inference

This section illustrates estimation and inference. Section 3.1 provides conditions for the existence of solutions to the optimisation problem; Section 3.2 describes uniqueness of the solution and practical implementation of the identification strategy.

3.1 Non-Emptiness

There is a trade-off between sharp identification and computation (Amir-Ahmadi & Drautzburg, 2021; Giacomini & Kitagawa, 2020; Giacomini et al., 2020; Gafarov et al., 2018; Granziera et al., 2018; Volpicella, 2021; Uhlig, 2017), especially when using inequality constraints. In fact, tight restrictions can potentially lead to unfeasible, or empty, regions, i.e., constraints (2.7) are rejected by data. It is therefore of paramount importance to distinguish when the identification is sharp and when constraints are too tight and lead to unfeasible sets. Thus, this section addresses the trade-off and provides sufficient conditions for assessing whether a solution to the optimization problem exists, i.e., the proposed identification strategy is not rejected by data.

In order to make this section self-contained, it is useful to remind that $\boldsymbol{q}_j^*$ for $j{-}1,\ldots,k$ denotes the j-th column of the identified matrix $\boldsymbol{Q}_{1:k}^*$. For $j=1$, given the constraints in (2.7)-(2.8), we define the following functions:

$$
\begin{aligned}
f_1 &= \frac{1}{2}\boldsymbol{q}_1' \left[\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{2}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{1}}(\boldsymbol{\phi})\right] \boldsymbol{q}_1, \\
f_2 &= \frac{1}{2}\boldsymbol{q}_1' \left[\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{3}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{1}}(\boldsymbol{\phi})\right] \boldsymbol{q}_1, \\
f_3 &= \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{q}_1 + \frac{1}{2}, \\
f_4 &= \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{q}_1 - \frac{1}{2}.
\end{aligned}
$$

Similar functions can be trivially defined for $j=2,\ldots,k$. We omit it for brevity. In the following we establish a Gordan type alternative theorem, which will be instrumental to obtain the existence result for our optimization problem.

Proposition 3.1 *Assume* $j=1$. *If* $\nexists \boldsymbol{\lambda} \in \mathcal{R}_+^4\backslash\{0\}$ *such that* $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$, $\boldsymbol{q}_1^*$ *exists.*

The proof is provided in the Appendix. This proposition rules out that, for a given shock, restrictions contradict each other and, more generally, that linear combinations of inequality constraints on the FEV violate restrictions. Put it another way, above proposition establishes existence ignoring orthogonality condition, which however is essential in identifying multiple shocks.

The next proposition establishes feasibility of the optimization problem, where $\boldsymbol{\sigma}$ indicates a permutation of $1,\ldots,k$ among the $k!$ possible permutations and $\boldsymbol{\sigma}(z)$ for $z=1,\ldots,k$ indicates the z-th element of the permutation $\boldsymbol{\sigma}$.

Proposition 3.2 *(Existence) If there exists a permutation* $\boldsymbol{\sigma}$ *such that*

i) for $j=\boldsymbol{\sigma}(1)$ *Proposition 3.1 is satisfied,*

ii) conditions in Proposition 3.1 are met for all $j=\boldsymbol{\sigma}(2),\ldots,\boldsymbol{\sigma}(k)$ *in the Nullspace of the previous* $j-1$ *shocks,*

then $\boldsymbol{Q}_{1:k}^*$ *exists.*

The Appendix provides technical details, here we point out the intuition. The permutation is instrumental to find at least one matrix $\boldsymbol{Q}_{1:k}^*$ such that its first k columns

$\boldsymbol{q}_1^*, \ldots, \boldsymbol{q}_k^*$, which satisfy restrictions as per Proposition 3.1, are orthogonal to each other. Proposition 3.2 is imaginably affected by a gray area, where sufficient conditions fail. However, in our application we have never met a single case where those assumptions do not hold; otherwise, a numerical procedure illustrated in Section 3.2 can be used to investigate feasibility of the optimization region. The above propositions can be easily adjusted to include sign restrictions, that are linear inequality constraints on $\boldsymbol{Q}_{1:k}$. Although those restrictions are not imposed in our empirical exercise, the extension allows users to combine the scheme here with more classical restrictions, depending on the application.

3.2 Uniqueness

This section provides the methodology and a practical algorithm for running identification and estimation. The optimization problem is non-convex, e.g., constraints are potentially non-convex. In principle, this would imply a multiplicity of solutions, and require complex and extremely time-consuming solvers, with possibility to find only an approximation of the true optimizer.

However, below we establish some sufficient conditions, which are remarkably easy to check and have an economic intuition, such that $\boldsymbol{Q}_{1:k}^*$ is unique and the problem gets tractable and computationally feasible.

Proposition 3.3 *(Uniqueness) Assume that $\boldsymbol{Q}_{1:k}^*$ exists and is orthogonal. If $\boldsymbol{c}'_{ih}(\boldsymbol{\phi})\boldsymbol{q}_j \geq 0$ for $i, j = 1, 2, 3$, $h = 0, \ldots, \tilde{h}$, then $\boldsymbol{Q}_{1:k}^*$ is unique.*

The formal proof is provided in the Appendix; here it is worth stressing the intuition. If there is a positive feedback (in both directions) between uncertainty and credit supply shocks, which is consistent with economic theory and previous empirical evidence, the orthogonal existent $\boldsymbol{Q}_{1:k}^*$ is selected over a closed convex feasibility region. Uniqueness follows.

In practice, in our application we have never found a case where those conditions are violated. Below Algorithm delivers the posterior of the impulse response functions (or any structural object) of interest. Note that if Proposition 3.3 is not satisfied, researchers can still implement the identification strategy, but they would need to check for multiple optima in Step 3 below.

Algorithm 3.1

Step 1: Draw $\boldsymbol{\phi}$ *from the posterior distribution of the reduced-form VAR.*

Step 2: Check that the problem is feasible (Proposition 3.2).

Step 3: Obtain $\boldsymbol{Q}^*_{1:k}$ *by solving the optimization problem and compute the impulse response functions as per (2.3).*

Step 4: Repeat Step 1-3 L times, e.g. $L = 1000$.

The algorithm depends on a conventional sampling from the posterior of reduced-form parameters (Step 1), the investigation of feasibility (Step 2) and a numerical optimization (Step 3). The latter consists of a quadratic objective function with quadratic constraints, which can be reduced to a much more tractable problem under Proposition 3.3. The estimation and inference is Bayesian as applied users tend to work with such tools.[11] However, since the optimization problem and subsequent results are conditional on the reduced-form parameters ($\boldsymbol{\phi}$) only and are not dependent on a specified prior over $\boldsymbol{Q}$, it is also frequentist friendly as Step 1 could be replaced by maximum likelihood estimate for $\boldsymbol{\phi}$ and previous properties could be derived conditioning on it.

Finally, when conditions in Proposition 3.2 do not hold, researchers can consider the optimization region unfeasible if Step 3 in Algorithm 3.1 cannot detect an interior solution for a variety of starting points. Alternatively, one can stick to the conventional literature and think of the feasibility region as empty whether, for many draws from the orthonormal space, an admissible rotation matrix $\boldsymbol{Q}$ cannot be discovered.

4 Monte-Carlo Exercise

This section provides some support to the identification assumptions. Also, here we show that our scheme recovers well the impulse response functions regardless the exogenous or endogenous feature of uncertainty in the DGP.

[11] The reasons, including curse of dimensionality and prior belief specification, have been long investigated and are not the scope of this paper.

4.1 Assessing the Identifying Restrictions

First, we aim evaluating our identification scheme by taking a model-based approach. In particular, the key assumption is that in the short-run a truly macro uncertainty shock explains the variation of macro uncertainty more than the fluctuations of financial uncertainty and credit spread (similarly for financial uncertainty and credit supply disturbances).

Since Bloom (2009) macroeconomic literature developed models where uncertainty is an exogenous source of business fluctuations (see discussion and references in Section 1). In this case, our identification assumptions for uncertainty disturbances are always satisfied; for instance, those frameworks, which usually consider a single source of uncertainty, mechanically suggest that macro uncertainty shock explains 100% of the within period variation of macro uncertainty. Recent efforts explicitly combine both (a single form of) uncertainty and credit supply shocks (Brianti, 2021; Gilchrist et al., 2014; Alfaro et al., 2018): while uncertainty is exogenous by construction, those models suggest that, although uncertainty impacts on credit spreads, within period the latter are mostly driven by credit supply shocks, which is fully consistent with our identification strategy. More formally, we can extend this result to a context with various sources of uncertainty; for example, Shin and Zhong (2020) built upon Basu and Bundick (2017) and Gertler and Karadi (2011) to construct a DSGE model with financial frictions (and credit supply shocks), exogenous macro uncertainty (as TFP volatility), and exogenous financial uncertainty (as capital quality volatility). Our identifying restrictions are confirmed by employing both the baseline parameterization in Shin and Zhong (2020)[12] and their battery of alternative calibrations[13]

On the other hand, literature started modeling endogenous uncertainty Van Nieuwerburgh and Veldkamp (2006); Bachmann and Moscarini (2011); Fajgelbaum et al. (2017); Brunnermeier and Sannikov (2014); Atkinson et al. (2021); Plante et al. (2018). In particular, Atkinson et al. (2021) departed from a Cobb-Douglas production function and suggested that complementarity between capital and labor inputs can generate endogenous uncertainty because the concavity in the production influences how output responds to productivity shocks. When matching labor share and uncertainty moments, they found

[12]See Table A-7 of their paper.
[13]See Section D.2.1 of the their paper.

16% of the volatility of uncertainty is endogenous in the short-run. However, even in that case credit shocks are not able to explain short-run fluctuations of the uncertainty proxy more than uncertainty disturbances, which is in line with our identification approach.

4.2 Simulation Results

This sections provides some evidence that our approach can recover the impulse response functions of DGP with different features in terms of endogenous/exogenous uncertainty. We first employ a SVAR with endogenous uncertainty as DGP and generate artificial data for industrial production (IP), financial uncertainty (uF*), credit spread (CS), price index (PCEPI), monetary policy rate (FFR), and macro uncertainty (uM*). In order to generate endogenous uncertainty, macro and financial uncertainty are ordered after the other covariates. In the baseline scenario of Figure 1, financial uncertainty is ordered before macro uncertainty, but the results still hold if we reverse the order between uncertainty disturbances. The DGP is parameterised at the Maximum Likelihood estimation with monthly US data, 1962 to 2016.

We then use artificial data to estimate the impulse response functions. For brevity, here we provide simulated results mostly for financial uncertainty shocks, but insights below apply to macro uncertainty and credit supply disturbances as well. Figure 1 shows that our identification scheme recovers well the impulse response functions of the DGP for real activity, uncertainty, and credit conditions to financial uncertainty disturbances. In each panel, the blue line denotes the DGP responses. Note the different scale across rows of the figure. In the first row, we employ our scheme to estimate the impulse responses (black lines). According to panels (a), (b) and (c), our strategy works well. In the second row (panels (a′), (b′) and (c′)), in the estimated model the macro uncertainty response is shut down in the short run (up to $h = 4$ months, but the results remain unchanged for $h = 0, 1, \dots, 6$); this corresponds to estimate a single source of uncertainty, i.e., financial uncertainty, while neglecting distinct forms of uncertainty in the DGP: both responses of real activity and credit spreads are biased. In the third row (panels (a″), (b″) and (c″)), the estimated model assumes that financial uncertainty is exogenous, i.e., no shock can contemporaneously affect it, leading to biased impulse responses. Put it another way, assuming uncertainty exogeneity while in the DGP there is some endogeneity causes misleading findings. In the fourth row (panels (a‴), (b‴) and (c‴)), impulse responses are estimated by replacing inequality constraints on the FEV with ordering restrictions between financial and uncertainty disturbances echoing the spirit of Caldara et al. (2016). However, impulse responses are biased, putting further evidence that inequality restrictions on the FEV are a useful identification tool. Overall, there is evidence that our identification strategy can successfully identify the uncertainty shocks when some degree of endogeneity occurs.

We now investigate the effectiveness of our scheme when uncertainty is an exogenous cause of macroeconomic fluctuations. Accordingly, here we consider a DGP where uncertainty is ordered before the other variables. Figure 2 provides the same insights as the previous simulation: our scheme recovers well the response of real activity and credit spreads to uncertainty shocks; on the other hand, omitting distinctive sources of uncertainty (second row), imposing endogeneity when this is absent in the DGP (third row), and removing the inequality constraints on the FEV (fourth row) lead to biased impulse responses.

Finally, if inequality restrictions on the FEV were enough to truly separate the three shocks, orthogonality conditions should not make a big difference. Thus, we reran the estimation without the orthogonality assumption, finding the same results (both quantitatively and qualitatively). This puts further (informal) evidence that inequality restrictions on the FEV are able to disentangle the competing disturbances and is confirmed in our empirical exercise as well.

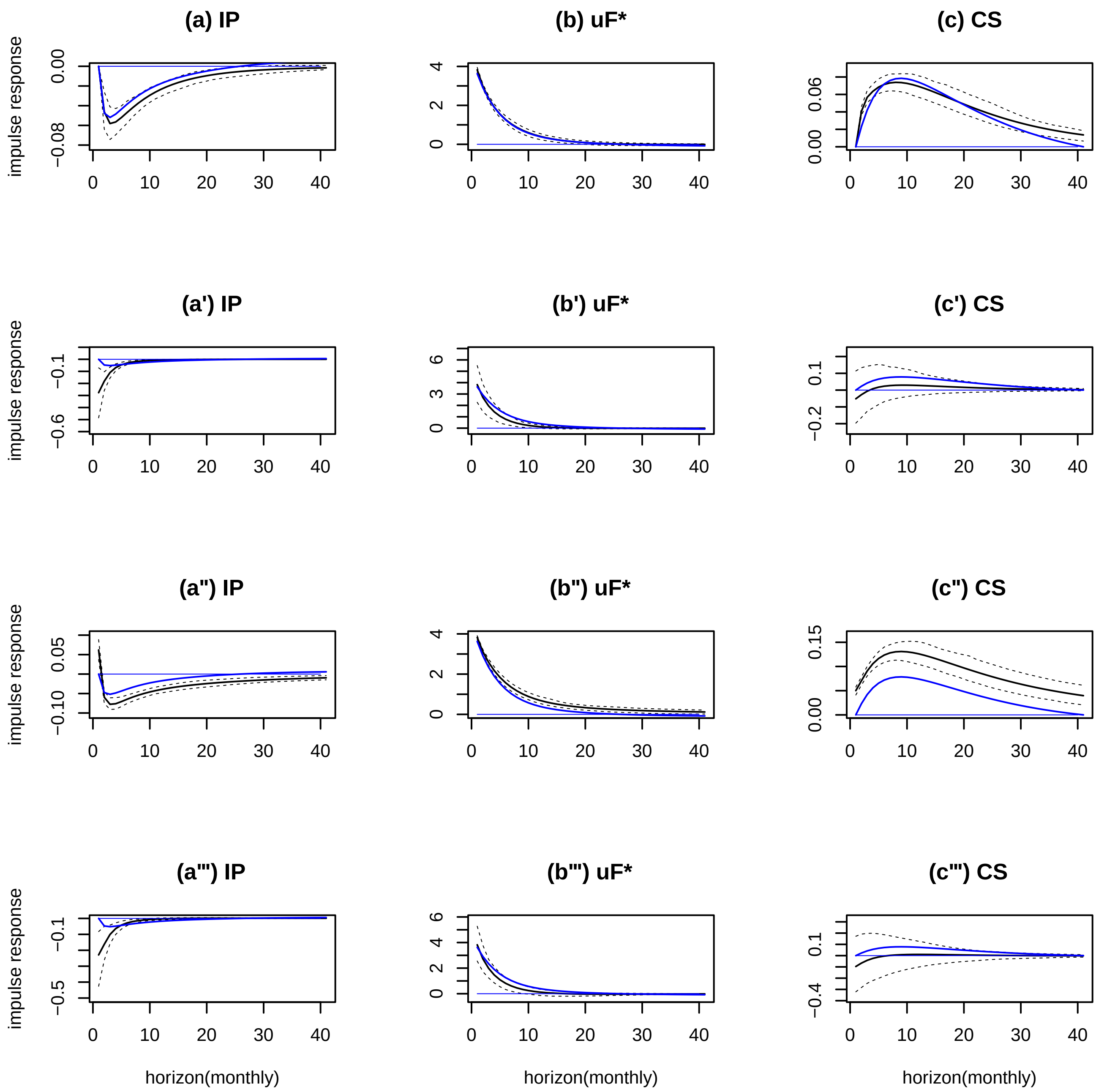

Figure 1: DGP with endogenous uncertainty: estimated responses to financial uncertainty shock

The blue line denotes the impulse responses to uncertainty shocks in the DGP. The black solid line represents the posterior mean of the estimated impulse responses, where in the first row ((a), (b), (c)) responses are identified through constraints on the FEV; in the second row ((a′), (b′), (c′)) the macro uncertainty response is shut down; in the third row ((a′′), (b′′), (c′′)) the uncertainty is estimated as exogenous; in the fourth row (panels (a′′′), (b′′′) and (c′′′)), impulse responses are estimated by replacing inequality constraints on the FEV with ordering restrictions between financial and uncertainty disturbances. The dashed black lines display the 68% Bayesian credibility region across replications. Shock size is set to 1 standard deviation.

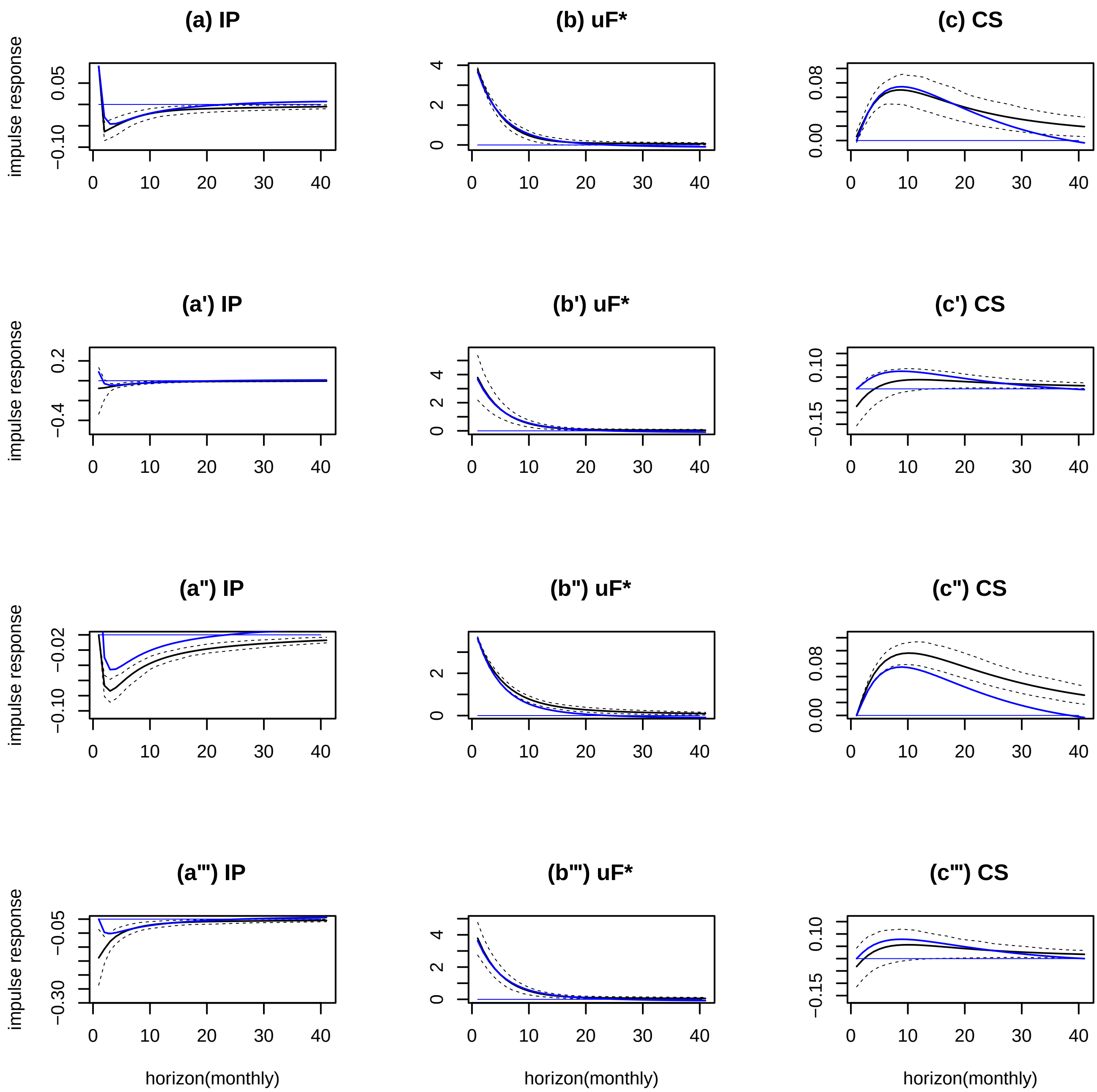

Figure 2: DGP with exogenous uncertainty: estimated responses to financial uncertainty shock

The blue line denotes the impulse responses to uncertainty shocks in the DGP. The black solid line represents the posterior mean of the estimated impulse responses, where in the first row ((a), (b), (c)) responses are identified through constraints on the FEV; in the second row ((a′), (b′), (c′)) the macro uncertainty response is shut down; in the third row ((a″), (b″), (c″)) the uncertainty is estimated as endogenous; in the fourth row (panels (a‴), (b‴) and (c‴)), impulse responses are estimated by replacing inequality constraints on the FEV with ordering restrictions between financial and uncertainty disturbances. The dashed black lines display the 68% Bayesian credibility region across replications. Shock size is set to 1 standard deviation.

5 Application: The Effects of Uncertainty and Financial Shocks

This section introduces the specification and data used for running the estimation (Section 5.1), illustrates the effect of uncertainty (Section 5.2) and credit supply shocks (Section 5.3), presents comparison with alternative identification strategies (Section 5.4) and describes the consequences of our findings for theoretical modelling (Section 5.5). Appendix B provides further evidence that the three shocks we identify are truly structural.

5.1 Specification and Data

Evaluating the relationship between economic variables and uncertainty needs selecting both a concept and metric of uncertainty. Several notions and measures of uncertainty have been put forward; accordingly, we consider a range of measures. In the baseline scenario, we employ one measure of financial uncertainty and one measure of macro uncertainty. For the former, we rely on the VXO, namely the Chicago Board Options Exchange S&P 100 Volatility Index; for the latter, we employ the measure of macro uncertainty developed by Jurado et al. (2015) (JLN hereafter). We have tested the robustness of our results to competing measures: for financial uncertainty, we also consider the measures of Carriero et al. (2018b) and Jurado et al. (2015); for macro uncertainty, we also make use of the measure of Carriero et al. (2018b).

We rely on VARs with US monthly data. The model includes 12 variables: macro uncertainty measure (JLN), financial uncertainty measure (VXO), credit spreads (CS), number of non-farm workers (PAYEM), industrial production (IP), weekly hours per worker (HOURS), real consumer spending (SPEND), real manufacturers' new orders (ORDER), real average earnings (EARNI), PCE price index (PCEPI), variation of federal funds rate (FFR), S&P 500 (S&P). The credit spread is measured as the difference between the BAA Corporate Bond Yield and the 10-year Treasury Constant Maturity rate; results are robust to employing the excess bond premium used in Caldara et al. (2016) and developed by Gilchrist and Zakrajšek (2012). We employ 7 lags,[14] a diffuse Normal Inverse Wishart prior and run the estimation over the sample from 1962m7 to

[14] This has been selected by maximizing the marginal likelihood.

2016m12. We have considered the first difference of logs, except for ORDER, PCEPI, FFR, CS, VXO, and JLN. All the variables are demeaned and the dataset is provided by Carriero et al. (2021). In order to make comparison with previous studies simpler, the impulse responses are cumulated and the units of the responses are percentage point changes (or rates for covariates not in log terms). Since some variables are differenced for stationarity, the long-run effects of transitory shocks do not vanish for those variables.

5.2 The Effects of Uncertainty Shocks

Figure 3 and Figure 4 show the impulse responses to macro and financial uncertainty shocks, respectively. Uncertainty has a strong recessionary effect on employment, industrial production, hours worked, consumer spending, investment, and earnings; the financial conditions also deteriorate, as shown by the response of stock market and credit spreads. Also, macro (financial) uncertainty increases financial (macro) uncertainty. Furthermore, the shock leads to expansionary monetary policy, that tries to counteract the depressive effect of uncertainty.

To facilitate comparisons, Figure 5 overlays the impulse responses shown in Figure 3 and 4. The effects of macro and financial uncertainty are qualitatively comparable; however, quantitatively some differences are substantial. The recessionary effect on real activity variables seem more pronounced for macro uncertainty, while credit spreads increase more with financial uncertainty shocks. Interestingly, we find a strong evidence in favor of a negative response of prices, that is short-lived for macro uncertainty and more persistent for financial uncertainty shocks, suggesting that uncertainty disturbances mimic demand shocks, namely they trigger a recession and a deflationary pressure on the economy. The slightly looser response of monetary policy for financial uncertainty is likely driven by the more significant drop in prices relative to macro uncertainty.

In the empirical literature the impact of uncertainty disturbances on inflation is mixed. Caggiano et al. (2014), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017) provided some empirical evidence that uncertainty is deflationary, while Mumtaz and Theodoridis (2015) found the opposite. Carriero et al. (2018b) and Katayama and Kim (2018) argued that the effect of uncertainty on prices is not significant; the international evidence in Carriero et al. (2018a) pointed out that the reaction of prices is country-specific and heterogeneous across the alternative measures of prices. In other contributions, the effect of uncertainty shocks on prices is not estimated (Jurado et al., 2015; Baker et al., 2016; Jo & Sekkel, 2017). While the effects of uncertainty on prices are substantially different across these articles, the common feature is the results strictly depend on some controversial and highly debatable recursive identification schemes; for most of them, uncertainty is exogenous. Interestingly, papers identifying uncertainty shocks without relying on exogeneity assumption do not provide clear-cut findings about the effects on prices. Angelini et al. (2019) and Ludvigson et al. (2021) have not included prices in the analysis. Carriero et al. (2021) found mixed results: macro uncertainty seems to be inflationary, while the response of prices to financial uncertainty (available in a previous working paper) is more heterogeneous as response of prices depends on data frequency.

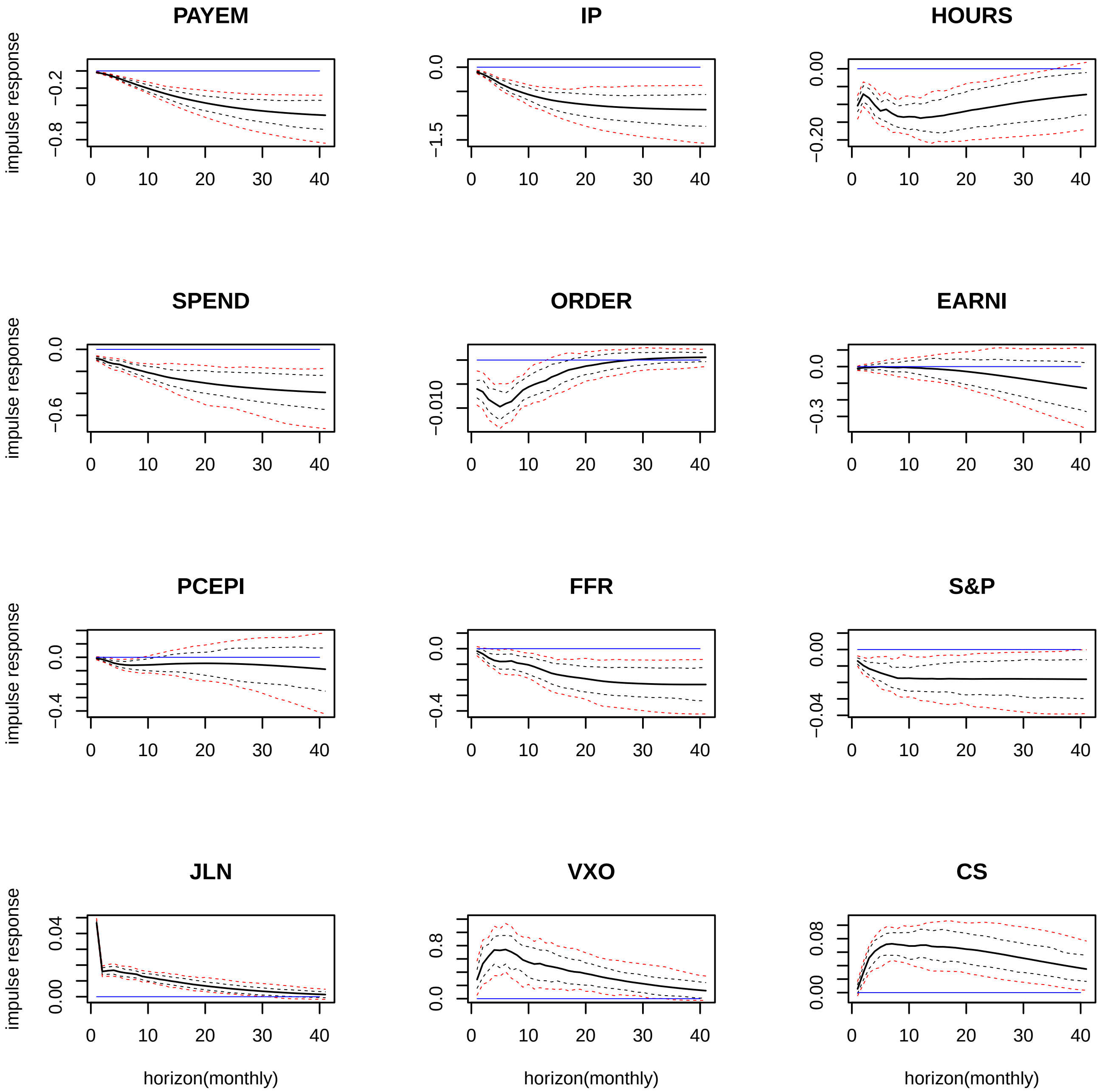

Figure 3: Responses to macro uncertainty shocks

Figure 3 reports the posterior median (black solid lines), the 68% Bayesian credibility region (black dashed lines), and the 90% Bayesian credibility region (red dashed lines) of the impulse response functions to macro uncertainty shocks. The blue solid line is the zero line. The shock size is set to one standard deviation.

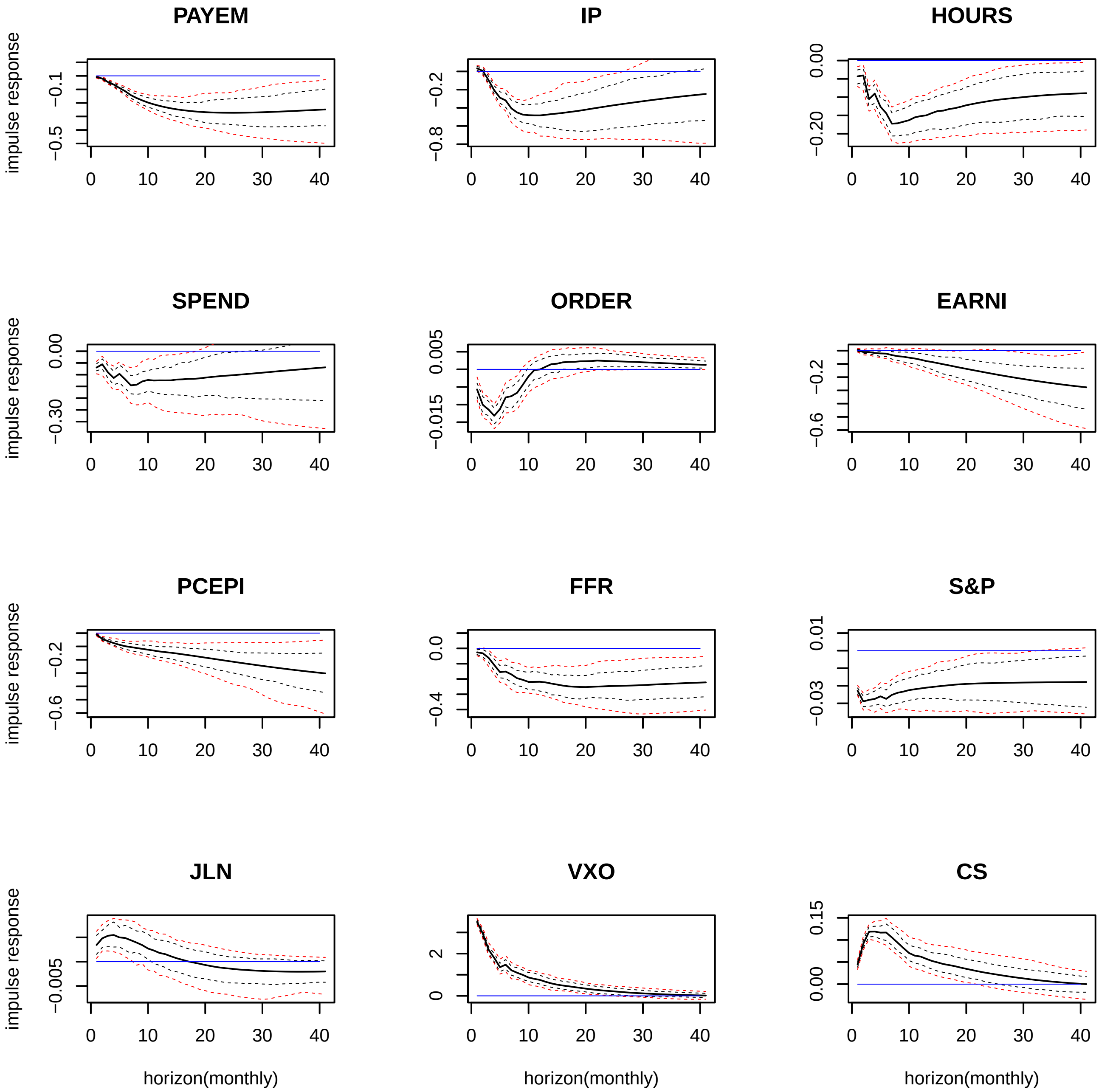

Figure 4: Responses to financial uncertainty shocks

Figure 4 reports the posterior median (black solid lines), the 68% Bayesian credibility region (black dashed lines), and the 90% Bayesian credibility region (red dashed lines) of the impulse response functions to financial uncertainty shocks. The blue solid line is the zero line. The shock size is set to one standard deviation.

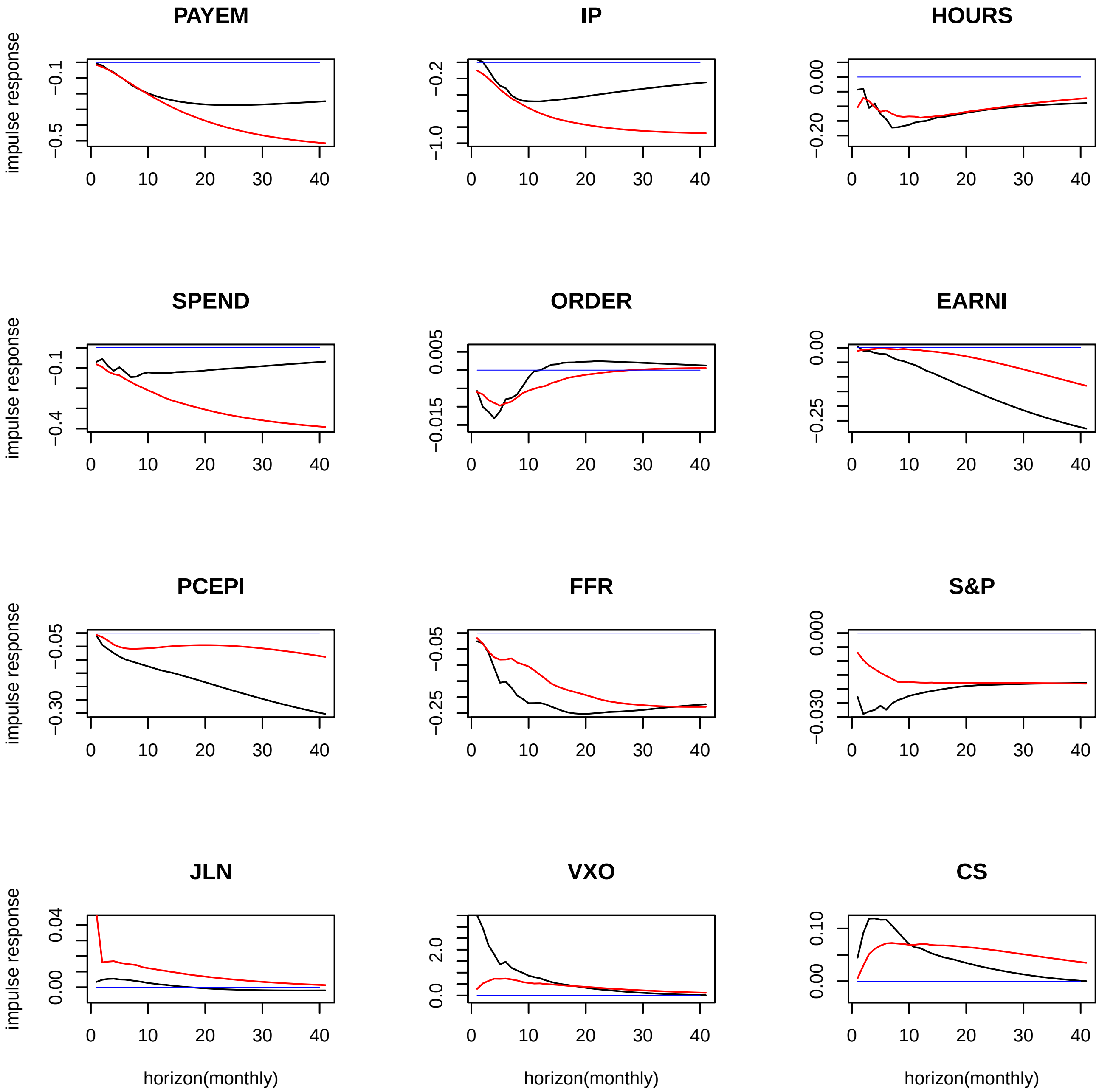

Figure 5: Comparison between macro and financial uncertainty shocks

Figure 5 reports the posterior median of the impulse response functions to macro (red line) and financial (black line) uncertainty shocks. The blue solid line is the zero line. The shock size is set to one standard deviation.

5.2.1 Endogenous Uncertainty?

Since our scheme does not restrict feedback from economic and financial variables to uncertainty, it provides a natural ground to look into endogeneity of uncertainty. We add the following restriction to our identification strategy: for each measure of uncertainty we assume that upon impact the latter cannot be affected by structural shocks other than uncertainty, i.e., uncertainty is contemporaneously exogenous. This is equivalent to order uncertainty first in a Cholesky decomposition scheme. Panels (a)-(l) in Figure 6 display the responses to macro uncertainty shocks for the baseline identification (black line) and when contemporaneous exogeneity for macro uncertainty is assumed (red line): imposing exogeneity clearly biases the impulse response functions for several variables. On the other hand, panels (a′)-(l′) display the responses to financial uncertainty shocks for the baseline identification (black line) and when contemporaneous exogeneity for financial uncertainty is assumed (red line): the responses seem relatively consistent with the belief of uncertainty as mostly exogenous phenomenon. Since our model is point-identified, we formally tested the exogeneity restrictions above, with the null being $(\boldsymbol{q}_1^*)'\boldsymbol{\Upsilon}_0^1(\boldsymbol{\phi})\boldsymbol{q}_1^* = 1$ for macro uncertainty and $(\boldsymbol{q}_2^*)'\boldsymbol{\Upsilon}_0^2(\boldsymbol{\phi})\boldsymbol{q}_2^* = 1$ for financial uncertainty. We find that exogeneity is rejected at 1% significance level for both macro and financial uncertainty. However, if we remove stock index and credit spreads, financial uncertainty turns out to be exogenous. Thus, we conclude that there is no feedback from real variables to financial uncertainty. This is in line with Ludvigson et al. (2021), who argued that, while financial uncertainty is mainly exogenous, macro uncertainty presents some endogeneity; Angelini et al. (2019) found that both macro and financial uncertainty are mostly exogenous. Carriero et al. (2021) pointed out that macro uncertainty displays some endogeneity, though more at quarterly than monthly frequency.[15] Overall, our empirical findings imply that there is some evidence for the endogeneity of uncertainty. In particular, dismissing the feedback effect from the economy to macro uncertainty biases the estimation. These results suggest that, to properly evaluate uncertainty and its impact, it is essential to depart from naive identification strategies and employ a more sophisticated scheme to identification, such as the one we build here.

5.2.2 Uncertainty and its Sources

In the next experiment, we want to evaluate what happens if we omit different sources of uncertainty, which is an assumption that is widespread in the existing literature. Panels (a)-(l) in Figure 7 display the responses to macro uncertainty shock for the baseline identification (black line) and when the response of financial uncertainty is muted for 6

[15] They mentioned that, in unreported results, news-based policy uncertainty as proxy for macro volatility turns out to be endogenous.

months (red line).[16] Similarly, panels (a′)-(l′) show the responses to financial uncertainty shock for the baseline identification (black line) and when the response of macro uncertainty is muted (red line). Remarkably, omitting multiple sources of uncertainty lead to distortions in the estimated effects of uncertainty on the economy. In particular, neglecting this channel seems to attenuate the impact of uncertainty. More formally, in our framework we always reject the hypothesis of zero impact response of macro (financial) uncertainty to financial (macro) disturbance.

[16] We tried horizons other than 6, and the results are qualitatively unchanged.

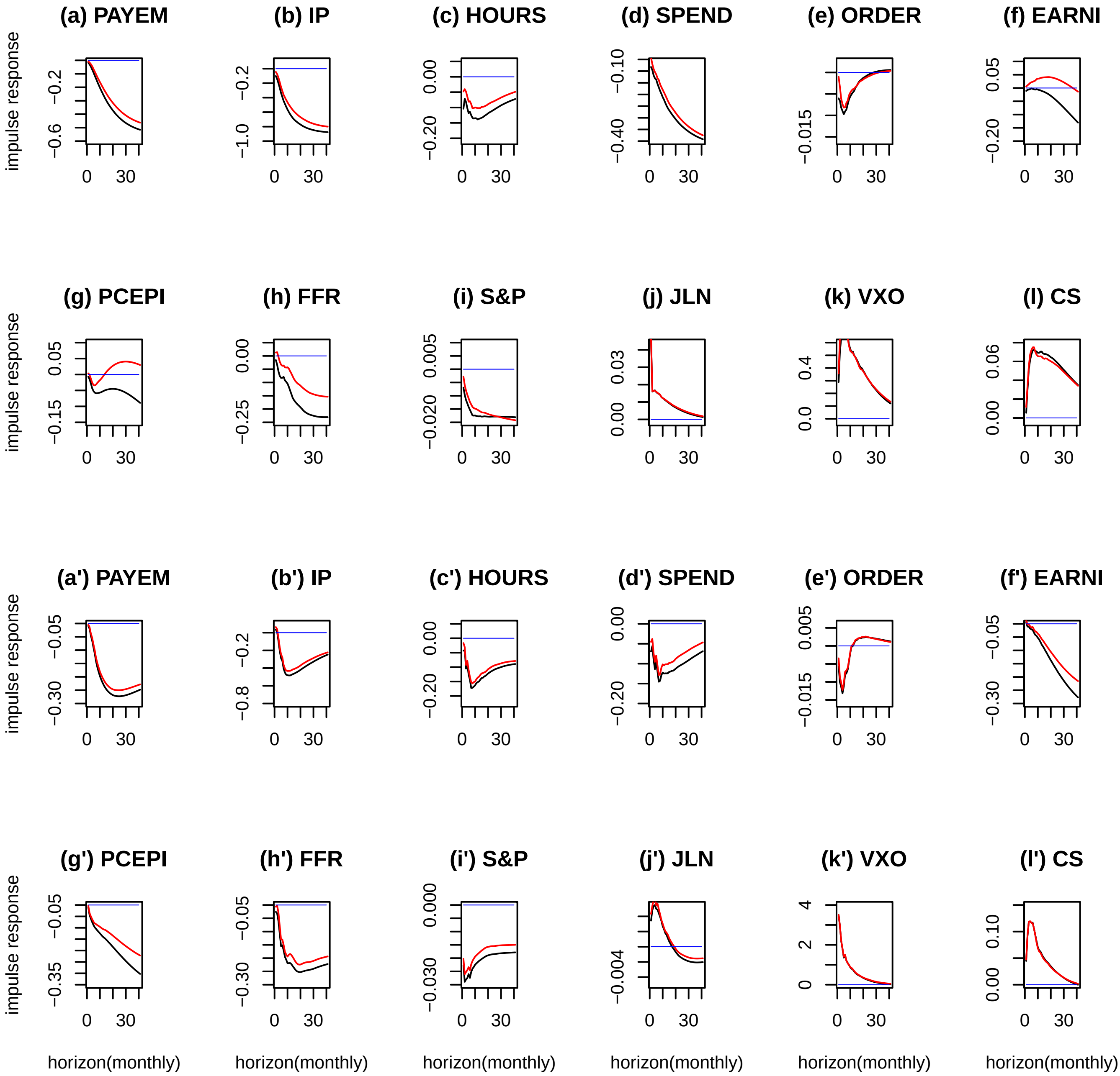

Figure 6: Baseline scenario vs exogenous uncertainty

Figure 6 reports the posterior median of the impulse response functions to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shocks for the baseline identification (black line) and when uncertainty is assumed to be exogenous (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

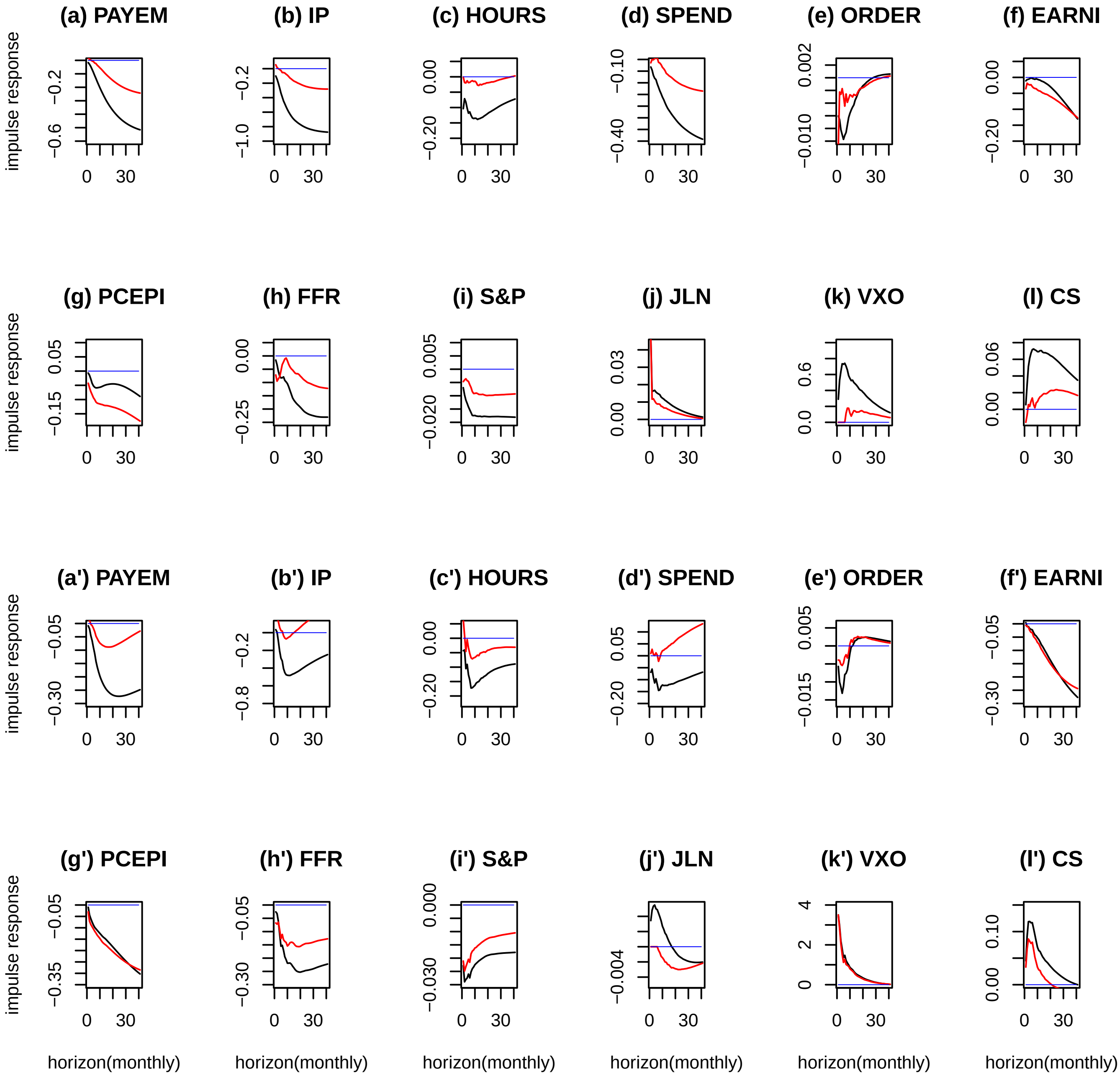

Figure 7: Baseline scenario vs shutting down the channel between macro and financial uncertainty

Panels (a)-(l) display the responses to macro uncertainty shock for the baseline identification (black line) and when the response of financial uncertainty is muted (red line). Panels (a′)-(l′) show the responses to financial uncertainty shock for the baseline identification (black line) and when the response of macro uncertainty is muted (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

5.2.3 The Financial Channel

Some literature has argued that financial conditions play a key role in amplifying and transmitting the uncertainty shocks. From a modeling standpoint, Arellano et al. (2018), Christiano et al. (2014), and Gilchrist et al. (2014) developed financial channel where the cost of external finance goes up in reaction to an increase in uncertainty; Alfaro et al. (2018) found that financial frictions can double the recessionary effect of uncertainty. On the other hand, Brunnermeier and Sannikov (2014) emphasized that a worsening of borrowers' financial position leads to higher uncertainty. Empirically, Caldara et al. (2016), Brianti (2021), and Caggiano et al. (2021) found evidence that deterioration of financial conditions magnify the impact of uncertainty shocks on the real activity.

Thus, it seems natural to investigate the role of financial channel within our identification scheme. Figure 8 compares the responses to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shock for the baseline identification (black line) and when the financial channel is shut down (red line) by imposing that there is no contemporaneous feedback between financial variables (credit spreads and stock market) and uncertainty. The overall picture is clearly one in which the financial channel seems present, and stronger for transmission of financial uncertainty than macro uncertainty.

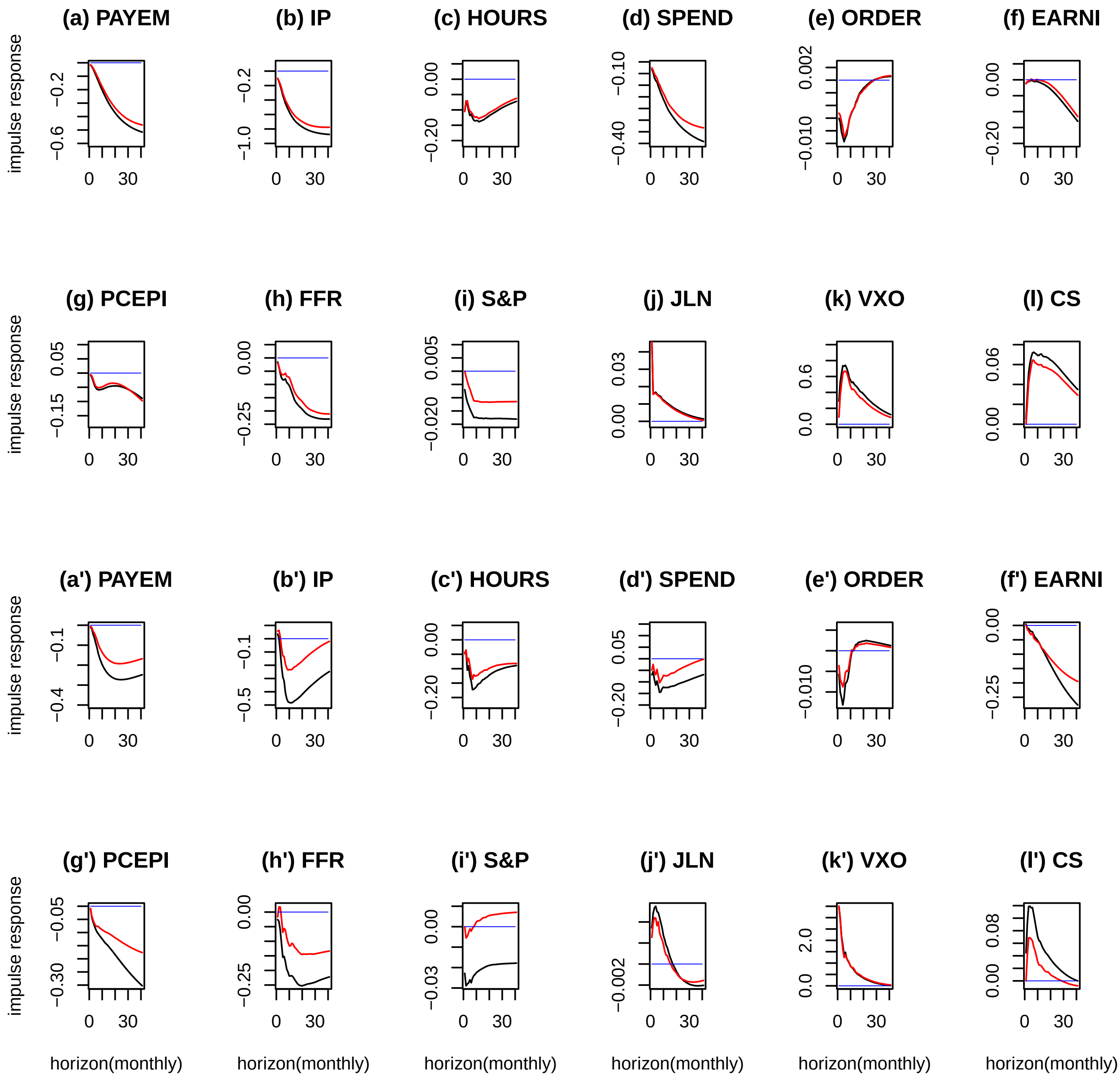

Figure 8: Financial channel

Figure 8 reports the posterior median of the impulse response functions to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shocks for the baseline identification (black line) and when the financial channel is shut down (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

5.3 The Effects of Financial Shocks

Prompted by the Great Recession and the debt crisis in the Euro-Area, a number of works has identified and estimated the effect of credit supply shocks. For instance, Peersman (2011), Bijsterbosch and Falagiarda (2014), Eickmeier and Ng (2015), and Gambetti and Musso (2017) employed sign restrictions; Gilchrist and Zakrajšek (2012) developed a measure of credit spreads based on firm level data, finding that a component of this index is an indicator for credit supply; alternative proxies of credit supply have been put forward by Kashyap and Wilcox (1993), Gertler and Gilchrist (1994), and Lown and Morgan (2006). The overall picture shows (i) the estimation of the effects of financial shocks on the economy are substantially sensitive to identification schemes and (ii) some identification strategies are likely to provide misleading results (discussion in Mumtaz et al. (2018)).

While identification of financial shocks and measurements of credit spreads significantly change across these contributions, the common feature is the absence of interaction between financial factors and uncertainty, in the sense that most of specifications exclude measures of uncertainty or overlook its role when trying to identify financial shocks. This is at odds with uncertainty literature, where, as shown, the influence of credit factors on the transmission of uncertainty disturbances has attracted prominent interest: the reverse mechanism, namely the role of uncertainty in transmitting credit supply shocks, has been overlooked in the literature. There are some exceptions, including Caldara et al. (2016), Furlanetto et al. (2017), Caggiano et al. (2021) and Brianti (2021), which will be analyzed in the next section.

Figure 9 provides some insights. The effect of an increase in credit spreads is depressive, deflationary, and leads to higher uncertainty, especially financial uncertainty. In the following exercise, for the initial six months we shut down the response of macro uncertainty to credit spreads shock (Figure 10, panels (a)-(l)).[17] The bias on the responses is significant, and is consistent with the endogenous features of macro uncertainty. Shutting down the response of financial uncertainty (Figure 10, panels (a′)-(l′)) dramatically mitigates the responses to financial shocks and leads to substantial bias. These findings confirm what we found in section 5.2.1, where although financial uncertainty is substantially exogenous with respect to macro variables, the feedback from financial conditions is

[17] We also constrained horizons other than 6, and the findings do not change.

paramount. Formal tests reject the zero impact response of uncertainty to credit spread shocks;[18] overall, the current section stresses that ignoring the role of uncertainty results in under-estimating (downward bias) the effects of credit shocks. Interestingly, omitting even a single source of uncertainty is enough to produce some bias.

[18]The null being $\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q^*}_3 = \boldsymbol{c}'_{20}(\boldsymbol{\phi})\boldsymbol{q^*}_3 = 0$.

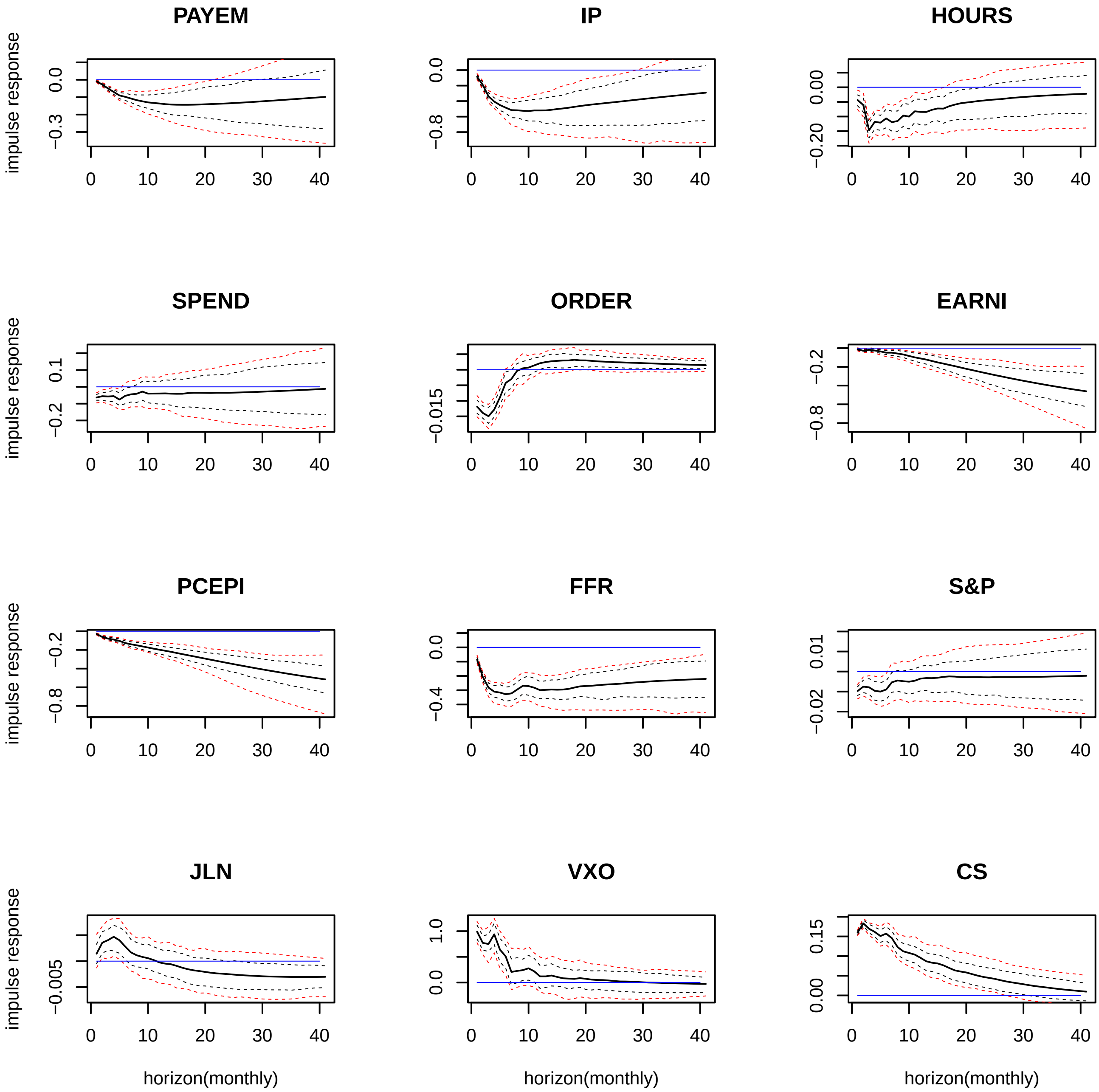

Figure 9: Responses to financial shocks

Figure 9 reports the posterior median (black solid lines), the 68% Bayesian credibility region (black dashed lines), and the 90% Bayesian credibility region (red dashed lines) of the impulse response functions to financial shocks. The blue solid line is the zero line. The shock size is set to one standard deviation.

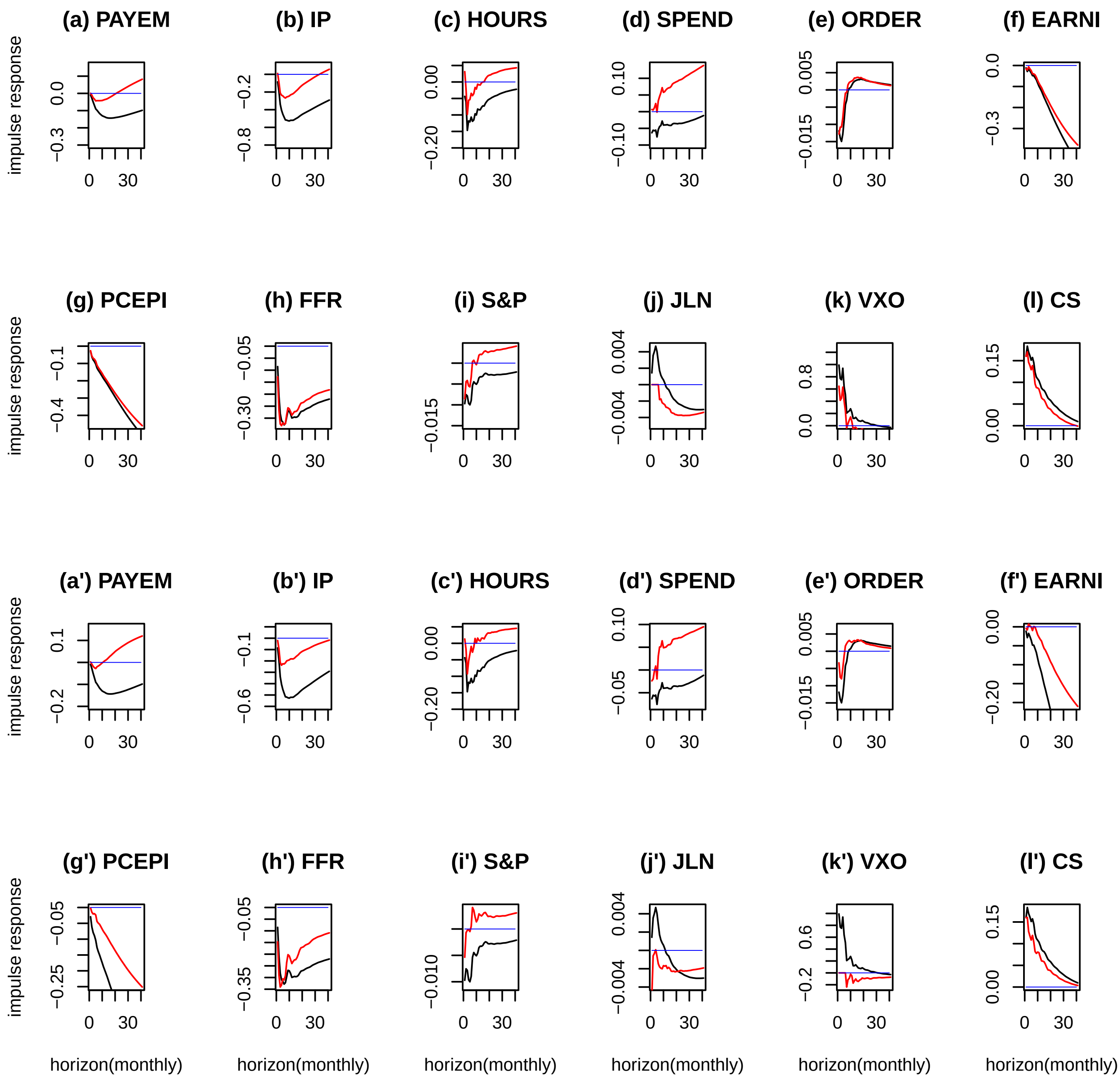

Figure 10: Financial shocks: shutting down uncertainty

Panels (a)-(l) report the posterior median of the impulse response functions to financial shocks for the baseline identification (black line) and when the response of macro uncertainty is shut down (red line). Panels (a′)-(l′) report the posterior median of the impulse response functions to financial shocks for the baseline identification (black line) and when the response of financial uncertainty is shut down (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

5.4 Comparison with Alternative Identification Strategies

The empirical contribution of this paper is to estimate the effects of uncertainty and financial disturbances while i) allowing for endogenous uncertainty, ii) separating real from financial uncertainty and iii) disentangling pure credit supply vs (macro and financial) uncertainty shocks. All studies of this nature require identifying assumptions. However, commonly employed SVARs are not well equipped to address the three challenges above. Recursive schemes rule out the chance that uncertainty and real activity could affect one another contemporaneously. Sign restrictions on impulse responses are ill equipped because theory is controversial about the sign of the relationship. Long run identification is hard to justify as the effects of uncertainty shocks over long horizon have not been theoretically characterized. Estimation with instrumental variables is difficult, since instruments that are convincingly exogenous are challenging to find for this application, especially when macro and financial uncertainty are separately identified (Ludvigson et al., 2021). While this is the first paper to address i), ii) and iii) in a unified framework, our article shares similarities and differences with a number of contributions.

Some scholars have (very) recently considered uncertainty as driver of real activity. Carriero et al. (2021) specified VARs with stochastic volatility in which one of the variable can affect both the mean and the variance of the other variables. They found that macro uncertainty has some endogenous features with quarterly data, and less with monthly data. In a previous draft, they also argued that financial uncertainty is likely to be endogenous. We thus briefly comment on the main differences. First, they placed the various uncertainty proxies in different VARs one at a time rather than including (and simultaneously identifying) them in a single framework. This is essential given that macro and financial uncertainty are strongly correlate; we believe that if we do not explicitly control for financial uncertainty in the same empirical system that incorporates macro uncertainty, and make them orthogonal, the latter could mimic the causal role of the former. It would be computationally challenging identifying both macro and financial uncertainty shocks in their framework. Second, while they included credit spreads and stock price index among the covariates, financial shocks are not explicitly identified. Third, ordering must be still imposed on the block of macro variables.

Angelini et al. (2019) used a small-scale model with macro uncertainty, financial uncertainty and real activity to identify macro and financial uncertainty disturbances by employing different volatility regimes over time. They found that both macro and financial uncertainty are exogenous. Differences with our approach are clear. First, a small-scale framework without proxies for financial conditions can omit important mechanisms. Second, they assumed that, in sub-sample before January 2008 (a break date that is exogenously determined), financial uncertainty shocks could neither contemporaneously impact on, nor have been impacted by, macro variables. They argued that financial regulation made the reaction of financial markets to non-financial disturbances slow-paced up to January 2008. This is an intriguing assumption because financial markets are usually expected to react fast to news (and this includes samples before 2008), while macro variables are relatively slower (Gertler & Karadi, 2015; Lettau et al., 2002). On the one hand, within our framework we tested and rejected the hypothesis that financial uncertainty can not jointly affect macro variables within period in the sub-sample up to 2008;[19] on the other hand, they in practice impose exogeneity of financial uncertainty to identify the model, while we let data speak about it.

Ludvigson et al. (2021) set-identified financial and macro uncertainty by using shock-based restrictions, namely event constraints, which require the size of the shocks to be consistent with some historical episodes, and correlation constraints, which require the shocks to be correlated with external instruments. They found that macro uncertainty is endogenous and financial uncertainty is mostly exogenous; however, they argued that financial uncertainty has a depressive effect, while macro uncertainty can also be expansionary. This major difference with our findings can be explained by illustrating the distinct identification schemes. Ludvigson et al. (2021) relied on event constraints that are used asymmetrically between the two disturbances: those restrictions are mainly imposed on financial uncertainty, and it is well-known that weakly restricted set-identified shocks can lead to loss of estimation precision. Here, we are free from a set of narrative-based restrictions and treat the shocks symmetrically. Also, they estimated a small-scale framework with macro uncertainty, financial uncertainty, and industrial production only: this could lead to some omitted variable bias. Financial and credit conditions are not modeled in their empirical framework: the only requirement for the shocks is to be negatively (positively) correlated with stock market index (gold price). Finally, it is known

[19]The null is $\boldsymbol{c}'_{IP0}(\boldsymbol{\phi})\boldsymbol{q}^*{}_2 = 0$.

that inference with set-identifying narrative restrictions is extremely tricky (Giacomini et al., 2021): Ludvigson et al. (2021) employed bootstrap to construct confidence intervals for the impulse response functions, but their frequentist validity is unknown. The fact that confidence intervals are presented for a specific point-estimate only (rather than for the identified sets as such) makes hard to evaluate the effect of sample bias and identification uncertainty in their setting. On the other hand, Bayesian inference naturally follows in our point-identified model.

It is also interesting to briefly compare our methodology and findings with other recent contributions. Shin and Zhong (2020) used sign restrictions with stochastic volatility to separate macro and financial uncertainty shocks, finding that the latter (former) has a strong (weak) recessionary effects on the economy. However, they assumed that uncertainty is exogenous and not contemporaneously affected by level shocks. Caggiano et al. (2021) identified financial uncertainty and pure financial shocks by employing a mix of sign restrictions, ratio restrictions, and narrative sign restrictions, finding that credit deterioration amplifies the negative response of output to uncertainty. Unlike our paper, their approach depends on several restrictions (for instance, responses of uncertainty and credit spread to uncertainty and financial shocks are sign-restricted), macro uncertainty is not explicitly identified, and the model is set-identified, i.e., it is not clear how much the posterior estimation is driven by the prior distributions (Baumeister & Hamilton, 2015; Giacomini & Kitagawa, 2020); similarly, Furlanetto et al. (2017) set-identified financial uncertainty and financial shocks by sign-restricting responses of real activity variables and employing ratio constraints. Brianti (2021) identified credit supply and macro uncertainty shocks relying on the qualitatively different responses of corporate cash holdings to a macro uncertainty shock (that pushes firms to increase their cash holdings for precautionary reasons) and a first-moment financial shock (that leads firms to reduce cash reserves as they lose access to external finance). However, i) those restrictions come from a theoretical framework with exogenous uncertainty and ii) the financial shocks as estimated by Brianti (2021) are a mix between first- and second-moment shocks within the financial sector, and as such cannot separate financial uncertainty shocks from pure credit supply disturbances. Caldara et al. (2016) identified level (financial) and second moment (uncertainty) shocks by employing a penalty function approach which relies on the ordering of the two first and second moment proxies and found that results are very sensitive to ordering. Also, they assumed a single source of uncertainty within the economy. The advantage of the identification scheme here is that we do not require any ordering and do not exclude multiple forms of uncertainty.

5.5 Relation with Theoretical Modeling of Uncertainty and Financial Shocks

Our results convey a mixed message for characterization of uncertainty and financial disturbances. New Keynesian models with Rotemberg-type sticky prices (Oh, 2020; Bonciani & Van Roye, 2016; Leduc & Liu, 2016; Basu & Bundick, 2017; Cesa-Bianchi & Fernandez-Corugedo, 2018; Katayama & Kim, 2018), or a framework with real frictions in the labor market and nominal rigidities, as shown by Leduc and Liu (2016) and Cacciatore and Ravenna (2020), have been proved to replicate the recessionary co-movement of output and prices after a shock to uncertainty. However, those models are mainly based on i) exogenous uncertainty and ii) a single source of uncertainty, which quantitatively distort the estimation of the effects of uncertainty on the economy. Furthermore, models with flexible prices, or Real Business Cycle (RBC) frameworks, are unable to replicate the recessionary effects of uncertainty shocks. For instance, Fernández-Villaverde and Guerrón-Quintana (2020) showed that a RBC framework with financial frictions leads to expansionary effects of uncertainty disturbances. Also, New Keynesian models with Calvo pricing, which imply a decrease in output and an inflationary pressure as response to uncertainty shocks, are inconsistent with the results here (Born & Pfeifer, 2014; Mumtaz & Theodoridis, 2015).[20] Finally, modeling credit supply shocks cannot disregard the role of uncertainty and, even more relevant, its features of (partial) endogeneity and originating from different sources.

6 Conclusions

In this paper, we developed a novel SVAR identification scheme based on constraints on the FEV decomposition to simultaneously identify a multiplicity of shocks. Identification is achieved by solving a quadratic optimization problem. We characterized the properties

[20] Fernández-Villaverde et al. (2015) found an inflationary effect of uncertainty disturbances in a Rotemberg-type framework. However, this outcome comes from the fact that the price adjustment cost directly affects firms' marginal costs.

of this approach, such as existence and uniqueness of solutions, and provided algorithms for estimation and inference. Although this approach is fully general and can be applied to any empirical application, a natural choice is to investigate the effects of uncertainty and financial shocks on the economy by allowing for endogeneity of uncertainty, disentangling real vs financial uncertainty, and separating uncertainty from pure financial shocks. We have illustrated the effectiveness of this approach via a Monte-Carlo exercise. Using US data, we found that uncertainty acts as a negative demand shock, but there are important differences between macro and financial uncertainty. Some variables have a significant contemporaneous feedback effect on macro uncertainty, and overlooking this endogenous channel can lead to distortions in the estimated effects of uncertainty on the economy. On the other hand, our empirical results suggest that financial uncertainty is likely to be an exogenous source of business cycle fluctuations. We also found that ignoring that uncertainty has various sources biases the estimation and that omitting the role of uncertainty leads to underestimate the effects of financial shocks on the economy.

Appendices

A Omitted Proofs

Proof of Proposition 3.1.

Note that the feasibility region for $\boldsymbol{q}_1$ is characterized by $f_i \leq 0$ for $i = 1, \ldots, 4$ in Section 3.1 of the main text. Let us write f_i for $i = 1, \ldots, 4$ more compactly:

$$f_1 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_1(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.1}$$

$$f_2 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_2(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.2}$$

$$f_3 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_3(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}, \tag{A.3}$$

$$f_4 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_4(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}, \tag{A.4}$$

where $\boldsymbol{A}_1(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})$, $\boldsymbol{A}_2(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})$, and $\boldsymbol{A}_3(\boldsymbol{\phi}) = \boldsymbol{A}_4(\boldsymbol{\phi}) = \boldsymbol{I}_n$, with $\boldsymbol{I}_n$ being the identity matrix. Note that $\boldsymbol{A}_i(\boldsymbol{\phi})$ for $i = 1, \ldots, 4$ is a square $n \times n$ matrix and can be as such decomposed into symmetric and skew-symmetric (or anti-symmetric) components (Toeplitz decomposition): $\boldsymbol{A}_i(\boldsymbol{\phi}) \equiv \boldsymbol{A}_{iS}(\boldsymbol{\phi}) + \boldsymbol{A}_{iAS}(\boldsymbol{\phi})$, where $\boldsymbol{A}_{iS}(\boldsymbol{\phi}) = \frac{\boldsymbol{A}_i(\boldsymbol{\phi}) + (\boldsymbol{A}_i(\boldsymbol{\phi}))'}{2}$ and $\boldsymbol{A}_{iAS}(\boldsymbol{\phi}) = \frac{\boldsymbol{A}_i(\boldsymbol{\phi}) - (\boldsymbol{A}_i(\boldsymbol{\phi}))'}{2}$ are the symmetric and antisymmetric components of $\boldsymbol{A}_i(\boldsymbol{\phi})$, respectively. It is trivial to show that $\boldsymbol{q}_1' \boldsymbol{A}_i(\boldsymbol{\phi})\boldsymbol{q}_1 = \boldsymbol{q}_1' \boldsymbol{A}_{iS}(\boldsymbol{\phi})\boldsymbol{q}_1$. As a result, we obtain

$$f_1 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_{1S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.5}$$

$$f_2 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_{2S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.6}$$

$$f_3 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_{3S}(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}, \tag{A.7}$$

$$f_4 = \frac{1}{2}\boldsymbol{q}_1' \boldsymbol{A}_{4S}(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}. \tag{A.8}$$

We now define the following objects:

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{A}_{1S} & 0 \\ 0 & 0, \end{bmatrix}$$

$$\boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{A}_{2S} & 0 \\ 0 & 0, \end{bmatrix}$$

$$\boldsymbol{H}_3 = \begin{bmatrix} \boldsymbol{A}_{3S} & 0 \\ 0 & 1, \end{bmatrix}$$

$$\boldsymbol{H}_4 = \begin{bmatrix} \boldsymbol{A}_{3S} & 0 \\ 0 & -1, \end{bmatrix}$$

Note that $\boldsymbol{H}_i$ for $i = 1, \ldots, 4$ is a Z-matrix, i.e., the off-diagonal elements of a symmetric matrix are non-positive. Define a set $\boldsymbol{\Omega}_0$:

$$\boldsymbol{\Omega}_0 := \{(\frac{1}{2}\boldsymbol{a'}\boldsymbol{H}_1\boldsymbol{a}, \ldots, \frac{1}{2}\boldsymbol{a'}\boldsymbol{H}_4\boldsymbol{a}) : \boldsymbol{a} \in \mathcal{R}^{n+1}\} + int\mathcal{R}^4_+. \tag{A.9}$$

It suffices to prove that if $\boldsymbol{q}_1^*$ does not exist, then $\exists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$. In doing so, we follow the argument in Jeyakumar et al. (2009), Theorem 5.2. Assume that $\boldsymbol{q}_1^*$ does not exist. This is equivalent to state that the following system has no solution: $\boldsymbol{q}_1 \in \mathcal{R}^n$, $f_i < 0$, $i = 1, \ldots, 4$. Introduce 4 homogeneous functions $\bar{f}_i : \mathcal{R}^{n+1} \to \mathcal{R}$, with $\bar{f}_i = \frac{1}{2}(\boldsymbol{q}, t)\boldsymbol{H}_i(\boldsymbol{q}, t)'$, where t is a scalar:

$$\bar{f}_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{1S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.10}$$

$$\bar{f}_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{2S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.11}$$

$$\bar{f}_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{3S}(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}t^2, \tag{A.12}$$

$$\bar{f}_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{4S}(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}t^2. \tag{A.13}$$

Note that $f_1 = \bar{f}_1$ and $f_2 = \bar{f}_2$

Then $\boldsymbol{0} \notin \boldsymbol{\Omega}_0$. Otherwise, there exists some $\boldsymbol{q} \in \mathcal{R}^n$ such that $f_i < 0$, $i = 1, \ldots, 4$, which is a contradiction (Jeyakumar et al. (2009) in Theorem 5.2 provide the technical proof of this). Since $\boldsymbol{H}_i$ for $i = 1, \ldots, 4$ are Z-matrices, $\boldsymbol{\Omega}_0$ is a convex set (see Theorem

5.1 in Jeyakumar et al. (2009)). By the convex separation theorem, there must exist $\boldsymbol{\lambda} \in \mathcal{R}^4 \backslash \{0\}$ such that for all $(y_1, \ldots, y_4) \in \boldsymbol{\Omega}_0$, $\sum_{i=1}^{4} \lambda_i y_i \geq 0$. In turn, this means that there must exist $\boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ and for all $(\boldsymbol{q}, t) \in \mathcal{R}^{n+1}$, $\sum_{i=1}^{4} \lambda_i \bar{f}_i \geq 0$. Setting $t = 1$ implies $f_i = \bar{f}_i$ for $i = 1, \ldots, 4$, namely $\exists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^{4} \lambda_i f_i \geq 0$.

■

Proof of Proposition 3.2.

Without loss of generality, assume a permutation of the set $\{1, \ldots, k\}$: for instance, $\boldsymbol{\sigma} = (1, \ldots, k)$. According to condition i), we then obtain that for $j = \boldsymbol{\sigma}(1) = 1$ Proposition 3.1 is satisfied, namely $\boldsymbol{q}_1$ satisfies optimization constraints. Consider the following projector operator: $proj_{\boldsymbol{q}}(\boldsymbol{v}) = \frac{\langle \boldsymbol{q}, \boldsymbol{v} \rangle}{\langle \boldsymbol{q}, \boldsymbol{q} \rangle} \boldsymbol{q}$, where $\langle \boldsymbol{q}, \boldsymbol{v} \rangle$ denotes the inner product of vectors $\boldsymbol{q}$ and $\boldsymbol{v}$, with $\boldsymbol{q}, \boldsymbol{v} \in \mathcal{R}^n$. Put it another way, we are projecting $\boldsymbol{v}$ orthogonally into the line spanned by $\boldsymbol{q}$. Given $\boldsymbol{q}_1$, assume the following Gram–Schmidt process for $\boldsymbol{q}_j$ with $j = \boldsymbol{\sigma}(2), \ldots, \boldsymbol{\sigma}(k)$:

$$\boldsymbol{q}_2 = \boldsymbol{v}_2 - proj_{\boldsymbol{q}_1}(\boldsymbol{v}_2) \tag{A.14}$$

$$\vdots \tag{A.15}$$

$$\boldsymbol{q}_k = \boldsymbol{v}_k - \sum_{j=1}^{k-1} proj_{\boldsymbol{q}_j}(\boldsymbol{v}_k). \tag{A.16}$$

Given $\boldsymbol{q}_1$, this corresponds to generate vectors $\boldsymbol{q}_j$ which are in the Nullspace of $\boldsymbol{q}_{j-1}$ for $j = 2, \ldots, k$, i.e., generating a series of orthogonal vectors. If $\boldsymbol{q}_j$ for $j = 1, \ldots, k$ satisfies Proposition 3.1, there must exist an orthogonal matrix $\boldsymbol{Q}_{1:k} = [\boldsymbol{q}_1, \ldots, \boldsymbol{q}_k]$ consistent with restrictions (2.7) and (2.8) in the main text. Existence follows.

Technical remark: the orthogonal vectors generated by the above Gram-Schmidt process can be easily adjusted to make them unit vectors, i.e. construct $\frac{\boldsymbol{q}}{||\boldsymbol{q}||}$. However, in our case this would be redundant as Proposition 3.1 imposes unit vectors condition.

■

Proof of Proposition 3.3.

Assume that $\boldsymbol{Q}^*_{1:k}$ exists and is orthogonal. This proof shows that under the conditions in Proposition 3.3, $\boldsymbol{q}^*_1, \ldots, \boldsymbol{q}^*_k$ are unique.

Without loss of generality, for $j = 1$ restrictions (2.7) are reduced to:

$$\boldsymbol{q}_1'[\boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})]\boldsymbol{q}_1 \geq 0 \tag{A.17}$$

$$\boldsymbol{q}_1'[\boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})]\boldsymbol{q}_1 \geq 0, \tag{A.18}$$

where $\boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}$. Let $\boldsymbol{\Upsilon}^{\mathbf{12}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}(\boldsymbol{\phi})\boldsymbol{c}_{1h}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}'(\boldsymbol{\phi})\boldsymbol{c}_{1h}(\boldsymbol{\phi})} - \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{2h}(\boldsymbol{\phi})\boldsymbol{c}_{2h}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{2h}'(\boldsymbol{\phi})\boldsymbol{c}_{2h}(\boldsymbol{\phi})}$ and $\boldsymbol{\Upsilon}^{\mathbf{13}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}(\boldsymbol{\phi})\boldsymbol{c}_{1h}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}'(\boldsymbol{\phi})\boldsymbol{c}_{1h}(\boldsymbol{\phi})} - \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{3h}(\boldsymbol{\phi})\boldsymbol{c}_{3h}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{3h}'(\boldsymbol{\phi})\boldsymbol{c}_{3h}(\boldsymbol{\phi})}$. Thus, restrictions on $\boldsymbol{q}_1$ are

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\mathbf{12}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \tag{A.19}$$

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\mathbf{13}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0. \tag{A.20}$$

For simplicity, and without loss of generality, assume $\tilde{h} = 0$:

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\mathbf{12}}_{\mathbf{0}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \tag{A.21}$$

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\mathbf{13}}_{\mathbf{0}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0, \tag{A.22}$$

where

$$\boldsymbol{\Upsilon}^{\mathbf{12}}_{\mathbf{0}}(\boldsymbol{\phi}) = \frac{\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi})}{\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})} - \frac{\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}'(\boldsymbol{\phi})}{\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})} \tag{A.23}$$

$$\boldsymbol{\Upsilon}^{\mathbf{13}}_{\mathbf{0}}(\boldsymbol{\phi}) = \frac{\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi})}{\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})} - \frac{\boldsymbol{c}_{30}(\boldsymbol{\phi})\boldsymbol{c}_{30}'(\boldsymbol{\phi})}{\boldsymbol{c}_{30}'(\boldsymbol{\phi})\boldsymbol{c}_{30}(\boldsymbol{\phi})}. \tag{A.24}$$

Thus, we obtain

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\mathbf{12}}_{\mathbf{0}}(\boldsymbol{\phi})\boldsymbol{q}_1 = \boldsymbol{q}_1'\left[m_1(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi}) - m_2(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}'(\boldsymbol{\phi})\right]\boldsymbol{q}_1 \tag{A.25}$$

$$= m_1(\boldsymbol{\phi})\boldsymbol{q}_1'\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1 - m_2(\boldsymbol{\phi})\boldsymbol{q}_1'\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{q}_1 \tag{A.26}$$

$$= m_1(\boldsymbol{\phi})\left(\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2 - m_2(\boldsymbol{\phi})\left(\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2, \tag{A.27}$$

where $m_1(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})}$ and $m_2(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})}$ are positive scalar. Similarly, we get

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\mathbf{13}}_{\mathbf{0}}(\boldsymbol{\phi})\boldsymbol{q}_1 = m_1(\boldsymbol{\phi})\left(\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2 - m_3(\boldsymbol{\phi})\left(\boldsymbol{c}_{30}'(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2, \tag{A.28}$$

where $m_3(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}'_{30}(\boldsymbol{\phi})\boldsymbol{c}_{30}(\boldsymbol{\phi})}$. Thus, for $j = 1$ restrictions (2.7) are equivalent to

$$m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2 - m_2(\boldsymbol{\phi}) \left(\boldsymbol{c}'_{20}(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2 \geq 0 \tag{A.29}$$

$$m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2 - m_3(\boldsymbol{\phi}) \left(\boldsymbol{c}'_{30}(\boldsymbol{\phi})\boldsymbol{q}_1\right)^2 \geq 0. \tag{A.30}$$

Recall that for $j = 1$, conditions in Proposition 3.3 are

$$\boldsymbol{c}'_{i0}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \text{ for } i = 1, \ldots, 3. \tag{A.31}$$

Combining (A.29)-(A.31) delivers the following restrictions for $j = 1$:

$$\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq \sqrt{\frac{m_2(\boldsymbol{\phi})}{m_1(\boldsymbol{\phi})}}\boldsymbol{c}'_{20}(\boldsymbol{\phi})\boldsymbol{q}_1 \tag{A.32}$$

$$\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq \sqrt{\frac{m_3(\boldsymbol{\phi})}{m_1(\boldsymbol{\phi})}}\boldsymbol{c}'_{30}(\boldsymbol{\phi})\boldsymbol{q}_1. \tag{A.33}$$

Thus, conditional on the existence of $\boldsymbol{q}_1^*$, constraints of the optimization problem become linear, and as such, convex for $\boldsymbol{q}_1$. Also, it is easy to observe that conditions in (A.31) make the objective function convex in $\boldsymbol{q}_1$. Since the problem is now convex, $\boldsymbol{q}_1^*$ must be unique. Extension to $\tilde{h} > 0$ is trivial. The same proof applies to $j = 2, 3$, i.e., $\boldsymbol{q}_2^*$ and $\boldsymbol{q}_3^*$ are unique. As a result, conditional on $\boldsymbol{q}_1^*$, $\boldsymbol{q}_2^*$ and $\boldsymbol{q}_3^*$ to exist and be orthogonal to each other, matrix $\boldsymbol{Q}_{1:k}^*$ is unique (with $k = 3$ in our setting).

■

B Shocks Series and Robustness Checks

Here we present some evidence that the three identified shocks are truly structural and exogenous to a set of structural shocks previously identified by the literature. We have re-estimated the impulse responses by explicitly controlling, i.e, imposing orthogonality condition, for military news (Ramey, 2016), expected tax (Leeper et al., 2013), unanticipated and anticipated tax (Mertens & Ravn, 2011), monetary policy (Romer & Romer, 1989), and technology surprise (Basu et al., 2006). All the results presented in the main text are robust to this additional control. Furthermore, we have computed the correlations between the three identified shocks and those disturbances, finding that correlations are never significant at 1% and 5% level.

Ludvigson et al. (2021) and Caggiano et al. (2021) stressed that credible identification regimes need to estimate shocks consistent with specific episodes in the history. In particular, we focus on three events: Black Monday (October 1987), Lehman collapse (September 2008), and Covid outbreak (March 2020). For the Black Monday, our estimated financial uncertainty shock is large,[21] while this is not the case for credit supply disturbances. This is in line with the narrative of Ludvigson et al. (2021) and Caggiano et al. (2021), where the Black Monday is featured by significant financial volatility but low credit conditions disruption. In September 2008, we find that our three estimated shocks are all large, which is consistent with the consensus view of the Great Recession as a mix of financial and uncertainty shocks. Also, we have extended our sample up to 2020: both macro and financial uncertainty shocks are large and bigger than pure financial shock in March 2020, which is compatible with the belief that the pandemic prompted a spike in uncertainty but significant fiscal and monetary policy interventions prevented credit supply deterioration.

The findings we have obtained are also robust to the following battery of checks, which are available upon request: lag length from three to twelve; selecting the prior tightness by maximizing the marginal likelihood rather than employing a flat specification; using the measures of Carriero et al. (2018b) and Jurado et al. (2015) as alternative proxies of financial uncertainty and the proposal in Carriero et al. (2018b) as proxy of macro uncertainty; employing the excess bond premium in Caldara et al. (2016) and Gilchrist and Zakrajšek (2012) as credit spreads. Moreover, we have run a quarterly specification, finding that results are equivalent to what shown so far.

Finally, a number of papers have pointed out that the effect of uncertainty shocks is more intense when the Zero Lower Bound (ZLB) holds (Caldara et al., 2016; Caggiano et al., 2014; Basu & Bundick, 2017; Johannsen, 2014). Thus, we have estimated the model over the sample up to 2008m9, which removes the years of the Great Recession where the ZLB binds. The results are qualitatively equivalent to Figure 3 and Figure 4; quantitatively, the response of the variables is slightly less pronounced. Since this is fully consistent with the previous literature, we omit it for brevity.

[21] Bigger than the median. This definition of large shocks is consistent with Ludvigson et al. (2021).

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