Corporate Legacy Debt, Inflation, and the Efficacy of Monetary Policy[*†]

Charles A.E. Goodhart [‡] M. Udara Peiris[§] Dimitrios P. Tsomocos[¶] Xuan Wang [‖]

January 27, 2022

Abstract

The COVID-19 pandemic has coincided with a rapid increase in indebtedness. Although the rise in public debt and its policy implications have received much attention recently, the rise in corporate debt has received less so. We argue that high levels of corporate debt may impede the transmission mechanism of monetary policy and make it less effective in controlling inflation. In an environment with working capital financing requirements, when firms' indebtedness is sufficiently high, the income effect of higher nominal interest rates offsets or even dominates its usual negative substitution effect on aggregate demand and is quantitatively important. This mechanism is independent of standard financial and nominal frictions and aggravates the trade-off between inflation and output stabilisation.

Keywords: Corporate indebtedness, debt inflation, working capital, monetary transmission mechanism, income effect, Taylor principle

JEL Codes: E31, E44, E52, G33

---

[*]We would like to thank participants of the 49th NES Research Conference, Konstanin Egorov for valuable comments, and Aleksandr Shirobokov for assistance in preparing this paper.

[†]Cite as CEPR DP16799, https://cepr.org/active/publications/discussion_papers/dp.php?dpno=16799 .

[‡]Financial Markets Group, London School of Economics and CEPR. Houghton Street, London, UK, WC2A 2AE. Email: caegoodhart@aol.com.

[§]ICEF, NRU Higher School of Economics, Moscow. Email: upeiris@hse.ru .

[¶]Saïd Business School and St Edmund Hall, University of Oxford. Park End Street, Oxford, UK. OX1 1HP. Email: dimitrios.tsomocos@sbs.ox.ac.uk.

[‖]Vrije Universiteit Amsterdam and Tinbergen Institute. De Boelelaan 1105, NL-1081HV, Amsterdam, Netherlands. Email: xuan.wang@vu.nl .

1 Introduction

Non-financial corporate debt-to-GDP ratio has risen globally since 2007. Corporate indebtedness in the Euro Area increased by almost 14% from an already high 93.3% in 2007, Sweden saw an increase of 26.8% from 125.2%, while in Canada the increase was almost 40%.[1][2] The COVID-19 pandemic crisis has led to a further sharp buildup of corporate debt. US corporate indebtedness rose by 12.5% between December 2018 and December 2020, much more than its total increase in the entire decade leading up to the COVID-19 pandemic. Meanwhile, inflation has been rising. We show that corporate debt poses a challenge for monetary policy in two ways. First, it introduces an additional *income effect* across heterogenous households that counteracts the traditional *substitution effect* and affects the overall effectiveness of interest rates. Second, it creates a more difficult trade-off between output and inflation stabilisation, requiring a reassessment of policy priorities.

While a growing literature on corporate indebtedness focuses on the implications for investment and aggregate demand (Abraham, Cortina Lorente and Schmukler, 2020; Bräuning and Wang, 2020; Brunnermeier and Krishnamurthy, 2020; Jordà, Kornejew, Schularick and Taylor, 2020; Goodhart and Pradhan, 2020), relatively less attention has been paid to how debt may hamper the transmission mechanism of monetary policy and its effectiveness in controlling inflation. In contrast, the buildup of public sector debt has led to concerns about the future path of inflation.[3] In this paper, we assess how effective monetary policy may be in controlling inflation when there is a large amount of corporate legacy debt in the economy.

We first build a flexible price static model and obtain the closed-form solution to show how legacy corporate debt causes monetary policy to have a redistributional effect on income, impeding the effect on inflation through the adjustment of money balances. We then extend the static model to a dynamic setting and show that this effect impedes the transmission of monetary policy through nominal rigidities. Our economy features two types of households, lender households, i.e. the bond holders, that accumulate safe corporate debt to save; owner households, the equity holders, that own firms that in turn issue the corporate debt. The differentiation between these two types of households is consistent with Fisher's (1910) narrative on the 'enterpriser-borrower' and the 'creditor, the salaried man, or the labourer'.[4] Firms face a working capital constraint à la Christiano, Eichenbaum and Evans (2005) in that they must borrow money to finance expenses for labour in advance of receiving income from production.[5]

[1] This has also occurred in emerging economies: China, Chile, Brazil, and Turkey have all seen more than a 50% rise during this period. Hong Kong's non-financial corporate debt-to-GDP ratio soared by over 77% to more than 200%.

[2] Throughout the paper, the term 'corporate debt' is used to refer to the debt of non-financial corporations.

[3] See, for example, Martin Wolf (2020),'Why inflation might follow the pandemic', *Financial Times*, May 19, 2020. In Q2 2021, US CPI inflation has jumped to 13-year high and the US economy saw the highest rate of core inflation since 1991, and UK inflation has topped the Bank of England's inflation target. The departing chief economist at the Bank of England, Haldane, has also warned of inflation rises in the UK.

[4] see Mankiw and Zeldes (1991); Toda and Walsh (2020) among many others as examples of more recent applications of this.

[5] Barth and Ramey (2001) provide empirical evidence for the working capital channel. There is also a long list of credit-channel papers offering evidence on the cost channel of monetary policy via firms borrowing from banks (see e.g., Kashyap, Stein, and Wilcox 1993, Kashyap, Lamont, and Stein, 1994 Gertler and Gilchrist, 1994, non-exhaustive.).

Our analytic results show that contractionary monetary policy reduces real wages, and that the effective elasticity of labour supply increases with the scale of legacy debt relative to working capital, and, as a consequence, varies during the business cycle through debt and working capital dynamics. When debt is below a threshold, the traditional Taylor principle holds: raising nominal interest rates lowers current inflation or prices. However, higher debt leads to a smaller fall in prices, meaning that monetary policy becomes less effective in controlling inflation. This is because the presence of debt triggers a heterogeneous income effect across households which affects the usual substitution effect. These results reinforce the importance of the heterogeneity of households and the relatively high pro-cyclicality of income and consumption expenditure of high income and high wealth households that own an overwhelmingly large share of stocks (see Parker, Vissing-Jorgensen, Blank and Hurst, 2010, for a deconstruction of the cyclical properties of household groups in the US).

Traditionally, the effect of raising nominal interest rates lowers aggregate demand and causes prices to fall through a substitution effect that puts downward pressure on aggregate demand. However, via the income effect through nominal corporate debt, we show the aggregate demand curve shifts less to the left, the aggregate supply curve becomes more elastic as a result of a higher effective labour elasticity, and it also moves in the same direction as the aggregate demand curve. Therefore, in equilibrium, although output falls responsively, prices only respond mildly. Interestingly, given working capital, when the level of corporate debt is above a threshold, the Taylor principle becomes inverted. Raising nominal rates actually *increases* inflation because the income effect dominates the substitution effect. Our result offers one rationalisation of the ‘price puzzle’, that prices generally respond less than output to monetary disturbances, increases with the level of corporate debt in the economy. At extreme levels of corporate debt, prices may even rise after a monetary contraction.[6]

We then extend our model to a dynamic New Keynesian framework with heterogeneous households and nominal rigidities via a Calvo Pricing assumption. We show that as the steady-state corporate debt-to-GDP ratio increases, the coefficient of monetary policy rate on the dynamic path of inflation declines, i.e., a weaker effect of monetary contractions in lowering inflation. Then in a numerical exercise, we compare the responses of the economy when the steady-state corporate debt-to-GDP ratio is low (benchmark) or high, describing the increase in corporate debt levels in the US over the last 15 years. This quantitative example considers both a contractionary monetary shock and a positive consumption demand shock with a standard benchmark Taylor rule. The choice of a positive demand shock is motivated by the question of how the recovery of demand post-pandemic could potentially affect the economy’s monetary profile and pose challenges to price stabilisation. Model simulations shed light on the cyclicality of the consumption expenditure of wealthy stockholding households and those who do not hold stocks. We find that the consumption expenditure of owner households, that is, the equity owners, tends to be highly pro-cyclical, whereas the expenditure of the lender households, those who do not own shares, is much less cyclical. As the debt level increases, the more pro-cyclical owner households’ consumption appears, and the more acyclical lender households’ consumption expenditure becomes.

[6] This price puzzle was first noted by Sims (1992) and has been documented and analysed extensively by subsequent work.

On the dynamic responses, after the monetary contraction, inflation falls on impact in both cases, before subsequently rising to the positive realm. Particularly, the subsequent rise in inflation is higher in the high debt case than the benchmark case, suggesting the higher corporate indebtedness is, the more challenging it is to rein in inflation. On the real side, output falls in both the high debt case and the benchmark case, but it falls more aggressively in the high debt case. With the positive consumption demand shock, unsurprisingly, inflation rises when demand picks up, and output also increases on impact. Notably, inflation is much higher in the high debt case than the benchmark case, and the subsequent drop in output and employment is also more severe in the high debt case.

Amid the pandemic crisis, monetary authorities may be inclined to re-evaluate the appropriate trade-off between the objectives of output and inflation stabilisation. In light of such considerations, we also conduct a counterfactual experiment where we consider a monetary authority who cares more about output stabilisation than our benchmark Taylor rule.[7] In this experiment, an output stabilisation Taylor rule could bring output back up to the steady state rather quickly, whereas our benchmark Taylor rule leads to greater and more persistent output and employment loss. Nevertheless, the output stabilisation Taylor rule leads to a much higher inflationary profile. Thus, the overall takeaway from this experiment is that the trade-off between inflation stabilisation and output stabilisation becomes acute with a large volume of corporate debt in the economy.

Related literature. There is a flourishing literature focusing on (corporate) debt and its implications for inflation and monetary policy (see e.g. Gomes, Jermann and Schmid, 2016; Ottonello and Winberry, 2020; Mian, Straub and Sufi, 2021). Much of the existing work has focused on the drag of debt on firm investment or aggregate demand, or the impact of unexpected inflation on the real burden of debt, but less attention has been paid to how nominal debt could affect the efficacy of monetary policy in controlling inflation. Our work serves to fill this gap in the literature.

More specifically, Gomes, Jermann and Schmid (2016) investigate how lower-than-expected inflation creates a debt overhang, and the authors focus on the macroeconomic responses to inflation changes. In contrast, we turn the question around by asking how debt hampers the ability of the central bank to control inflation. What distinguishes our model is that we show nominal debt may even be a source of inflation even when the central bank tries to use contractionary monetary policy to combat inflation.

In a similar spirit, Ottonello and Winberry (2020) and Mian, Straub and Sufi (2021) show that monetary policy is less powerful when the debt level is high or the distance to default is low. In Ottonello and Winberry (2020), the authors show that the investment of low debt firms or those with high distance to default is more responsive to expansionary monetary shocks, while the investment of high debt firms with high default risks is less so; the concern there is not with contractionary monetary policy controlling inflation. Mian, Straub and Sufi (2021) focus on household debt, propose a theory of indebted demand,

[7] This is supported by policy makers' public speech as that, for example, the FOMC's 'balanced approach' of accommodative policy is more consistent with a Taylor rule that includes a much higher output coefficient (see Bernanke, 2015; Yellen, 2012).

and show that large household debt lowers aggregate demand and the natural rate of interest.[8] Whereas both Mian, Straub and Sufi (2021) and our paper suggest monetary policy has limited ammunition in the presence of large debt, the policy angles and the mechanisms differ. In Mian, Straub and Sufi (2021) the policy angle is on accommodative monetary policy supporting aggregate demand, whereas in our model, it is on the general ability of monetary policy to target inflation. Furthermore, the mechanism in Mian, Straub and Sufi (2021) works through the demand side where the assumption of nonhomothetic preferences generates the property that large debt levels weigh negatively on aggregate demand. Our mechanism works through the income effect of nominal debt shifting the aggregate supply curve in addition to aggregate demand; the income effect of debt flattens the aggregate supply curve, which blunts contractionary monetary policy in lowering inflation. In this regard, we see our work as complementary.

Other salient examples of corporate indebtedness in the macroeconomy include but are not limited to Farhi and Tirole (2009), Bhamra, Fisher and Kuehn (2011), Occhino and Pescatori (2014, 2015), Greenwald (2019), Darmouni, Giesecke and Rodnyansky (2020), and Lakdawala and Moreland (2021).[9] While our paper shares the similarity with many papers in this literature that inflation reduces the real burden of corporate debt, it differs because of our general equilibrium channel through legacy debt and heterogeneous households. Much of the empirical literature on corporate debt investigates the real consequences of corporate debt on investment, output, or tail risks (see for example, Mian, Sufi and Verner, 2017; Jordà, Kornejew, Schularick and Taylor, 2020), but there is limited work concerning how corporate debt affects the monetary transmission mechanism and whether it hampers the monetary authority's ability to control inflation, for which our model provides testable implications. Nevertheless, our results echo a similar point in Schularick and Taylor (2012) that credit and money deserve to be watched carefully when implementing monetary policy rules.

Papers including Gomes, Haliassos and Ramadorai (2020) emphasise the skewed cross-sectional distribution of stock ownership, while the strong cylicality of bank versus bond financing of corporate liabilities is documented in Becker and Ivashina (2014), and Adrian, Colla and Song Shin (2013) among others. This suggests that the cyclicality of aggregate savings is important to the understanding of corporate leverage implications. Furthermore, we argue that the distinction between households that own equity and the lender/worker households that save, either through the banking system or through non-bank financial intermediaries is important. First, empirical evidence suggests that the top rich invest relatively more in stocks (see Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Parker, 2001; Carroll, 2002; Vissing-Jorgensen, 2002; Campbell, 2006; Wachter and Yogo, 2010; Bucciol and Miniaci, 2011; Calvet and Sodini, 2014; Gârleanu and Panageas, 2015), and moreover, a significant proportion of safe corporate debt are held either by households directly, or through bank deposits, or in mutual funds, ETFs, life insurance, pension funds, which the 'salaried creditors' indirectly hold. As Campbell (2006) shows, low wealth households hold overwhelmingly large proportions of liquid or safe assets and do

[8] Mian, Straub and Sufi (2021) point out that indebted demand is primarily driven by household debt, not corporate debt.

[9] The implication of high levels of corporate debt has also been studied extensively in the corporate finance literature focusing solely on optimal firm decisions (see Myers (1977) as a classic example), but usually in real models that do not focus on monetary policy transmission mechanisms.

not participate in the risky stock markets. Second, Toda and Walsh (2020) also differentiate households as an equity holder and a bond holder. Based on their model, Toda and Walsh (2020) provide empirical evidence that suggests that the portfolio share of the 1% income earners in the United States concentrates in stocks and that when the income share of the top 1% rises, the subsequent 1-year excess stock market return falls on average. Toda and Walsh (2020) also show that this finding is not specific to the US. Third, that the lender households supply labour and do not participate in the equity market is also consistent with Benzoni, Collin-Dufresne and Goldstein, 2007, who show that zero equity allocations can be obtained where labour income risks are highly correlated with stocks and produce results consistent with empirical observation.

More broadly, our paper connects with the classic literature on inside money in general equilibrium that dates back at least to Grandmont and Younes, 1972, 1973; Shapley and Shubik, 1977. In this literature, money is inside because it enters the economy issued against an offsetting loan, and the repayment of the loan guarantees money's departure, and, together with a non-Ricardian seigniorage transfer, the price level is determined in equilibrium and money is non-neutral, even with flexible prices. This literature includes Dubey and Geanakoplos (2003); Tsomocos (2003); Bloise and Polemarchakis (2006); Goodhart, Sunirand and Tsomocos (2006).

The next section provides some motivating facts, and Section 3 presents a static model and obtains closed-form solutions for equilibrium analysis. Section 4 extends the static model to a dynamic setting while Section 4.9 presents a quantitative example to illustrate the analytic results. Section 5 concludes.

2 Motivating facts

2.1 Rise of corporate debt

Following Goodhart and Pradhan (2020), Table 1 documents the non-financial corporate indebtedness of both advanced economies and emerging economies in Q4 2007, Q4 2018, and Q4 2020. Two observations emerge: in the decade since the onset of the Global Financial Crisis leading up to the COVID-19 pandemic, there was already a significant increase in non-financial corporate indebtedness across both advanced and emerging economies. Between Q4 2018 and Q4 2020, the rise in corporate debt has been even more pronounced, primarily due to the pandemic crisis.

Table 1: Indebtedness of non-financial corporations

	Advanced Economies					
	US	EA	SWE	CAN	UK	JPN
Dec-07	70	93.3	125.2	81.7	82.1	99.5
Dec-18	75.2	106.2	158.8	114.3	76.1	99
Dec-20	84.6	115.1	175.3	132.4	80	115.6
	Emerging Economies					
	CHN	KOR	HK	CHL	BRA	TUR
Dec-07	94.3	84.8	124	65.2	29.7	29.6
Dec-18	149.1	95.6	219.5	100.2	46.3	68.1
Dec-20	160.7	111.1	246.8	115.9	54	72.1

Source: BIS. Numbers express non-financial corporate debt as % of GDP.

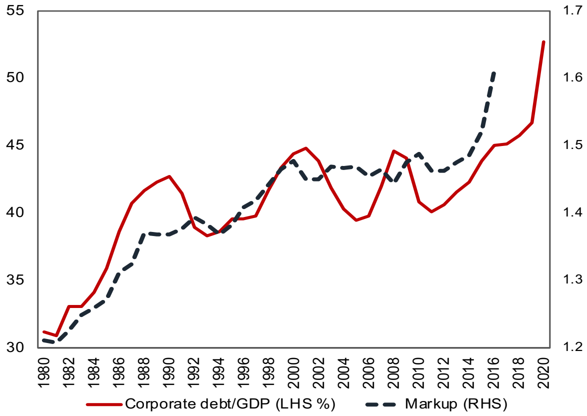
2.2 Corporate debt, markup and inflation

Since 1980, the non-financial corporate debt-to-GDP ratio and the aggregate markup in the US have both been on the rise.[10] Figure 1a plots the time series of corporate debt-to-GDP ratio and the aggregate markup in the US. Such a trend may raise concerns that given high debt and higher debt servicing cost, firms with market power may raise product prices in order to reduce the real burden of debt, so that post COVID-19 debt inflation becomes a possibility. The recent uptick in core inflation is very pronounced, as can be seen in Figure 1b, which documents trends of core inflation and non-financial corporate indebtedness in the US. The left-side graph of Figure 1b plots the quarterly time series of core inflation (i.e., CPI excluding food and energy) and the de-trended corporate leverage since 2000. To capture these features, the model set up in the next section features corporate debt holders and those who owe the debt, and also follows the New Keynesian tradition by modelling non-competitive firms that charge markups.

[10] De Loecker, Eeckhout and Unger (2020) find that the results hold across industries and sizes though are higher in smaller firms. Moreover, Díez, Fan and Villegas-Sánchez (2021) provide comprehensive empirical evidence suggesting the decline in competition at the global level.

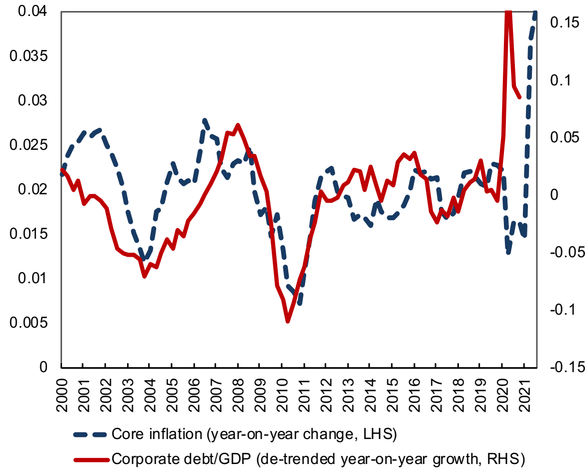
Figure 1: Aggregate markup, corporate debt, and core inflation in the US

(a) Markup and non-financial corporate debt-to-GDP



Source: The markup data is from De Loecker et al. (2020). Data on corporate debt and GDP are from Board of Governors of the Federal Reserve System (US) and U.S. Bureau of Economic Analysis, retrieved from FRED, Federal Reserve Bank of St. Louis.

(b) Core inflation and de-trended corporate debt/GDP



Source: Board of Governors of the Federal Reserve System (US) and U.S. Bureau of Economic Analysis, retrieved from FRED, Federal Reserve Bank of St. Louis, and authors' calculation.

3 Static Model

In this section, we present a stylised flexible price one-period general equilibrium with money in order to fix ideas on how corporate debt, in the presence of a working capital channel, generates an additional income effect of monetary policy. In Section 4 we extend the static model to a familiar New Keynesian dynamic setting with nominal rigidities to show the implications of this income effect on the trade-off between output and inflation stabilisation.

It is our thesis that the accretion of corporate debt makes models that assume no such historical legacy to be inappropriate for assessing current conditions. That said, however, the introduction of history and time makes it more complicated to apply static one-period models. In particular, we assume that there are two types of households: the first are 'owner households' that own firms, which is in accord with the usual assumptions, or the 'enterpriser-borrower' à la Fisher (1910). But the innovation in our paper is that we assume that the counterpart to the historical debt owed by firms is held in funds to which 'lender and worker households' were required to contribute from previous periods, and this type of households is essentially Fisher's 'creditor, the salaried man, or the labourer'. These funds pay out a proportion of their accumulated returns from corporate debt, D, depending on past interest rates, R. Because it is a one-period static model, we then assume that both owner households and lender households seek to use all their available funds in this period for consumption. In the subsequent dynamic setting, we relax this assumption and model the saving decision of the lender households, where both the quantity and the price of debt are endogenised.

For the rest, the underlying assumptions are more standard. The static model illustrates a one-period production economy and the period is divided into sub-periods, morning and evening. A unit measure of firms produce different types of consumption goods, so firms possess market power. A central bank exists to supply liquidity against offsetting credits and sets the policy rate, which we take as the borrowing cost in the money market. Owner households are endowed with a monetary (fiat) endowment and all private agents can borrow inside money against an offsetting credit from the money market should they wish. Lender households supply labour endogenously. There are two transaction moments in the period, which we term, 'morning' and 'evening'. In the morning, firms borrow from the money market to obtain liquidity and pay wages. This is the working capital financing in advance constraint that follows a long tradition in the literature on the cost channel of the monetary transmission mechanism (see Blinder, 1987; Farmer, 1984, 1988*a*,*b*; Fuerst, 1992; Christiano et al., 2005, 2015, non-exhaustive). Production then takes place. All output is then sold in the evening. Households carry their wealth and income into the evening to purchase goods. Firms repay debt that comes due in the evening.

3.1 Households

Owner households and lender households are indexed by $h \in \{o, l\}$ respectively and they demand a consumption bundle C^h, given by

$$C^h \equiv \left(\int_0^1 (c_j^h)^{1-\frac{1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

with c_j^h representing the quantity of goods variety j consumed by the household, and $\theta > 1$ being the elasticity of substitution between goods varieties. A lower θ leads to a higher markup set by the firms.

The price index is given by

$$P \equiv \left(\int_j (p_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}. \tag{2}$$

Owner households are shareholders of the firms, and the rest of the households are lenders to the firms. Each owner household is endowed with a monetary (fiat) endowment $m^o \geq 0$. We now outline the maximisation programme for the owner households and the lender households.

3.1.1 Owner Households

Owner households have a monetary endowment of m^0 and profits of Π from all firms as income. They spend the income on consumption c^o. Their preference is represented by (3),

$$U = c^o. \tag{3}$$

In the morning, initial cash balances are simply carried over till the evening without earning interest. In the evening, the owner household receives the profits of the firm and spends any unspent money on goods. Their flow constraint is (4),

$$Pc^o = \Pi + m^o. \tag{4}$$

3.1.2 Lender Households

Lender households have wage income of wL^l, and net debt repayment ψRD as income sources, where w denotes the nominal wage, L^l is the labour supply, R is the gross interest rate of the debt, D is the total stock of debt firms owe to the Lender Households, and ψ is the proportion of debt that comes due in the evening. We refer to D as the legacy debt, and the repayment of debt principal ψD and its interest rate R will be made endogenous in the dynamic section of the model.

Lender households' preference is represented by (5),[11] and they choose consumption and the supply of labour,

$$U = log(c^l) - L. \tag{5}$$

In the morning the lender households obtain their labour income and carry the money till the evening

$$\hat{m}^l = wL^l. \tag{6}$$

In the evening they receive the debt repayment. They spend the repayment of debt and their labour income on goods. Their effective flow budget constraint is (7)

$$Pc^l = wL^l + \psi RD. \tag{7}$$

3.2 Firms

A unit measure of firms is owned by owner households. Firm j produces good j according to a linear production function as below, where y_j is firm j's output, l_j is the labour it demands and A denotes technology.

$$y_j = Al_j. \tag{8}$$

Let b_j be the amount of liquidity the firm obtains from the money market by borrowing, and i be the monetary policy rate. Equation (9) is the liquidity constraint firm j faces in the morning. It states that firm j uses the money market b_j to pay for wages, essentially the working capital financing constraint. Equation (10) states that at the end of the period the firm uses the sales proceeds to pay back money market credit $b_j(1+i)$, repay the legacy debt plus interest due (ψRD) and distribute profits π_j. As we assume strictly positive interest rates, each constraint binds.

Firm j maximises profits π_j from the perspective of owner households by choosing labour l_j and money market liquidity b_j, and most crucially, by setting the price of its own goods variety p_j monopolistically.

[11] This specification was simple enough to incorporate meaningful substitution between consumption and leisure and still permit analytic results. Nevertheless, in the New Keynesian extension in the next section we use more standard preferences.

The morning constraint is

$$wl_j = b_j, \tag{9}$$

the evening constraint is

$$\pi_j + \psi RD + b_j(1+i) = p_j y_j, \tag{10}$$

and the flow budget constraint is:

$$\pi_j + (1+i)wl_j + \psi RD = p_j y_j. \tag{11}$$

3.3 Equilibrium

Equilibrium is defined as an allocation of resources and positive prices, given a positive monetary policy rate and monetary endowment, and legacy debt such that

(i) firms set prices while taking into account the price impact on demand,

(ii) agents maximise subject to their budget and liquidity constraints,

(iii) goods market, labour market, and money market clear, and expectations are rational.

We now characterise the equilibrium to show that the combination of legacy debt and working capital can provide clear monetary transmission mechanisms, even when allowing prices to adjust. To start with, Lemma 1 below summarises how real wage and the effective labour supply elasticity respond after a contractionary monetary policy shock (see Appendix A for the proof).

Lemma 1.

1. *Contractionary monetary policy reduces real wages.*
2. *Given the price level, the effective labour supply elasticity with respect to real wages is increasing on the real value of legacy debt and deceasing on the real value of working capital.*

The above lemma first shows that real wages fall in response to a contractionary monetary policy shock. Furthermore the markup, σ interacts with the policy rate positively. Through the working capital channel alone, the fall in real wages is unambiguous, in contrast to canonical sticky wage models. Furthermore, Lemma 1 implies that the effective labour supply elasticity in our model depends not only on preferences but also depends on the state of the economy through legacy debt and the working capital used in the economy. In contrast, in Christiano et al. (1997) the labour supply elasticity only depends on the parameter for leisure in preferences and their model's empirical performance depends sensitively on this parameter.

3.4 Distribution of Income and Aggregate Demand

Aggregate profits Π can be derived from (11) as

$$\frac{\Pi}{P} = Y - (1+i)\tilde{w}L - \psi\frac{RD}{P}. \tag{12}$$

The income, and hence, the demand from the owner household can be obtained by substituting (68) and (65). The equilibrium expression for income, $\frac{m^o}{P} + \frac{\Pi}{P}$, can be represented as

$$\frac{m^o}{P} + Y - \frac{A}{\sigma} + i\psi\frac{RD}{P}. \tag{13}$$

From this it follows that raising interest rates actually increases demand from owner households. This is because raising interest rates lowers the demand for labour, and the wage bill for the firm decreases sufficiently which puts upward pressure on profits.

We can combine the owner and lender households' budget constraints ($\frac{1}{P}(\psi RD + wL + \Pi + m)$) to obtain the expression (14) for aggregate demand in equilibrium,[12]

$$\frac{m}{P} + Y + i\left\{\psi\frac{RD}{P} - \frac{A}{\sigma(1+i)}\right\}. \tag{14}$$

From (14) we can see two effects of monetary policy. Contractionary monetary policy that increases i may increase or decrease aggregate demand depending on how large legacy debt is. On the one hand, higher interest rates increase the financing cost of labour and less is demanded. As a result, real wages decrease, causing downward pressure on aggregate demand. This is the usual substitution effect. On the other hand, the presence of legacy debt renders labour supply more elastic (see Lemma 1), so that the increase in i causes the decrease in wage expenditure to dominate the increase the financing costs. This leads to upward pressure on profits and owner households' income, and hence, aggregate demand. This is the income effect through legacy debt. We collect the insights so far in the following proposition.

Proposition 1. *In equilibrium, the response of aggregate demand to contractionary monetary policy (increasing i) depends positively on legacy debt.*

The income effect of monetary policy crucially depends on the presence of legacy debt and heterogeneous households. This can also be seen through the supply of labour which depends on the distribution of income (and hence demand) through legacy debt ($L = 1 - \frac{\psi\frac{RD}{P}}{\frac{1}{\sigma}\frac{1}{1+i}A}$). With a representative household the income effect disappears even when legacy debt is present, and contractionary monetary policy always decreases aggregate demand. To see this, we compare the model with the outcome if we had a

[12] $m = m^o$ denotes the aggregate monetary endowment of households.

representative agent combining both owner and lender households. Aggregate income would become

$$\tilde{w}L + \psi\frac{RD}{P} + \frac{m}{P} + \frac{\Pi}{P}, \tag{15}$$

and substituting in aggregate profits, aggregate demand becomes

$$\frac{m}{P} + Y - i\frac{A}{\sigma(1+i)}. \tag{16}$$

Comparing (14) and (16), given a price level, raising interest rates only reduces aggregate demand in the representative agent case. This is because in the representative agent case, the distribution of income does not matter, the upward pressure on profits from lower wage expenditure is exactly offset by the increase in financing costs, and hence, the income effect is no longer present.

Building on the above analysis, we derive the closed-form solution for the price level and allocation in Appendix B. The derivation steps to obtain the closed-form solution lead to the following corollary.

Corollary 1. *In equilibrium, both nominal profits and real profits fall when nominal interest rates rise.*

Even though the rise of nominal interest rates reduces wage expenditure, it also causes revenue to go down due to the drop in labour supply. In equilibrium firm profits unambiguously fall when nominal interest rates rise, and vice versa, which is consistent with the empirical facts documented in Christiano, Eichenbaum and Evans (2005) and Christiano, Eichenbaum and Evans (1997).

We now characterise the transmission mechanism of monetary policy and state the central result in the following proposition (see the proof in Appendix C).

Proposition 2. *In equilibrium*

1. *when legacy debt is sufficiently low ($i\psi RD < b$),*
   - *(a) the standard Taylor principle applies,*
   - *(b) the higher debt is, the less effective is raising interest rates in lowering current inflation;*
2. *when legacy debt is sufficiently high ($i\psi RD > b$),*
   - *(a) the Taylor principle is inverted - raising interest rates increases current inflation,*
   - *(b) the higher debt, the worse inflation caused by raising interest rates.*

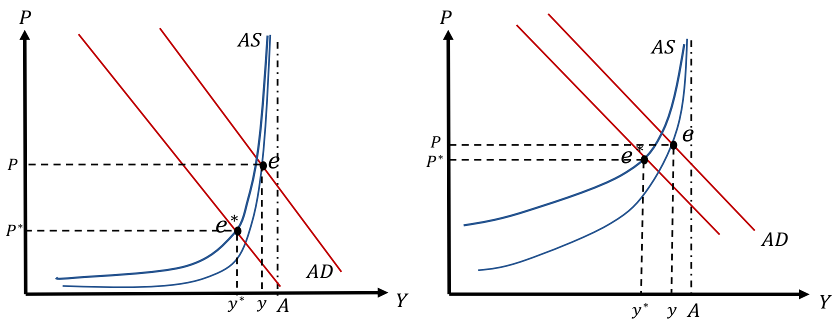
Proposition 2 highlights that the transmission of monetary policy crucially depends on the amount of legacy debt ψRD relative to working capital b. The standard Taylor principle holds ($\epsilon_{Pi} < 0$) *iff* $i\psi RD < b$. When $i\psi RD < b$ holds, an increase in legacy debt increases the labour supply elasticity and thus flattens the AS curve. So when nominal rates rise, current inflation falls less than in the case with lower debt. Hence, the higher the debt is, the weaker prices respond, and prices respond less than output to monetary disturbances, which provides an alternative mechanism to rationalise the price puzzle.

When $i\psi RD > b$ the Taylor principle is inverted and $\epsilon_{Pi} > 0$. That is, if debt is extremely high relative to working capital liquidity, raising interest rates *raises* the rate of inflation. This phenomenon is also documented in the empirical literature.

To reinforce this intuition, we use an aggregate supply AS and aggregate demand AD diagram for the goods market to illustrate a low debt scenario and a high debt scenario with a rise in the policy rate. For this AS-AD diagram, we have factored in the clearing of labour market and money market, but not the goods market (P, y); therefore, we are able to express the AS and AD as functions of output y, price level P, and exogenous parameters $m, i, D, \sigma, A, \psi, R$. The aggregate demand is expressed in (14). As can be seen in (14), with the rise in i, the substitution effect shifts the AD curve to the left, but the income effect through debt offsets the shift; thus, the high debt scenario sees the AD shift less to the left than the low debt case. To obtain the AS curve, we combine the producer's optimality condition for labour demand (65), the labour supply curve (68), and the production function, and we get

$$y = A - \sigma(1+i)\psi\frac{RD}{P}. \tag{17}$$

As can be seen in (17), an increase in i reduces aggregate supply, and a higher debt renders the AS curve more elastic.



(a) Low debt

(b) High debt

Figure 2: AS-AD diagram: a rise in policy rate

The left diagram (a) illustrates a low debt scenario. The right diagram (b) illustrates a high debt scenario. Equilibrium e is the equilibrium before the rise in the policy rate, and equilibrium e^* is the equilibrium after the rise in the policy rate. The vertical line at A is the output when there is no debt in the economy.

Figure 2 displays the AS-AD diagram to qualitatively show the equilibrium changes when the central bank raises interest rates. The left diagram (a) illustrates a low debt case, and the right diagram (b) shows a high debt case. In the low debt case, the rise in the policy rate significantly reduces inflation, whereas in the high debt case, the rise in the policy rate only moderately reduces inflation but output falls more responsively. This is because the high debt case shifts the AD to the left less and the AS curve also becomes more elastic due to the income effect through debt. Indeed, if the debt level is exceptionally high, the rise in the policy rate would even increase inflation, as proved in the inverted Taylor principle case in Proposition 2.2.

4 Dynamic Model

We now show that the intuition and mechanisms illustrated in the static model hold in an environment based on the canonical New Keynesian framework with nominal rigidities (via Calvo pricing) and an endogenous monetary policy rule (Taylor rule). The dynamic version distinguishes wholesale producers from intermediate goods producers. Wholesale producers are price-takers and can access short-term financing from the money market. Intermediate goods producers are static price-setters with market power. We assume a steady-state level of legacy debt which wholesale firms choose to roll over at prevailing interest rates. Wholesale firms solve a dynamic problem by maximising the discounted value of real profits, equated at the owner household marginal utility. We also replace the monetary endowment of households with central bank open market operations in the bond market.

4.1 Owner Households

Owner households own both wholesale and intermediate goods firms, and they maximise their expected inter-temporal utility

$$U^o = \sum_t \mathbb{E}_t \beta^t exp(\epsilon_t^d) log(c_t^o), \tag{18}$$

where ϵ_t^d is a normally distributed demand shock[13]. Preferences are subject to their flow budget constraint written in real terms as follows:

$$c^o + k' = \tilde{\pi}_W + \tilde{r}_k k + \int_j \tilde{\pi}_j, \tag{19}$$

where $\tilde{\pi}_W$ are profits from wholesale producers, $\tilde{\pi}_j$ are profits from intermediate goods producers each period. Optimality with respect to capital gives

$$\frac{1}{c^o} = \beta \mathbb{E} \frac{1}{c^{o'}} (\tilde{r}_k'). \tag{20}$$

4.2 Lender Households

Similar to owner households, lender households maximise

$$U^l = \sum_t \mathbb{E} \beta^t \left\{ exp(\epsilon_t^d) log(c_t^l) - \frac{\kappa}{2} l^2 \right\}. \tag{21}$$

and are subject to the budget constraint written in real terms

$$\tilde{q}\tilde{d}' + \frac{\phi_d}{2} \tilde{q}(\tilde{d}' - \bar{d})^2 + c^l = \tilde{w} l + \frac{\tilde{d}}{1+\eta}, \tag{22}$$

[13]We suppress notation for this for the sake of brevity and reintroduce it in the quantitative simulation. Nevertheless, the shock should appear wherever the marginal utility of households appears, including in the forward looking equations of the firms.

where $\bar{d}$ is the steady state value of debt and $\frac{\phi_d}{2}\tilde{q}(\tilde{d}' - \bar{d})^2$ is a quadratic adjustment cost for debt and η is the net rate of inflation. The optimality condition with respect to labour is

$$\frac{\tilde{w}}{c^l} = \kappa l, \tag{23}$$

while the optimality condition with respect to debt is

$$\frac{\tilde{q}}{c^l}(1 + \phi_d(\tilde{d}' - \bar{d})) = \beta \mathbb{E} \frac{1}{c^{l'}} \frac{1}{1+\eta'}. \tag{24}$$

4.3 Wholesale Firms

Wholesale firms are price takers, and maximise the present discounted value of real value profits valued at the owner's marginal utility

$$\sum_t \beta^t \mathbb{E} \frac{1}{c_t^o} \tilde{\pi}_{W,t}. \tag{25}$$

They have a production function with capital k and labour l being the inputs and A being productivity:

$$y_W = A k^\alpha l^{1-\alpha}. \tag{26}$$

Capital is rented from the owner households while labour is rented from the lenders. As in the static model, firms face a morning budget constraint and an evening one. In equilibrium, these can be represented as the working capital constraint and the flow budget constraint respectively. The nominal working capital constraint is represented by eq (27), and the end-period nominal constraint is represented by eq (28).

$$wl = b \tag{27}$$

$$\pi_W + r_k k + d_W + b(1+i) = p_W y_W + q d'_W, \tag{28}$$

where p_W is the nominal value of a unit of wholesale goods, and b is the money that wholesale firms borrow from short-term money market at nominal interest rate i. d'_W is the nominal value of inter-temporal bonds sold at price q, and which is repaid one period in the future. Define the real value of short-term borrowing as $\tilde{b} = \frac{b}{P}$, the real value of inter-temporal bonds as $\tilde{d}'_W = \frac{d'_W}{P}$, and recall that inflation is given by $1 + \eta = \frac{P}{P_{-1}}$. With this, we obtain the real flow budget constraints as follows:

$$\tilde{w} l = \tilde{b}, \tag{29}$$

$$\tilde{\pi}_W + \tilde{r}_k k + \frac{1}{1+\eta}\tilde{d}_W + \tilde{b}(1+i) = \tilde{p}_W y_W + \tilde{q} d'_W. \tag{30}$$

Optimality with respect to debt gives

$$\tilde{q}\frac{1}{c^o} = \beta\mathbb{E}\frac{1}{c^{o'}}\frac{1}{1+\eta'}. \tag{31}$$

Optimality with respect to capital and labour are

$$\tilde{r}_k = \alpha\tilde{p}_W y_W/k, \tag{32}$$

$$\tilde{w} = \frac{1}{1+i}(1-\alpha)\tilde{p}_W y_W/l. \tag{33}$$

Using these optimality conditions we obtain the expression for the price of wholesale goods,

$$\tilde{p}_W = \frac{1}{A}(1-\alpha)^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{\alpha}}((1+i)\tilde{w})^{1-\alpha}(r_k)^{\alpha}. \tag{34}$$

4.4 Intermediate Goods Firms

Intermediate goods firms purchase goods from wholesale firms, and have a simple linear production function. They each have differentiated goods and sell that to the consumer, setting the price of the goods they sell. The marginal cost of each firm is $\tilde{p}_W$.

These constraints can be summarised in a nominal flow budget constraint.

$$\tilde{\pi}_j = \frac{1}{P}\left\{p_j y_j - p_W y_j\right\} \tag{35}$$

substituting in the demand function $y_j = \left(\frac{p_j}{p}\right)^{-\theta} y$,

$$\tilde{\pi}_j = \left(\frac{p_j}{p}\right)^{1-\theta} y - \tilde{p}_W\left(\frac{p_j}{p}\right)^{-\theta} y \tag{36}$$

where $\tilde{p}_W$ is the real marginal cost.

Let ϕ be the probability that a intermediate goods firm does not change its price each period.

This gives us the following expression for the price of the firms that re-set their price each period

$$p_j^{\#} = \sigma\frac{X_1}{X_2} \tag{37}$$

where $\sigma = \frac{\theta}{\theta-1}$ and

$$X_1 = \frac{1}{c^o}\tilde{p}_W P^{\theta} y + \phi\beta\mathbb{E}X_1' \tag{38}$$

$$X_2 = \frac{1}{c^o}P^{\theta-1} y + \phi\beta\mathbb{E}X_2'. \tag{39}$$

With flexible prices it follows that

$$p_j^{\#} = \sigma P \tilde{p}_W. \tag{40}$$

Finally aggregate profits of this sector are

$$\tilde{\pi} = \int_0^1 \tilde{\pi}_j dj = y \int_0^1 \left\{ \left(\frac{p_j}{p}\right)^{1-\theta} - \tilde{p}_W \left(\frac{p_j}{p}\right)^{-\theta} \right\} dj \tag{41}$$

$$= y - \tilde{p}_W \nu y, \tag{42}$$

where ν is price dispersion.

4.5 Final Goods Firm

The final goods firm's problem is exactly the same as in the standard literature. Each period the final consumption good, y is produced by a perfectly competitive, representative final goods firm firm. The firm produces the final good by combining a continuum of intermediate goods, indexed by $j \in (0,1)$, using the technology

$$y = \left(\int_0^1 y_j^{1-\frac{1}{\theta}} \right)^{\frac{\theta}{\theta-1}} dj. \tag{43}$$

Optimality implies

$$y_j = \left(\frac{p_j}{p}\right)^{-\theta} y, \tag{44}$$

and

$$P = \left[\int_0^1 p_j^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{45}$$

and note that integration of (44) using the production function of the intermediate goods firm gives

$$y_W = \nu y = \int_0^1 \left(\frac{p_j}{p}\right)^{-\theta} y dj. \tag{46}$$

4.6 Monetary Policy

The monetary authority sets the short-term interest rate of the money market according to a Taylor rule. It also trades inter-temporal bonds in its regular open market operation. Let the *overline* symbol denote the steady-state real value and let $\rho_y, \rho_i, \rho_\eta$ be the Taylor rule coefficients, and the Taylor rule is specified as follows:

$$\frac{1+i}{1+\bar{i}} = (\frac{y}{\bar{y}})^{\rho_y}(\frac{1+i_{-1}}{1+\bar{i}})^{\rho_i}(\frac{1+\eta}{1+\bar{\eta}})^{\rho_\eta}e^{\epsilon_i}, \tag{47}$$

where ϵ_i is a Normally distributed shock.

A meaningful trade-off between inflation and output stabilisation requires a real rigidity in the canonical New Keynesian model (Blanchard and Galí, 2007 call this this absence of the 'divine coincidence').[14] What is the appropriate output target is also unclear (Woodford, 2001, Garín, Lester and Sims, 2016). We include the log deviation of output from its trend in the Taylor rule. We do this because the nominal interest rate enters as a direct working capital financing cost, as well as because of the additional transmission mechanism we obtain through corporate debt. These reasons imply that monetary policy can meaningfully target overall output fluctuations and not only its deviation from the flexible price equilibrium.

Given the nominal interest rate specified from the Taylor rule, the monetary authority supplies money on demand in the money market $\tilde{M}$. This is interpreted as discount window actions. In addition, the monetary authority commits to trade a constant real amount of inter-temporal bonds $\tilde{\mu}$, and we interpret the trading of inter-temporal bonds as open market operations. These actions result in a public flow balance equation,

$$\tilde{M}i + \frac{\tilde{\mu}}{1+\eta} - \tilde{q}\tilde{\mu}' = 0, \tag{48}$$

where the interest rate i is given by the monetary policy rule, $\tilde{M}$ is supplied endogenously to clear the money market.

4.7 Market Clearing and Equilibrium

Below we summarise the market clearing conditions for final goods, money market, and the inter-temporal bond market:

- The market clearing condition for final goods is

$$Y = C^o + C^l + K' + \frac{\phi_d}{2}\tilde{q}(\tilde{D}' - \bar{D})^2. \tag{49}$$

- The money market clearing condition is

$$\tilde{B} = \tilde{M}. \tag{50}$$

[14] Ravenna and Walsh (2006) show that the presence of a working capital or cost channel alters the trade-off between inflation and output stabilisation. We show that this trade-off depends on the quantity of corporate debt in the economy and that the mechanism hinges on the income effect through corporate debt or bond, which differs from working capital credit. The higher the level of corporate debt is, the more difficult this trade-off becomes.

- The inter-temporal bond market clearing condition is

$$\tilde{D}'_W = \tilde{D}' + \tilde{\mu}'. \tag{51}$$

Note that the upper case variables coincide with the aggregate value of the population share. In the quantitative simulations we calibrate our economy such that the population share of the owner households is smaller than that of the workers. For analysis and derivations of the model we will assume that each household type is of unit measure and so we use the lower case variables to denote aggregate quantities.

In addition, the labour market, capital rental market, wholesale goods markets clear. For the sake of brevity we have assumed clearing in the description of the problem in the previous sections. Equilibrium is defined as a sequence of quantities and prices, given the monetary policy rule, and the real quantity of inter-temporal bonds traded by the monetary authority ($\tilde{\mu}$), such that

(i) the monetary authority supplies real money balances on demand ($\tilde{M} = \tilde{b}$),

(ii) intermediate goods firms set prices while taking into account the price impact on demand,

(iii) agents maximise subject to their budget and liquidity constraints,

(iv) goods market, labour market, capital market, corporate bond market, and money market clear, and expectations are rational.

Summing up the flow of funds constraint of the economy, we note that the interest payment of the monetary market equals the trading cost in the open market operation, i.e., $i\tilde{b} = q\tilde{\mu}' - \frac{\tilde{\mu}}{1+\eta}$. Let $m \equiv q\tilde{\mu}' - \frac{\tilde{\mu}}{1+\eta}$, it follows that $\tilde{M} = \frac{\tilde{m}}{i}$, and variable $\tilde{M}$ refers to the real value of money balance. The system of equations that summarise equilibrium together with the closed-form solution for the steady state and linearised dynamic equations are presented in Appendix D - F. Proposition 3 characterises the real effects of money and legacy debt in the steady state.

Proposition 3. *In the steady state,*

a More legacy debt decreases real money balance and output;

b An increase in the nominal interest rate reduces real money balance, but such reduction is weaker the higher legacy debt is;

c Changing the nominal interest rate exerts real effects in the steady state when debt $\bar{d} \neq 0$, but is neutral when debt $\bar{d} = 0$.

4.8 Dynamic Properties

In this section, we study the effects of legacy debt on the dynamic properties of the model and on the monetary transmission mechanism away from the steady state. Using (158) and (159) from Appendix F,

$$\hat{x}_1 - \hat{x}_2 = (1 - \phi\beta)\hat{p}_W + \phi\beta(\widehat{1 + \eta'}) + \phi\beta(\hat{x}'_1 - \hat{x}'_2), \tag{52}$$

and then using (156) and (157) from Appendix F we obtain the Phillips curve:

$$(1 \hat{+} \eta) = \frac{(1-\phi)(1-\phi\beta)}{\phi}\hat{p}_W + \beta(1 \hat{+} \eta'). \tag{53}$$

where the marginal cost is given by[15]

$$\hat{p}_W = -\frac{(1 \hat{+} \eta) + \bar{q}\hat{q}}{1-\bar{q}} - \frac{(1 \hat{+} i)}{((1 \bar{+} i)-1)}\left\{1 - \frac{(1 \bar{+} i)(1-\alpha)\bar{d}(1-\bar{q})}{2(\bar{w}\bar{l} + \bar{d}(1-\bar{q}))}\right\} - \hat{A} - \alpha\hat{k} - \frac{(1-\alpha)\bar{d}\left\{\bar{q}\hat{d}' - \hat{d}\right\}}{2(\bar{w}\bar{l} + \bar{d}(1-\bar{q}))}. \tag{54}$$

As the steady state level of legacy debt increases, the absolute value of the coefficient of interest rates on the path of inflation declines, i.e. changes in interest rates have a smaller negative effect on inflation. This is summarised in the following proposition.

Proposition 4. *Given monetary policy, as the steady-state debt level increases, the effectiveness of interest rates on the path of inflation declines.*

Here we show that the lack of 'divine coincidence' depends, in part, on the level of legacy debt. We can see this by taking the labour first order condition and the working capital constraint, (150), (148) and for analytical convenience set $\phi_d = 0$,

$$\hat{p}_W = \hat{b} + (1 \hat{+} i) - \hat{y} \tag{55}$$

Hence the Phillips' curve becomes

$$(1 \hat{+} \eta) = \frac{(1-\phi)(1-\phi\beta)}{\phi}(\hat{b} + (1 \hat{+} i) - \hat{y}) + \beta(1 \hat{+} \eta'). \tag{56}$$

We can now substitute the linearised form of our term structure expression, $i\tilde{b} = q\tilde{\mu}' - \frac{\tilde{\mu}}{1+\eta}$,

$$\hat{b} + (1 \hat{+} i) = \frac{\bar{\mu}(\bar{q}\hat{q} + (1 \hat{+} \eta))}{\bar{b}((1 \bar{+} i)-1)} - \frac{\bar{b}}{\bar{b}((1 \bar{+} i)-1)}(1 \hat{+} i). \tag{57}$$

This shows that the Philips' curve depends on the bond price $\hat{q}$ and working capital $\hat{b}$. In turn, through the lender household's budget constraint, the bond price will depend on the steady state level of debt. Putting this together, the path of interest rates that stabilises the path of inflation may cause instability in output directly through instability in working capital that indirectly causes instability in the path of intertemporal debt.

[15] The derivation is in Appendix G.

4.9 Quantitative Example

We now present our simulation, calibrated to the US. We take the population share of the owners to be 10% (and the worker-lenders to be 90%) to broadly match known distributions in financial asset holdings, in particular equity (see Toda and Walsh, 2020 and Campbell, 2006, for example). Other than corporate leverage, we appeal to standard calibrated parameters from recent literature (see Table 2). The model period is one quarter, and we set the discount factor β to 0.99, the same as in Ottonello and Winberry (2020). We set the markup parameter to 1.25, which is at the low end of the estimated markup in De Loecker, Eeckhout and Unger (2020) but at the high end of the value conventionally used in the New Keynesian literature. Regarding the monetary policy rule, the response to inflation is set to 1.5 while the smoothing parameter is set to be 0.5, similar to Gomes, Jermann and Schmid (2016) and in line with the literature. Following Christiano, Trabandt and Walentin (2010), we set the output coefficient to 0.2 as our benchmark.

A crucial calibration in this economy is the value of the corporate debt-to-GDP ratio at the steady state, i.e. the steady state leverage. This parameter matters for the wealth distribution of the 'enterpriser-borrower' and the 'salaried creditor'. We set the benchmark leverage to a 75% corporate debt-to-GDP ratio at the steady state. We then set a high debt leverage to 100%. In our numerical illustrations, we compare the macroeconomic responses between the benchmark case and the high debt case. Our choice of leverage is based on the ratio of the US non-financial corporate debt to the quarterly revenue of non-financial corporate business from 2001 to date, which fluctuates between 3 and 4 (or 75% and 100% annualised) and has been trending up in the recent decade. We believe our choice of leverage is reasonable, in view of the trend of corporate debt-to-GDP ratios in various economies documented in Section 2.1. In fact, our high debt leverage, i.e. 100% corporate debt-to-GDP ratio at the steady state, can be considered conservative. For example, the total non-financial business debt in the US stands at a historically high level of around 130% of GDP in 2020 (see Jordà, Kornejew, Schularick and Taylor, 2020 and Federal Reserve Board Financial Accounts of the United States 2020). In Appendix E, we report the steady state values in Table 4.

Table 2: Calibration

Parameter	A	α	β	i	σ	κ	ϕ	ϕ_d	ρ_y	ρ_η	ρ_i
Value	100	0.33	0.99	0.01	1.25	0.1	0.7	0.001	0.2	1.5	0.5

We simulate the model with two shocks, a positive shock to interest rates, and a positive demand shock. The former we assume to have no persistence while the latter a persistence of 0.9. A consumption demand shock gives us an insight into the response of policy in a post-pandemic recovery.

The model simulation sheds light on the cyclicality of the consumption expenditure of the households that own large shares of equity and those who do not. Table 3 presents the correlation matrix of key variables with output. The consumption expenditure of owner households, that is, the equity owners, tends to be highly pro-cyclical, whereas the expenditure of the lender households, those who do not own shares, is much less cyclical. Moreover, both working capital and labour income appear highly pro-cyclical. As the debt level increases, the more pro-cyclical owner households' consumption appears, and the more acyclical lender households' consumption expenditure becomes. This result connects with the literature on the high sensitivity of consumption growth of wealthy stockholders to the stock market and aggregate fluctuations. For example, Malloy, Moskowitz and Vissing-Jørgensen (2009) find higher sensitivity of the consumption growth of wealthy stockholders to both the stock market and to aggregate consumption growth, and Parker and Vissing-Jorgensen (2009) show that consumption growth of high-consumption and high-income households is significantly more exposed to aggregate fluctuations, among others (see Mankiw and Zeldes, 1991; Parker, 2001).

Table 3: Cyclical properties: correlations with output

	c^o	c^l	b	l	d
y (BMK lev)	0.73	0.38	0.96	0.93	-0.76
y (High lev)	0.88	0.20	0.99	0.97	-0.86

BMK lev refers to the benchmark leverage of 75% (annual), or $\bar{b}/\bar{y} = 3$. High lev refers to the high debt leverage of 100% (annual), or $\bar{b}/\bar{y} = 4$. c^o is the consumption of owner households, c^l is the consumption of lender households, b is working capital in real terms, l is labour, d is debt in real terms, and y is real output.

4.9.1 The Effect of Monetary Contractions

The tightening monetary policy shock we introduce is of 0.025 standard deviation in the nominal policy rate, which leads to an endogenous increase in the policy rate of around 1 percentage point. Figure 3 shows the dynamic responses to the monetary contraction shock, where the blue line represents benchmark leverage, or corporate debt-to-output ratio, of 75%, while the red line represents high debt leverage of 100%. As it shows, after a monetary contraction, inflation falls on impact in both cases, before subsequently rising to the positive realm. Particularly, the subsequent rise in inflation is higher in the high debt case than the benchmark case, suggesting the higher corporate indebtedness is, the more challenging it is to rein in inflation. On the real side, output falls in both the high debt case and the benchmark case. However, output responds much more aggressively in the high debt case. This is because the presence of corporate debt triggers the income effect of rising interest rates, labour supply becomes more elastic, which implies that in the high debt case the AS curve is more elastic than that in the low debt case. The positive shock to the nominal interest rates dampens both aggregate demand and aggregate supply, and with a more elastic AS curve, inflation, although it falls on impact, can even increase slightly after a monetary contraction (see Proposition 2.2).

Our impulse responses for real wages and labour confirm that Lemma 1 also holds on the dynamic path (that the effective labour supply elasticity depends on legacy debt and working capital and that, given working capital, higher legacy debt leads to higher effective labour supply elasticity). A monetary contraction increases the borrowing cost of financing the working capital, driving down real wages. With a high effective labour supply elasticity, the decrease in wages should drive down labour supply significantly. As can be seen in the high debt case, labour decreases more than in the benchmark case.

Moreover, corporate profits fall after a monetary contraction.

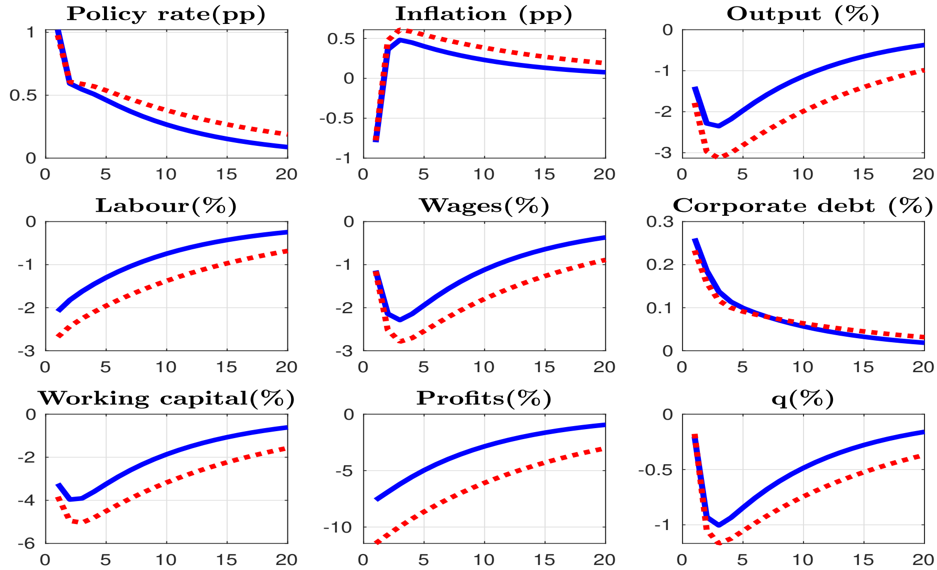


Figure 3: Tightening shock to nominal policy rate i.

Blue line is 75% leverage and red line is 100% leverage.

y-axis is % change and x-axis is the number of periods, and q is the real price of bond. Other than inflation and policy rate, all variables are in real terms

4.9.2 Output Stabilisation Taylor Rule

We now compare how legacy debt affects output-inflation stabilisation trade-offs and show that the trade-off between inflation stabilisation and output stabilisation becomes more acute with a large volume of corporate debt in the economy. With high levels of corporate debt, if the monetary authority is more concerned about output and employment stabilisation, the inflationary pressure then is high; if the monetary authority is strictly sticking to its price stability mandate, it could bring down inflation on impact but at the cost of hurting output and employment persistently.

Figure 4 shows different Taylor rule coefficients (in which the output coefficient is set to 0.2 or 0.9 and the inflation coefficient is kept at 1.5) in a high leverage regime (100% debt to GDP). The counterfactual experiment we consider is between a monetary authority who cares more about output stabilisation than our benchmark Taylor rule. To model this, we increase the Taylor rule output coefficient to 0.9, which is among the high range estimated in the literature (see e.g. Clarida, Gali and Gertler, 2000), and suggested by policy makers (see Bernanke, 2015; Yellen, 2012).[16]

[16] As Bernanke (2015) pointed out that 'in principle, the relative weights on the output gap and inflation should depend on, among other things, the extent to which policymakers are willing to accept greater variability in inflation in exchange for greater stability in output'. Moreover, according to Bernanke (2015), the FOMC pays closer attention to variants of the Taylor rule that include the higher output coefficient, and that Janet Yellen has also suggested that the FOMC's 'balanced approach' is more consistent with an output coefficient of 1.

In Figure 4 the solid line corresponds to the benchmark Taylor rule $\rho_y = 0.2$ and the dashed line corresponds to the output stabilisation Taylor rule $\rho_y = 0.9$. Compared with the benchmark Taylor rule, the output stabilisation Taylor rule ($\rho_y = 0.9$) brings output back up to the steady state within seven quarters whereas with the benchmark Taylor rule the loss of output is greater and much more persistent. Furthermore, the benchmark Taylor rule also sees more persistent loss in employment and business profits than the output stabilisation Taylor rule. Nevertheless, the output stabilisation Taylor rule leads to a much higher inflationary profile.

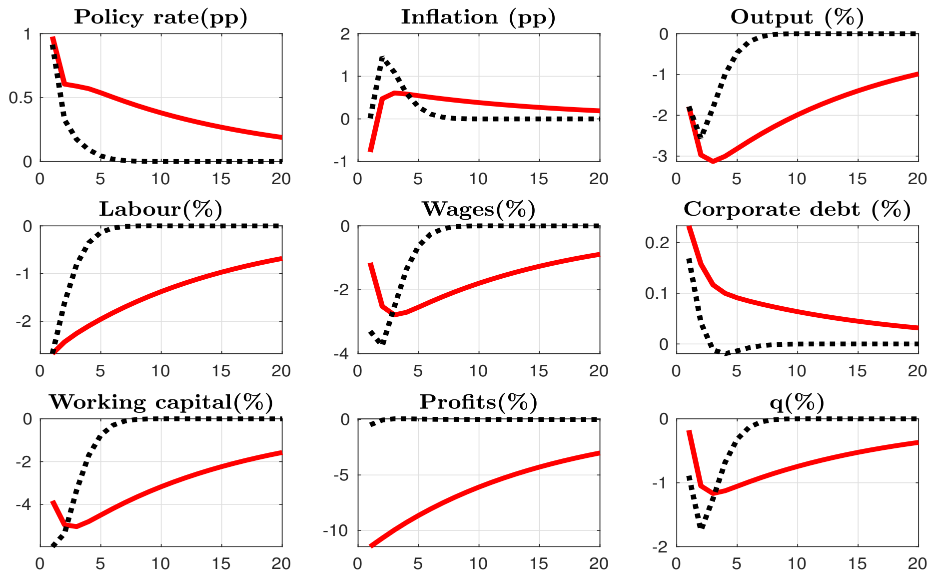


Figure 4: Tightening shock to nominal policy rate i with no output stabilisation.
Red solid line is the benchmark Taylor rule ($\rho_y = 0.2$) and the dashed black line is the output stabilisation Taylor rule ($\rho_y = 0.9$). y-axis is % change and x-axis is the number of periods, and q is the real price of bond. Other than inflation and policy rate, all variables are in real terms

4.9.3 The Effect of a Positive Demand Shock

We now study a positive demand shock of 0.05 standard deviation and an autoregressive coefficient of 0.9. Figure 5 demonstrates the dynamic responses with the positive demand shock and our benchmark Taylor rule. Unsurprisingly, inflation rises when demand picks up, and output also increases on impact. The monetary authority responds by tightening monetary policy, so the policy rate increases. As the policy rate increases, the cost channel of monetary policy starts to dampen aggregate supply, and with the income effect of debt, the aggregate supply curve shifts inward and becomes more elastic, leading to a subsequent drop in output. Notably, inflation is much higher in the high debt case than the benchmark case, and the subsequent drop in output is more severe in the high debt case than the benchmark case, for reasons already explained. Relatedly, employment in the high debt case subsequently drops but it holds up well in the benchmark case, which also suggests that high level of corporate debt increases the effective labour supply elasticity.

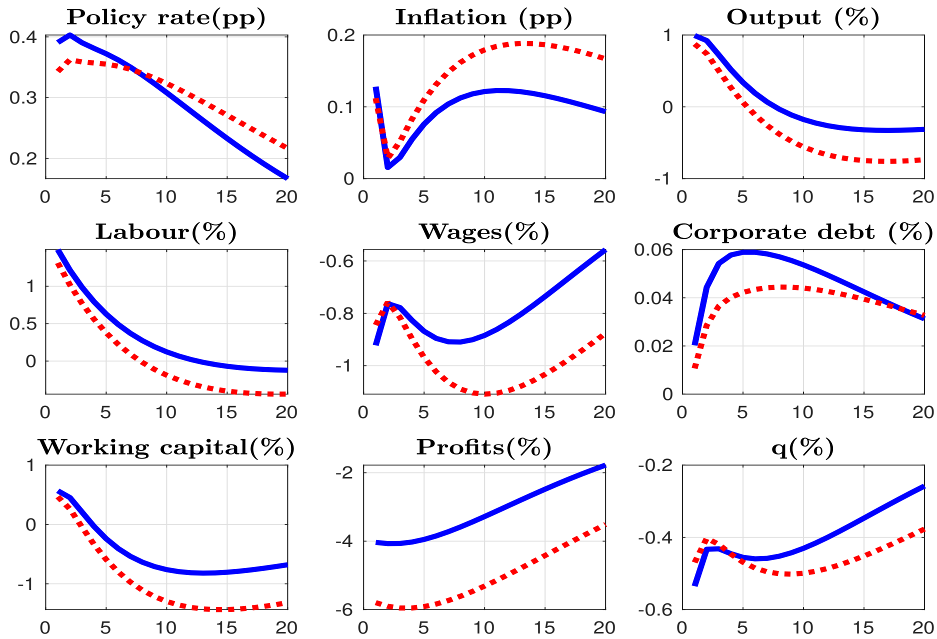


Figure 5: A positive consumption demand shock.

Blue line is 75% leverage and red line is 100% leverage.
y-axis is % change and x-axis is the number of periods, and q is the real price of bond. Other than inflation and policy rate, all variables are in real terms

5 Conclusion

We have presented a general equilibrium model to study the effect of corporate indebtedness on the monetary transmission mechanism. We highlight the result that high corporate debt levels render contractionary monetary policy less effective in controlling inflation. When the level of corporate debt is sufficiently high, contractionary monetary policy even increases inflation. While Irving Fisher's narrative is that booms and busts are caused by changes in the relative wealth of the 'enterpriser-borrower' and the 'creditor, the salaried man, or the labourer', our focal point is on the impact of such relative wealth on the efficacy of monetary policy in controlling inflation.

The model, both static and dynamic, is kept simple to derive intuitive. In the dynamic model, we derive the Phillips curve augmented with corporate debt, which shows analytically that the effectiveness of interest rates on the path of inflation declines as the steady-state debt level increases. Then a quantitative example is given to illustrate that the key results hold on the dynamic path away from the steady state. More generally, our results echo a similar point in Schularick and Taylor (2012) and Jordà, Schularick and Taylor (2013) that credit and money deserve to be watched carefully when implementing monetary policy rules

The mechanism of our central result is independent of standard financial and nominal frictions. We acknowledge that the attempt to write a tractable model to unpack the main mechanism and logic of debt inflation unavoidably leaves out many other features (such as financial frictions and labour market frictions) from the dynamic model. As the particular features of this model are not suited to quantitatively evaluate the effectiveness of policy, future research within a fully-fledged medium-scale quantitative models is warranted. Nevertheless, our result that monetary policy effectiveness depends on corporate debt levels adds support to the argument in papers including Curdia and Woodford (2010) that monetary policy should be conducted taking into account financial market conditions.

References

Abraham, Facundo, Juan Jose Cortina Lorente and Sergio L Schmukler (2020), 'Growth of global corporate debt: Main facts and policy challenges', *World Bank Policy Research Working Paper* (9394). 1

Adrian, Tobias, Paolo Colla and Hyun Song Shin (2013), 'Which financial frictions? parsing the evidence from the financial crisis of 2007 to 2009', *NBER Macroeconomics Annual* **27**, 159–214.
URL: *https://doi.org/10.1086/669176* 1

Barth, Marvin J and Valerie A Ramey (2001), 'The cost channel of monetary transmission', *NBER Macroeconomics Annual* **16**, 199–240. 5, 17

Becker, Bo and Victoria Ivashina (2014), 'Cyclicality of credit supply: Firm level evidence', *Journal of Monetary Economics* **62**, 76–93.
URL: *https://www.sciencedirect.com/science/article/pii/S0304393213001372* 1

Benzoni, Luca, Pierre Collin-Dufresne and Robert S Goldstein (2007), 'Portfolio choice over the life-cycle when the stock and labor markets are cointegrated', *The Journal of Finance* **62**(5), 2123–2167. 1

Bernanke, Ben S (2015), 'The taylor rule: A benchmark for monetary policy?', *Ben Bernanke's Blog* **28**. 7, 4.9.2, 16

Bhamra, Harjoat S, Adlai J Fisher and Lars-Alexander Kuehn (2011), 'Monetary policy and corporate default', *Journal of Monetary Economics* **58**(5), 480–494. 1

Blanchard, Olivier and Jordi Galí (2007), 'Real wage rigidities and the new keynesian model', *Journal of Money, Credit and Banking* **39**, 35–65.
URL: *http://www.jstor.org/stable/4123055* 4.6

Blinder, Alan S (1987), 'Credit rationing and effective supply failures', *The Economic Journal* **97**(386), 327–352. 3

Bloise, Gaetano and Herakles M Polemarchakis (2006), 'Theory and practice of monetary policy: Guest editors' introduction', *Economic Theory* pp. 1–23. 1

Bräuning, Falk and J Christina Wang (2020), 'The great leverage 2.0? a tale of different indicators of corporate leverage', *A Tale of Different Indicators of Corporate Leverage (April, 2020). Federal Reserve Bank of Boston Research Paper Series Current Policy Perspectives Paper* (87795). 1

Brunnermeier, Markus and Arvind Krishnamurthy (2020), 'The macroeconomics of corporate debt', *The Review of Corporate Finance Studies* **9**(3), 656–665. 1

Brunnermeier, Markus, Darius Palia, Karthik A Sastry and Christopher A Sims (2021), 'Feedbacks: financial markets and economic activity', *American Economic Review* **111**(6), 1845–79.

Bucciol, Alessandro and Raffaele Miniaci (2011), 'Household portfolios and implicit risk preference', *Review of Economics and Statistics* **93**(4), 1235–1250. 1

Buiter, W. H. (2002), 'The fiscal theory of the price level: A critique', *The Economic Journal* **112**, 459–480. 18

Calvet, Laurent E and Paolo Sodini (2014), 'Twin picks: Disentangling the determinants of risk-taking in household portfolios', *The Journal of Finance* **69**(2), 867–906. 1

Campbell, John Y (2006), 'Household finance', *The Journal of Finance* **61**(4), 1553–1604. 1, 4.9

Carroll, Christopher D (2002), 'Portfolios of the rich in household portfolios: Theory and evidence'. 1

Christiano, Lawrence J, Martin Eichenbaum and Charles L Evans (1997), 'Sticky price and limited participation models of money: A comparison', *European Economic Review* **41**(6), 1201–1249. 3.3, 3.4

Christiano, Lawrence J, Martin Eichenbaum and Charles L Evans (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of Political Economy* **113**(1), 1–45. 1, 3, 3.4, A

Christiano, Lawrence J, Martin S Eichenbaum and Mathias Trabandt (2015), 'Understanding the great recession', *American Economic Journal: Macroeconomics* **7**(1), 110–67. 3

Christiano, Lawrence J, Mathias Trabandt and Karl Walentin (2010), Dsge models for monetary policy analysis, *in* 'Handbook of monetary economics', Vol. 3, Elsevier, pp. 285–367. 4.9

Clarida, Richard, Jordi Gali and Mark Gertler (2000), 'Monetary policy rules and macroeconomic stability: evidence and some theory', *The Quarterly journal of economics* **115**(1), 147–180. 4.9.2

Curdia, Vasco and Michael Woodford (2010), 'Credit spreads and monetary policy', *Journal of Money, Credit and Banking* **42**(s1), 3–35.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1538-4616.2010.00328.x* 5

Darmouni, Olivier, Oliver Giesecke and Alexander Rodnyansky (2020), 'The bond lending channel of monetary policy', CEPR Discussion Paper No. DP14659. 1

De Loecker, Jan, Jan Eeckhout and Gabriel Unger (2020), 'The rise of market power and the macroeconomic implications', *The Quarterly Journal of Economics* **135**(2), 561–644. 10, 1a, 4.9

Díez, Federico J, Jiayue Fan and Carolina Villegas-Sánchez (2021), 'Global declining competition?', *Journal of International Economics* p. 103492. 10

Drèze, J. H. and H. Polemarchakis (2000), Monetary equilibria, *in* G.Debreu, W.Neufeind and W.Trockel, eds, 'Economic Essays: A Festschrift in Honor of W. Hildenbrand', Springer, pp. 83–108. 18

Dubey, Pradeep and John Geanakoplos (2003), ‘Inside and outside fiat money, gains to trade, and is-lm’, *Economic Theory* **21**(2), 347–397. 1, 18

Farhi, Emmanuel and Jean Tirole (2009), ‘Leverage and the central banker’s put’, *American Economic Review* **99**(2), 589–93. 1

Farmer, Roger EA (1984), ‘A new theory of aggregate supply’, *The American Economic Review* pp. 920–930. 3

Farmer, Roger EA (1988*a*), ‘Money and contracts’, *The Review of Economic Studies* **55**(3), 431–446. 3

Farmer, Roger EA (1988*b*), ‘What is a liquidity crisis?’, *Journal of Economic Theory* **46**(1), 1–15. 3

Fisher, Irving (1910), *Introduction to economic science*, Macmillan. 3

Friedman, Benjamin M (1990), ‘Implications of corporate indebtedness for monetary policy’, *NBER Working Paper* (w3266).

Fuerst, Timothy S (1992), ‘Liquidity, loanable funds, and real activity’, *Journal of Monetary Economics* **29**(1), 3–24. 3

Garín, Julio, Robert Lester and Eric Sims (2016), ‘On the desirability of nominal gdp targeting’, *Journal of Economic Dynamics and Control* **69**, 21–44.
URL: *https://www.sciencedirect.com/science/article/pii/S0165188916300665* 4.6

Gârleanu, Nicolae and Stavros Panageas (2015), ‘Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing’, *Journal of Political Economy* **123**(3), 670–685. 1

Gertler, Mark and Simon Gilchrist (1994), ‘Monetary policy, business cycles, and the behavior of small manufacturing firms’, *The Quarterly Journal of Economics* **109**(2), 309–340. 5

Gomes, Francisco, Michael Haliassos and Tarun Ramadorai (2020), ‘Household finance’, *Journal of Economic Literature, forthcoming* . 1

Gomes, Joao, Urban Jermann and Lukas Schmid (2016), ‘Sticky leverage’, *American Economic Review* **106**(12), 3800–3828. 1, 4.9

Goodhart, Charles AE, Pojanart Sunirand and Dimitrios P Tsomocos (2006), ‘A model to analyse financial fragility’, *Economic Theory* **27**(1), 107–142. 1

Goodhart, Charles and Manoj Pradhan (2020), *The Great Demographic Reversal*, Springer International Publishing. 1, 2.1

Grandmont, Jean-Michel and Yves Younes (1972), ‘On the role of money and the existence of a monetary equilibrium’, *The Review of Economic Studies* **39**(3), 355–372. 1

Grandmont, Jean-Michel and Yves Younes (1973), ‘On the efficiency of a monetary equilibrium’, *The Review of Economic Studies* **40**(2), 149–165. 1

Greenwald, Daniel (2019), *Firm debt covenants and the macroeconomy: The interest coverage channel*, MIT Sloan School of Management. 1

Haliassos, Michael and Carol C Bertaut (1995), 'Why do so few hold stocks?', *The Economic Journal* **105**(432), 1110–1129. 1

Jordà, Òscar, Martin Kornejew, Moritz Schularick and Alan M Taylor (2020), Zombies at large? corporate debt overhang and the macroeconomy, Technical report, National Bureau of Economic Research. 1, 4.9

Jordà, Òscar, Moritz Schularick and Alan M Taylor (2013), 'When credit bites back', *Journal of Money, Credit and Banking* **45**(s2), 3–28. 5

Jordà, Òscar, Sanjay R Singh and Alan M Taylor (2020), 'The long-run effects of monetary policy', National Bureau of Economic Research.

Kaplan, Greg, Benjamin Moll and Giovanni L Violante (2018), 'Monetary policy according to hank', *American Economic Review* **108**(3), 697–743.

Kashyap, Anil K, Jeremy C Stein and David W Wilcox (1993), 'Monetary policy and credit conditions: Evidence from the composition of external finance', *The American Economic Review* **83**(1), 78–98.

Kashyap, Anil K, Owen A Lamont and Jeremy C Stein (1994), 'Credit conditions and the cyclical behavior of inventories', *The Quarterly Journal of Economics* **109**(3), 565–592.

Lakdawala, Aeimit and Timothy Moreland (2021), 'Monetary policy and firm heterogeneity: The role of leverage since the financial crisis', *Available at SSRN 3405420* . 1

Malloy, Christopher J, Tobias J Moskowitz and Annette Vissing-Jørgensen (2009), 'Long-run stockholder consumption risk and asset returns', *The Journal of Finance* **64**(6), 2427–2479. 4.9

Mankiw, N Gregory and Stephen P Zeldes (1991), 'The consumption of stockholders and nonstockholders', *Journal of Financial Economics* **29**(1), 97–112. 4, 1, 4.9

Mian, Atif, Amir Sufi and Emil Verner (2017), 'Household debt and business cycles worldwide', *The Quarterly Journal of Economics* **132**(4), 1755–1817. 1

Mian, Atif, Ludwig Straub and Amir Sufi (2021), 'Indebted demand', *The Quarterly Journal of Economics* **136**(4), 2243–2307. 1, 8

Mian, Atif R, Ludwig Straub and Amir Sufi (2020), The saving glut of the rich, Working Paper 26941, National Bureau of Economic Research.
URL: *http://www.nber.org/papers/w26941*

Minsky, Hyman P (1980), 'The federal reserve: between a rock and a hard place', *Challenge* **23**(2), 30–36.

Myers, Stewart C (1977), 'Determinants of corporate borrowing', *Journal of Financial Economics* **5**(2), 147–175. 9

Occhino, Filippo and Andrea Pescatori (2014), 'Leverage, investment, and optimal monetary policy', *The BE Journal of Macroeconomics* **14**(1), 511–531. 1

Occhino, Filippo and Andrea Pescatori (2015), 'Debt overhang in a business cycle model', *European Economic Review* **73**, 58–84. 1

Ottonello, Pablo and Thomas Winberry (2020), 'Financial heterogeneity and the investment channel of monetary policy', *Econometrica* **88**(6), 2473–2502. 1, 4.9

Parker, Jonathan A (2001), 'The consumption risk of the stock market', *Brookings Papers on Economic Activity* **2001**(2), 279–333. 1, 4.9

Parker, Jonathan A and Annette Vissing-Jorgensen (2009), 'Who bears aggregate fluctuations and how?', *American Economic Review* **99**(2), 399–405. 4.9

Parker, Jonathan A., Annette Vissing-Jorgensen, Rebecca M. Blank and Erik Hurst (2010), 'The increase in income cyclicality of high-income households and its relation to the rise in top income shares [with comments and discussion]', pp. 1–70.
URL: *http://www.jstor.org/stable/41012844* 1

Ravenna, Federico and Carl E Walsh (2006), 'Optimal monetary policy with the cost channel', *Journal of Monetary Economics* **53**(2), 199–216. 14

Schularick, Moritz and Alan M Taylor (2012), 'Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008', *American Economic Review* **102**(2), 1029–61. 1, 5

Shapley, Lloyd and Martin Shubik (1977), 'Trade using one commodity as a means of payment', *Journal of Political Economy* **85**(5), 937–968. 1

Sims, Christopher A (1992), 'Interpreting the macroeconomic time series facts: The effects of monetary policy', *European Economic Review* **36**(5), 975–1000. 6

Toda, Alexis Akira and Kieran James Walsh (2020), 'The equity premium and the one percent', *The Review of Financial Studies* **33**(8), 3583–3623. 4, 1, 4.9

Tsomocos, Dimitrios P (2003), 'Equilibrium analysis, banking and financial instability', *Journal of Mathematical Economics* **39**(5-6), 619–655. 1

Vissing-Jorgensen, Annette (2002), 'Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures', National Bureau of Economic Research Cambridge, Mass., USA. 1

Wachter, Jessica A and Motohiro Yogo (2010), 'Why do household portfolio shares rise in wealth?', *The Review of Financial Studies* **23**(11), 3929–3965. 1

Woodford, Michael (2001), 'The taylor rule and optimal monetary policy', *The American Economic Review* **91**(2), 232–237.
URL: *http://www.jstor.org/stable/2677765* 4.6

Yellen, Janet L (2012), 'The economic outlook and monetary policy', *Board of Governors of the Federal Reserve System, Speech* . 7, 4.9.2

Appendix

A Proof of Lemma 1

Note that households' optimisation gives $\int_j c_j^h = \int_j \left(\frac{p_j}{P}\right)^{-\theta} C^h$, goods market clearing gives $c_j^o + c_j^l = c_j = y_j$ and hence $\int_j y_j = Y \int_j \left(\frac{p_j}{P}\right)^{-\theta}$ where Y is the aggregate bundle of goods produced. The aggregate goods market clearing is $c^o + c^l = Y$.

Substituting in the demand function $y_j = \left(\frac{p_j}{P}\right)^{-\theta} Y$ and $l_j = \frac{1}{A}\left(\frac{p_j}{P}\right)^{-\theta} Y$ into (11):

$$\pi_j = (p_j)^{1-\theta} P^\theta Y - \psi RD - (1+i)(w p_j^{-\theta} P^\theta \frac{Y}{A}). \tag{58}$$

We now break the firm's problem into one of minimising cost and then of setting the price. This will help us to illustrate the working capital channel in place.

Cost Minimisation

From 11, Firms solve

$$\begin{aligned} min_{l_j} &(1+i) w l_j \\ s.t. A l_j &\geq \left(\frac{p_j}{P}\right)^{-\theta} Y. \end{aligned} \tag{59}$$

The solution to this satisfies

$$\hat{mc}_j = \frac{(1+i)}{A}\tilde{w}, \tag{60}$$

where $\hat{mc}_j$ is the real marginal cost and $\tilde{w}$ the real wage. This is the expression for the working capital channel of Christiano et al. (2005). We show below that the presence of debt and household heterogeneity amplify the working capital channel.

Price Setting

Take the first-order condition for optimal profits with respect to price and substitute 60:

$$0 = (1-\theta)(p_j)^{-\theta} P^\theta Y - (1+i)(-\theta w (p_j)^{-1-\theta} P^\theta l_j), \tag{61}$$

$$0 = (1-\theta)A - (1+i)(-\theta w (p_j)^{-1}) \tag{62}$$

$$p_j = \sigma P \hat{mc}_j \tag{63}$$

Where $\sigma = \frac{\theta}{\theta - 1}$ is the markup, where a higher value of σ means greater market power. This shows that the real marginal cost is constant and equal to the inverse of σ in this example. Monetary policy has an effect through the wage rate affecting labour supply and the corresponding effect on the distribution of income across household types. Although a direct effect of an increase in the monetary policy rate is to increase marginal cost via the financial cost of working capital (as can be seen in 60), the increase in the monetary policy rate reduces the real wage. This leads to an indirect effect pushing down marginal cost. In the general equilibrium these two effects are cancelled out. As we shall shortly prove, even in this case when monetary policy has no effect on the real marginal cost, it is possible for prices to respond much less than output to monetary disturbances, simply owing to the income effect through corporate debt.

Aggregate prices

Use $p_j = P$, and substitute $l_j = L$,

$$0 = (1-\theta)Y + (1+i)(\theta \tilde{w} L), \tag{64}$$

equivalent to

$$\tilde{w} = \frac{A}{\sigma(1+i)}. \tag{65}$$

Labour Supply

The optimality conditions for the Lender Households' labour supply gives

$$\begin{aligned} \tilde{w} &= c_L && (66)\\ &= \tilde{w}L + \psi \frac{RD}{P} && (67)\\ \tilde{w}L &= \tilde{w} - \psi \frac{RD}{P}. && (68) \end{aligned}$$

The above equation shows that the presence of debt flattens the labour supply curve and supports the high effective labour supply elasticity emphasised in the cost channel of monetary policy literature.[17] This high elasticity may dampen the response of prices in the presence of monetary disturbances, even though output remains responsive. Given the price level, the elasticity of labour supplied ϵ_L is

$$\epsilon_L = \frac{\frac{\partial L}{\partial \tilde{w}}}{\frac{L}{\tilde{w}}} = \frac{\psi RD}{P\tilde{w}L} = \frac{\psi}{\tilde{b}} \frac{RD}{P}. \tag{69}$$

B Proof of Corollary 1

To derive the closed-form solution for the price level, we simply equate Aggregate Demand and Supply and obtain (70):

[17] See Barth and Ramey (2001) for the aggregate and industry-level evidence on the strength of monetary disturbances as a cost shock.

$$P = \frac{m + \psi i RD}{\frac{1}{\sigma}\frac{i}{1+i}A}. \tag{70}$$

To obtain the closed-form solution for allocation, we combine all flow of funds constraints of households (4) and (7) and of the firms (11). This leads to (71), showing that when the working capital liquidity that was injected in the morning exits the economy, the net interest payment of the working capital liquidity bi equates the aggregate monetary endowment m - an outstanding liability of the monetary-fiscal authority, which becomes monetary authority's seigniorage profits. In the dynamic model, nominal seigniorage profits are transferred to the next period.[18]

$$bi = m. \tag{71}$$

The total money lent by the monetary-fiscal authority (inside money) is given by $M = \frac{m}{i}$. This is because the seigniorage profits of the monetary-fiscal authority is m and the total money supply is $M + m$, the inside money plus outside money. Substituting $b = wL$ and (65) into (71), we obtain

$$L = \frac{m}{iP}\left(\frac{A}{\sigma(1+i)}\right)^{-1}. \tag{72}$$

Combine the above equation with (70) and $Y = AL$, we have the closed-form solution for output:

$$Y = \frac{A}{1 + \frac{i\psi RD}{m}}. \tag{73}$$

We obtain nominal profits from 12

$$\begin{aligned}
\Pi &= P\frac{A}{1 + \frac{i\psi RD}{m}} - (1+i)P(\frac{A}{\sigma(1+i)} - \psi\frac{RD}{P}) - \psi RD \\
&= \frac{1+i}{i}(m(\sigma - 1) - \psi i RD) + i\psi RD \\
&= \frac{1+i}{i}m(\sigma - 1) - \psi RD.
\end{aligned} \tag{74}$$

It follows that $\partial\Pi/\partial i = -i^{-2}m(\sigma - 1)$. Since $\sigma > 1$, $\partial\Pi/\partial i < 0$.

Moreover, given we have obtained the close form for the price level (70), the expression for real profits $\tilde{\Pi}$ is as follows:

$$\tilde{\Pi} = \frac{\frac{\sigma-1}{\sigma}mA - \psi RD(1 - \frac{1}{1+i})\frac{A}{\sigma}}{m + \psi RDi}. \tag{75}$$

As can be seen from the previous equation, real profits decrease when i increases.

[18]Fiscal policy is non-Ricardian. The monetary transfer is a government liability that is recovered through seigniorage profits at a unique price level (as in the Fiscal Theory of the Price Level). See Drèze and Polemarchakis (2000), Buiter (2002), and Dubey and Geanakoplos (2003) among others.

C Proof of Proposition 2

Let ϵ_{P_i} be the elasticity of the price level with respect to the monetary policy rate. We use (70) to derive ϵ_{P_i} to obtain the following:

$$\epsilon_{Pi} = \frac{\frac{\partial P}{\partial(1+i)}}{\frac{P}{1+i}} = \frac{i\psi RD - b}{(m + i\psi RD)}. \tag{76}$$

Therefore, $\epsilon_{Pi} < 0$ (the standard Taylor principle) holds *iff* $i\psi RD < b$,[19]. Otherwise, the Taylor principle is inverted and $\epsilon_{Pi} > 0$. That is, if debt is extremely high relative to working capital requirements, raising interest rates *raises* the rate of inflation.

The effectiveness of monetary policy can be seen when we rearrange (76) as

$$\epsilon_{Pi} = 1 - \frac{1+i}{i} \frac{1}{\frac{i\psi RD}{m} + 1}. \tag{77}$$

It is straightforward that ϵ_{Pi} is higher when D is larger. Hence when legacy debt is below the threshold that $i\psi RD < b$, then $\epsilon_{Pi} < 0$ and the absolute value of ϵ_{Pi} is smaller the higher debt is. When debt is above the threshold that $i\psi RD > b$, the opposite is true.

D Equilibrium Equations

Based on the analysis so far, the equilibrium of the dynamic economy is characterized by the following system of equations consisting optimality conditions, market-clearing conditions, and the monetary policy rule:

[19] In terms of primitives, the condition can be written as $i\psi RD < \frac{m}{i}$.

$$c^o + k' = \tilde{\pi}_W + \tilde{r}_k k + \tilde{\pi}, \tag{78}$$

$$\frac{1}{c^o} = \beta \mathbb{E} \frac{1}{c^{o'}} (\tilde{r}'_k), \tag{79}$$

$$\tilde{q}\tilde{d}' + \frac{\phi_d}{2}\tilde{q}(\tilde{d}' - \bar{d})^2 + c^l = \tilde{w}l + \frac{\tilde{d}}{1+\eta}, \tag{80}$$

$$\frac{\tilde{w}}{c^l} = \kappa l, \tag{81}$$

$$\frac{\tilde{q}}{c^l}(1 + \phi_d(\tilde{d}' - \bar{d})) = \beta \mathbb{E} \frac{1}{c^{l'}} \frac{1}{1+\eta'}, \tag{82}$$

$$y_W = A l^\alpha k^{1-\alpha}, \tag{83}$$

$$\tilde{w}l = \tilde{b}, \tag{84}$$

$$\tilde{\pi}_W + \tilde{r}_k k + \frac{1}{1+\eta}\tilde{d}_W + \tilde{w}l(1+i) = \tilde{p}_W y_W + \tilde{q}\tilde{d}'_W, \tag{85}$$

$$\tilde{w} = \frac{1}{1+i}(1-\alpha)\tilde{p}_W y/l, \tag{86}$$

$$\tilde{r}_k = \alpha \tilde{p}_W y/k \tag{87}$$

$$\tilde{q}\frac{1}{c^o} = \beta \mathbb{E} \frac{1}{c^{o'}} \frac{1}{1+\eta'}, \tag{88}$$

$$\tilde{\pi} = y - \nu y \tilde{p}_W, \tag{89}$$

$$y_W = \nu y, \tag{90}$$

$$\tilde{p}_W = \frac{1}{A}(\frac{(1+i)\tilde{w}}{1-\alpha})^{1-\alpha}(\frac{r_k}{\alpha})^\alpha, \tag{91}$$

$$(1+\eta) = \left[(1-\phi)(1+\eta^{\#})^{1-\theta} + \phi\right]^{\frac{1}{1-\theta}} \tag{92}$$

$$(1+\eta^{\#}) = \frac{\theta}{\theta-1}(1+\eta)\frac{x_1}{x_2} \tag{93}$$

$$x_1 = \frac{1}{c^o}\tilde{p}_W y + \phi\beta\mathbb{E}(1+\eta')^\theta x'_1 \tag{94}$$

$$x_2 = \frac{1}{c^o} y + \phi\beta\mathbb{E}(1+\eta')^{\theta-1} x'_2, \tag{95}$$

$$\nu = (1-\phi)(1+\eta^{\#})^{-\theta}(1+\eta)^\theta + (1+\eta)^\theta \phi \nu_{-1} \tag{96}$$

$$\frac{1+i_t}{1+\bar{i}} = (\frac{y_t}{\bar{y}})^{\rho_y}(\frac{1+i_{t-1}}{1+\bar{i}})^{\rho_i}(\frac{1+\eta_t}{1+\bar{\eta}})^{\rho_\eta} e^{\epsilon_i}, \tag{97}$$

$$\tilde{d}_W = \tilde{d} + \tilde{\mu}, \tag{98}$$

$$y = c^o + c^l + k' + \frac{\phi_d}{2}\tilde{q}(\tilde{d}' - \bar{d})^2. \tag{99}$$

E Steady State

The equations required to solve the zero inflation steady state are:

$$\bar{c}^o + \bar{k} = \bar{\pi}_W + \bar{r}_k\bar{k} + \bar{\pi}, \tag{100}$$

$$1 = \beta\bar{r}_k, \tag{101}$$

$$\bar{q}\bar{d} + \bar{c}^l = \bar{w}\bar{l} + \bar{d}, \tag{102}$$

$$\frac{\bar{w}}{\bar{c}^l} = \kappa\bar{l}, \tag{103}$$

$$\bar{y}_W = A\bar{k}^\alpha\bar{l}^{1-\alpha}, \tag{104}$$

$$\bar{w}\bar{l} = \bar{b}, \tag{105}$$

$$\bar{\pi}_W + \bar{r}_k\bar{k} + \bar{d}_W + \bar{w}\bar{l}(1+\bar{i}) = \bar{p}_W\bar{y}_W + \bar{q}\bar{d}_W, \tag{106}$$

$$\frac{\alpha}{1-\alpha}(1+\bar{i})\bar{w}\bar{l} = \bar{r}_k\bar{k}, \tag{107}$$

$$\bar{q} = \beta, \tag{108}$$

$$\bar{\pi} = \bar{y} - \nu\bar{y}\bar{p}_W, \tag{109}$$

$$\bar{y}_W = \nu\bar{y}, \tag{110}$$

$$\bar{p}_W = \frac{1}{A}(\frac{(1+i)\bar{w}}{1-\alpha})^{1-\alpha}(\frac{\bar{r}_k}{\alpha})^\alpha, \tag{111}$$

$$1 = \frac{\theta}{\theta-1}\frac{x_1}{x_2}, \tag{112}$$

$$\bar{x}_1 = \frac{\frac{1}{\bar{c}^o}\bar{p}_W\bar{y}}{1-\phi\beta}, \tag{113}$$

$$\bar{x}_2 = \frac{\frac{1}{\bar{c}^o}\bar{y}}{1-\phi\beta}, \tag{114}$$

$$\nu = 1 \tag{115}$$

$$\bar{d}_W = \bar{d} + \bar{\mu}, \tag{116}$$

$$\eta = 0 \tag{117}$$

$$\bar{y} = \bar{c}^o + \bar{c}^l + \bar{k}. \tag{118}$$

Aggregate demand at the steady state is $\bar{c}^o + \bar{k} + \bar{c}^l$. Substitute in households' and firms' flow of funds constraints into aggregate demand for output, with the market-clearing condition for final output $\bar{y} = \bar{c}^o + \bar{c}^l + \bar{k}$, we obtain the following:

$$\bar{y} = \bar{c}^o + \bar{k} + \bar{c}^l = -\bar{w}\bar{l}\bar{i} + \bar{y} + \bar{m}, \tag{119}$$

$$\bar{w}\bar{l} = \frac{\bar{m}}{\bar{i}}, \tag{120}$$

$$= \bar{M}. \tag{121}$$

From the marginal cost of the firm we get that $\bar{p}_W = \frac{1}{\sigma}$ in the steady state (simply by combining (112), (113), and (114)). Combine (111), (101), and $\bar{p}_W = \frac{1}{\sigma}$, we obtain the analytic expression for real wage at the steady state (122). We can see that contractionary monetary policy reduces real wages in the steady state.

$$\bar{w} = \frac{1}{1+\bar{i}} \left\{ \frac{A(\beta\alpha)^\alpha (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}}, \tag{122}$$

To obtain the closed-form solution for labour in the steady state, we combine (122) and (121):

$$\bar{l} = \frac{\bar{M}(1+\tilde{i})}{\left\{ \frac{A(\beta\alpha)^\alpha (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}}}. \tag{123}$$

Combine the lenders' first order condition for labour (103) and their budget constraint (102):

$$\bar{w} = \kappa \bar{l}(\bar{w}\bar{l} + \bar{d}(1-\bar{q})), \tag{124}$$

$$\bar{l} = \frac{\bar{w}}{\kappa(\frac{\bar{m}}{\bar{i}} + \bar{d}(1-\bar{q}))}. \tag{125}$$

Now we make use of the steady state equations to prove Proposition 3. We combine (107), (101), and (122):

$$\frac{\bar{k}}{\bar{l}} = \beta\frac{\alpha}{1-\alpha}(1+\bar{i})\bar{\bar{w}} \tag{126}$$

$$= \frac{\beta\alpha}{1-\alpha} \left\{ \frac{A(\beta\alpha)^\alpha (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}} \tag{127}$$

$$= \left\{ (\frac{\beta\alpha}{1-\alpha})^{1-\alpha} \frac{A(\beta\alpha)^\alpha (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}} \tag{128}$$

$$= \left\{ \frac{A\beta\alpha}{\sigma} \right\}^{\frac{1}{1-\alpha}} \tag{129}$$

and so the steady state level of output is

$$\bar{y} = A(\frac{\bar{k}}{\bar{l}})^\alpha \bar{l}$$

$$= A \left\{ \frac{A\beta\alpha}{\sigma} \right\}^{\frac{\alpha}{1-\alpha}} \bar{l}$$

$$= A \left\{ \frac{A\beta\alpha}{\sigma} \right\}^{\frac{\alpha}{1-\alpha}} \frac{\bar{M}(1+\bar{i})}{\left\{ \frac{A(\beta\alpha)^\alpha (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}}} \tag{130}$$

$$= \frac{\sigma}{1-\alpha}\bar{M}(1+\bar{i}) \tag{131}$$

$$= \frac{\sigma}{1-\alpha}\frac{\bar{m}}{1-\frac{1}{1+\bar{i}}}. \tag{132}$$

This is independent of household preferences. Keeping $\bar{i}$ unchanged, the ratio of real money balance to output is constant. We can now solve for the steady state real money balance. Note that (124) can be re-expressed as follows:

$$(\bar{w})^2 = \kappa\bar{M}(\bar{M} + \bar{d}(1-\bar{q})) \tag{133}$$

$$\frac{1}{(1+\bar{i})^2}\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{2}{1-\alpha}} = \kappa\bar{M}(\bar{M} + \bar{d}(1-\bar{q})) \tag{134}$$

$$\tag{135}$$

Suppose that $\bar{d} = 0$. In this case, $\bar{M} = \kappa^{-.5}\frac{1}{1+\bar{i}}\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{1}{1-\alpha}}$ and the nominal interest rate has an inverse relationship with the steady state level of money balance. As legacy debt $\bar{d}$ increases, the steady state level of money decreases. Furthermore, as the nominal interest rate increases, due to the presence of the legacy debt, money balance decreases to a less degree.

Note that when $\bar{d} = 0$, $\bar{y} = \frac{\sigma}{1-\alpha}\kappa^{-.5}\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{1}{1-\alpha}}$, so money is neutral in the steady state. When $\bar{d} \neq 0$, money is non-neutral in the steady state.

It is convenient to denote legacy debt in terms of leverage: $lev = \frac{\bar{d}}{\bar{y}}$.

$$\frac{1}{(1+\bar{i})^2}\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{2}{1-\alpha}} = \kappa\frac{\bar{M}}{1+\bar{i}}(\frac{\bar{M}}{1+\bar{i}} + \bar{y}lev(1-\bar{q})) \tag{136}$$

$$= \kappa\frac{\bar{M}}{1+\bar{i}}(\frac{\bar{M}}{1+\bar{i}} + \frac{\sigma}{1-\alpha}\bar{M}lev(1-\bar{q})) \tag{137}$$

$$= \kappa(\frac{\bar{M}}{1+\bar{i}})^2(1 + \frac{\sigma}{1-\alpha}(1+\bar{i})lev(1-\bar{q})) \tag{138}$$

$$\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{2}{1-\alpha}} = \kappa(\bar{M})^2(1 + \frac{\sigma}{1-\alpha}(1+\bar{i})lev(1-\beta)) \tag{139}$$

$$\bar{M} = \left\{\frac{\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{2}{1-\alpha}}}{\kappa(1 + \frac{\sigma}{1-\alpha}(1+\bar{i})lev(1-\beta))}\right\}^{\frac{1}{2}} \tag{140}$$

$$\bar{M} = \frac{\left\{\frac{A(\beta\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\sigma}\right\}^{\frac{1}{1-\alpha}}}{\left\{\kappa(1 + \frac{\sigma}{1-\alpha}(1+\bar{i})lev(1-\beta))\right\}^{\frac{1}{2}}}. \tag{141}$$

The expression above implies that as leverage increases, the quantity of real money balance decreases.

Given our parameterisation in Table 2, below Table 4 displays the model steady state values with quantity variables normalised by output.

Table 4: Steady state values

	$\bar{c}^0/\bar{y}$	$\bar{c}^l/\bar{y}$	$\bar{k}/\bar{y}$	$\bar{b}/\bar{y}$	$\bar{\pi}/\bar{y}$	$\bar{d}/\bar{y}$	$\bar{q}$	$\bar{r}_k$
BMK lev	0.178	0.558	0.264	0.587	0.175	3	0.990	1.01
High lev	0.168	0.568	0.264	0.587	0.165	4	0.990	1.01

BMK lev refers to the benchmark leverage of 75% (annual), or $\bar{b}/\bar{y} = 3$. *High lev refers to the high debt leverage of 100% (annual), or* $\bar{b}/\bar{y} = 4$.

F Dynamic Equations

$$
\begin{aligned}
\bar{c}^o\hat{c}^o + \bar{k}\hat{k}' &= \bar{\pi}_W\hat{\pi}_W + \bar{r}_k\bar{k}(\hat{r}_k + \hat{k}) + \bar{\pi}\hat{\pi}, && (142)\\
\hat{c}^{o'} - \hat{c}^o &= \hat{r}_k', && (143)\\
\bar{q}\bar{d}(\hat{q} + \hat{d}') + \phi_d\bar{q}\bar{d}\hat{d}' + \bar{c}^l\hat{c}^l &= \bar{w}\bar{l}(\hat{w} + \hat{l}) + \bar{d}(\hat{d} - (1 \hat{+} \eta)), && (144)\\
\hat{w} - \hat{c}^l &= \hat{l}, && (145)\\
\hat{q} - \hat{c}^l + \phi\bar{d}\hat{d}' &= -(\hat{c}^{l'} + (1 \hat{+} \eta)) && (146)\\
\hat{y}_W &= \hat{A} + \alpha\hat{k} + (1-\alpha)\hat{l}, && (147)\\
\hat{w} + \hat{l} &= \hat{b}, && (148)\\
\bar{\pi}_W\hat{\pi}_W + \bar{k}\bar{r}_k(\hat{r}_k + \hat{k}) + \bar{d}_W(\hat{d}_W - (1 \hat{+} \eta)) + \bar{w}\bar{l}(1 \bar{+} i)(\hat{w} + \hat{l} + (1 \hat{+} i)) & && \\
&= \bar{p}_W\bar{y}_W(\hat{p}_W + \hat{y}_W) + \bar{q}\bar{d}_W(\hat{q} + \hat{d}_W'), && (149)\\
\hat{w} &= -(1 \hat{+} i) + \hat{p}_W + \hat{y} - \hat{l}, && (150)\\
\hat{r}_k &= \hat{p}_W + \hat{y} - \hat{k} && (151)\\
\hat{q} - \hat{c}^o &= -\hat{c}^{o'} - (1 \hat{+} \eta'), && (152)\\
\bar{\pi}\hat{\pi} &= \bar{y}\hat{y} - \nu\bar{y}\bar{p}_W(\hat{y} + \hat{p}_W), && (153)\\
\hat{y}_W &= \hat{\nu} + \hat{y}, && (154)\\
\hat{p}_W &= -\hat{A} + (1-\alpha)(1 \hat{+} i) + (1-\alpha)\hat{w} + \alpha\hat{r}_k, && (155)\\
(1 \hat{+} \eta) &= (1-\phi)(1 \hat{+} \eta^{\#}) && (156)\\
(1 \hat{+} \eta^{\#}) &= (1 \hat{+} \eta) + \hat{x}_1 - \hat{x}_2 && (157)\\
\hat{x}_1 &= (1-\phi\beta)(-\hat{c}^o + \hat{p}_W + \hat{y}) + \theta\phi\beta(1 \hat{+} \eta') + \phi\beta\hat{x}_1, && (158)\\
\hat{x}_2 &= (1-\phi\beta)(-\hat{c}^o + \hat{y}) + \phi\beta((\theta-1)(1 \hat{+} \eta') + \hat{x}_2'), && (159)\\
\hat{\nu} &= 0 && (160)\\
(1 \hat{+} i) &= \rho_i(1 \hat{+} i_{-1}) + \rho_y\hat{y} + \rho_\eta(1 \hat{+} \eta) + \epsilon_i, && (161)\\
\bar{d}_W\hat{d}_W &= \bar{d}\tilde{d}, && (162)\\
\bar{y}\hat{y} &= \bar{c}^o\hat{c}^o + \bar{c}^l\hat{c}^l + \bar{k}\hat{k}' + \phi_d\bar{q}\bar{d}\hat{d}'. && (163)
\end{aligned}
$$

G Proof of Proposition 4

Recall the public balance equation (48). After substituting the working-capital constraint, and the constant purchases of intetemporal bonds, this becomes

$$\tilde{w}li + \bar{\mu}(\frac{1}{1+\eta} - \tilde{q}) = 0, \tag{164}$$

When we linearise, this becomes

$$\bar{\mu}(\bar{q}\hat{q} + (1 \hat{+} \eta)) = \bar{w}\bar{l}((1 \bar{+} i) - 1)(\hat{w} + \hat{l}) + \bar{w}\bar{l}(1 \bar{+} i)(1 \hat{+} i). \tag{165}$$

Simplifying

$$\hat{w} + \hat{l} = \frac{\bar{\mu}(\bar{q}\hat{q} + (1 \hat{+} \eta)) - \bar{w}\bar{l}(1 \bar{+} i)(1 \hat{+} i)}{\bar{w}\bar{l}((1 \bar{+} i) - 1)}, \tag{166}$$

where

$$\bar{w}\bar{l} = \bar{\mu}\frac{\bar{q} - 1}{\bar{i}}. \tag{167}$$

We can now solve for labour supply from (144) and (145)

$$\hat{l} = \frac{1}{2\bar{c}^l}\left\{\bar{q}\bar{d}(\hat{q} + \hat{d}') + \phi_d\bar{q}\bar{d}\hat{d}' + (\bar{c}^l - \bar{w}\bar{l})(\hat{w} + \hat{l}) - \bar{d}(\hat{d} - (1 \hat{+} \eta))\right\}. \tag{168}$$

With this in hand, we can obtain an expression for output 147:

$$\hat{y}_W = \hat{A} + \alpha\hat{k} + (1-\alpha)\frac{1}{2\bar{c}^l}\left\{\bar{q}\bar{d}(\hat{q} + \hat{d}') + \phi_d\bar{q}\bar{d}\hat{d}' + (\bar{c}^l - \bar{w}\bar{l})(\hat{w} + \hat{l}) - \bar{d}(\hat{d} - (1 \hat{+} \eta))\right\}, \tag{169}$$

Take 150, and for analytical convenience set $\phi_d = 0$,

$$\begin{aligned}
\hat{p}_W &= \hat{l} + \hat{w} + (1 \hat{+} i) - \hat{y} && (170)\\
&= \hat{l} + \hat{w} + (1 \hat{+} i) - \hat{A} - \alpha\hat{k}\\
&- (1-\alpha)\frac{1}{2\bar{c}^l}\left\{\bar{q}\bar{d}(\hat{q}+\hat{d}') + \phi_d\bar{q}\bar{d}\hat{d}' + (\bar{c}^l - \bar{w}\bar{l})(\hat{w}+\hat{l}) - \bar{d}(\hat{d} - (1 \hat{+} \eta))\right\} && (171)\\
&= (\hat{l}+\hat{w})\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}(\bar{c}^l - \bar{w}\bar{l})\right\} + (1 \hat{+} i) - \hat{A} - \alpha\hat{k}\\
&- (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}\left\{\bar{q}(\hat{q}+\hat{d}') - \hat{d}\right\} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}(1 \hat{+} \eta) && (172)\\
&= \frac{\bar{\mu}(\bar{q}\hat{q} + (1 \hat{+} \eta)) - \bar{w}\bar{l}(1 \bar{+} i)(1 \hat{+} i)}{\bar{w}\bar{l}((1 \bar{+} i) - 1)}\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}(\bar{c}^l - \bar{w}\bar{l})\right\} + (1 \hat{+} i) - \hat{A} - \alpha\hat{k}\\
&- (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}\left\{\bar{q}(\hat{q}+\hat{d}') - \hat{d}\right\} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}(1 \hat{+} \eta) && (173)\\
&= (1 \hat{+} \eta)\left\{\frac{\bar{\mu}\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\}}{\bar{w}\bar{l}((1 \bar{+} i) - 1)} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}\right\}\\
&+ \bar{q}\hat{q}\left\{\frac{\bar{\mu}\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\}}{\bar{w}\bar{l}((1 \bar{+} i) - 1)} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}\right\}\\
&+ (1 \hat{+} i)\left\{1 - (1 \bar{+} i)\frac{\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\}}{((1 \bar{+} i) - 1)}\right\} - \hat{A} - \alpha\hat{k} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l}\left\{\bar{q}\hat{d}' - \hat{d}\right\} && (174)
\end{aligned}$$

where $\bar{c}^l = \bar{w}\bar{l} + \bar{d}(1-\bar{q})$. Consider the coefficient in front of $(1 \hat{+} i)$

$$\left\{1 - (1 \bar{+} i)\frac{\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\}}{((1 \bar{+} i) - 1)}\right\} = \frac{-1}{((1 \bar{+} i) - 1)}\left\{1 - (1 \bar{+} i)(1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\} \quad (175)$$

As $(1 \bar{+} i)(1-\alpha)\frac{\bar{d}(1-\bar{q})}{2\bar{c}^l} < 1$ holds, it follows that higher steady state levels of legacy debt, $\bar{d}$, makes the coefficient of $(1 \hat{+} i)$ closer to 0 in absolute value.

Similarly we can simplify the expression in front of the inflation term, $(1 \hat{+} \eta)$, and bond price term $\bar{q}\hat{q}$,

$$\begin{aligned}
\frac{\bar{\mu}\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\}}{\bar{w}\bar{l}((1 \bar{+} i) - 1)} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l} &= \frac{\left\{1 - (1-\alpha)\frac{1}{2\bar{c}^l}\bar{d}(1-\bar{q})\right\}}{\bar{q}-1} - (1-\alpha)\frac{\bar{d}}{2\bar{c}^l} && (176)\\
&= -\frac{1}{1-\bar{q}}. && (177)
\end{aligned}$$

This allows us to obtain the following expression for the marginal cost

$$\hat{p}_W = -\frac{\widehat{(1+\eta)} + \bar{q}\hat{q}}{1-\bar{q}} - \frac{\widehat{(1+i)}}{(\overline{(1+i)}-1)}\left\{1 - \frac{\overline{(1+i)}(1-\alpha)\bar{d}(1-\bar{q})}{2(\bar{w}\bar{l}+\bar{d}(1-\bar{q}))}\right\} - \hat{A} - \alpha\hat{k} - \frac{(1-\alpha)\bar{d}\left\{\bar{q}\hat{d}' - \hat{d}\right\}}{2(\bar{w}\bar{l}+\bar{d}(1-\bar{q}))}. \tag{178}$$

To summarise, higher steady state legacy debt reduces the direct effect of interest rates on marginal cost and increases the sensitivity of changes in debt.