Subsidies to Innovation with Endogenous Uncertainty*

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Abstract

How should firms be incentivized to switch to new technologies when the information they possess about the profitability of such technologies is endogenous? We study a model of investment under uncertainty in which the return to investment depends on both unknown fundamentals and the investment decisions of other firms. We show that, when information is exogenous, inefficiencies in investment can be corrected with a simple (constant) subsidy to innovative firms, along with a subsidy that corrects for firms' market power. When, instead, firms must also be incentivized to collect information about the new technology, inefficiencies in both the acquisition of information and the subsequent investment decisions must be corrected with a Pigouvian policy that conditions subsidies on the aggregate investment in the new technology. Finally, we show how the insights extend to richer economies with both nominal and real rigidities in which firms make investment decisions and set prices under endogenous dispersed information.

Keywords: endogenous information, strategic complementarity/substitutability, efficiency, welfare

JEL classification: D62, D83, E50

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1 Introduction

When deciding whether to adopt new technologies such as a new operating system, or a new production process, firms typically face uncertainty about the returns to their investments. Such uncertainty is largely endogenous, as firms have the possibility to acquire information about relevant fundamentals affecting the profitability of the new technologies. Furthermore, such a profitability may depend on the investment decisions of other firms as the benefit from switching to new technologies often depends on whether such technologies eventually become widely used. The information that firms collect prior to investing may thus also help them predict the spillovers that come with their investments.

The combination of firms' market power with the externalities originating in firms' investment spillovers makes it unlikely that firms will acquire information efficiently and then use it in the best interest of society. In such contexts, how should policy be designed to alleviate the inefficiencies?

In this paper, we present a simple model that permits us to address the above question. The model features a continuum of firms with market power making investment decisions under uncertainty. Both the significance of the investment spillovers and the relative merits of the new technology vis-a-vis the old one are unknown to the firms at the time they make their decisions. Such uncertainty is however endogenous as firms can collect information about fundamentals affecting the differential returns to the new technology.

We start by considering a stylized version of the model in which the managers of the firms are risk neutral and prices are flexible, so that the only decisions that firms make under dispersed (endogenous) information is whether to adopt the new technology. We show that, if the information the firms possess were exogenous, then efficiency in investment decisions could be induced by combining familiar subsidies correcting for firms' market power with an additional constant subsidy to innovating firms appropriately designed to make them use their available information in a socially optimal way. When, instead, information is endogenous, such simple policies fail to induce the firms to collect information efficiently. We show that efficiency in both information acquisition and information usage can, however, be induced by providing the innovative firms with a subsidy that depends on the aggregate investment in the new technology. Such a subsidy operates as a Pigouvian correction realigning the private value of information to its social counterpart by inducing the firms to internalize the externality that their investment decisions impose on others. That Pigouvian taxes/subsidies can correct externalities under complete information is known. Our contribution is in showing that a specific version of such policies also creates the right incentives for information acquisition.

We then show that the above insights extend to richer economies with both nominal and real rigidities in which managers are risk averse and firms also set prices under (endogenous) dispersed information. In such richer economies, the above Pigouvian taxes must be paired with an appropriate monetary rule that induces firms to set prices that are invariant to the firms' private information. By appropriately conditioning the supply of money to the fundamentals, such a policy implements the same equilibrium allocations as in the flexible-price equilibrium by making the demand of each firm respond to the fundamentals, but with constant prices. That such a monetary policy removes inefficiencies due to price rigidity is known. Our contribution is in showing that such a rule, when paired with the Pigouvian policy described above, remains optimal also when there are complementarities in technology adoption and firms' dispersed information is endogenous.

We expect similar policies to induce efficiency in both the collection and the usage of information also in economies in which the relevant externalities originate in pollution, as when firms must be incentivized to switch to more environmentally friendly technologies but face uncertainty about the returns to the newer 'greener' technologies. Furthermore, while the model we consider is fairly stylized, it features many ingredients that seem relevant also for the analysis of optimal monetary and fiscal policy over the business cycle, an angle that we plan to explore in more detail in future work.

Plan. The rest of the paper is organized as follows. We wrap up the Introduction below with a concise description of the most relevant literature. Section 2 contains the model. Section 3 characterizes the efficient acquisition and usage of information. Section 4 characterizes the equilibrium acquisition and usage of information and discusses policy interventions implementing the efficient allocations as a decentralized equilibrium. Section 5 discusses how the key results extend to richer economies with risk-averse entrepreneurs and nominal price frictions. Section 6 offers a few concluding remarks. All proofs are either in the Appendix at the end of the document or in the Online Supplement.

Related literature.

The paper is related to various streams of the literature. The first one is the literature on investment under uncertainty (see the book by Dixit and Pindyck, 1994).

The second one is the literature on subsidies to innovation (see, for example, the recent work by Akcigit, Soler, Miguelez, Stantcheva, and Sterzi (2018), as well as Akcigit, Hanley, and Stantcheva (2021) and Akcigit, Grigsby, Nicholas, and Stantcheva (2022). Our contribution relative to the above two lines of work is in introducing endogenous dispersed information about both primitive fundamentals as well as the spillovers associated with the relevant investment decisions.

A third line of related work is the one on optimal monetary and fiscal policy over the business cycle (see, among others, Angeletos and La'O, 2021 and the references therein). The key contribution relative to this body of work is in introducing spillovers in investment decisions and in showing that, while simple policies induce efficiency in information usage when information is dispersed but exogenous, richer policies whereby subsidies to innovating firms have a Pigouvian flavor become necessary when firms must also be incentivized to acquire information efficiently.

2 The Model

The economy is populated by (i) a measure-1 continuum of monopolistically-competitive firms each producing a differentiated intermediate good, (ii) a competitive retail sector producing a final good using the intermediate goods as inputs, (iii) a measure-1 continuum of homogenous workers, and (iv) a benevolent planner.

Each firm is run by a single entrepreneur who must decide whether to retain an old technology or adopt a new one. Indexing firms by $i \in [0,1]$, we denote by $n_i = 1$ the decision by firm i to adopt the new technology and by $n_i = 0$ the decision to retain the old technology. Next, let

$$N = \int n_i di.$$

denote the aggregate investment in the new technology (that is, the total mass of firms adopting the new technology) and $l_i \in \mathbb{R}_+$ the amount of labor employed by firm i. The amount of the intermediate good produced by firm i is given by

$$y_i = \begin{cases} \gamma\Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 1 \\ \Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 0 \end{cases} \tag{1}$$

with $\gamma > 1$, $\beta \geq 0$, $\alpha \geq 0, \psi \leq 1$. The variable Θ proxies for various fundamentals that are unknown at the time the firms' investments decisions are made. The parameter γ scales the return differential between the two technologies, whereas the parameters α and β control for the returns to scale and the intensity of the investment spillovers, respectively. Finally, the parameter ψ captures the returns to scale in the use of labor. Note that the variable Θ influences both the output differential under the two technologies, and the magnitude of the investment spillover, that is, the effect of aggregate investment N on individual output.

The decision of whether or not to adopt the new technology is made under incomplete information about Θ (described below). After choosing which technology to operate, each entrepreneur learns Θ and N, and then chooses the price p_i for the intermediate good it produces. Finally, given Θ, N, and the realized demand for its intermediate good, firm i employs labor l_i on a competitive market to meet its demand. Labor is supplied by the continuum of measure-one workers.

Adopting the new technology costs the firm $k > 0$. Such a cost can be interpreted as the disutility the entrepreneur suffers from familiarizing herself with the new technology. More broadly, it can be interpreted as managerial effort. What matters for the purposes of our analysis is that such a cost is not mediated by a market aggregating the entrepreneurs' dispersed information about Θ.

The dependence of the production function on the aggregate investment in the new technology N captures the idea that each entrepreneur benefits from the adoption of the new technology by the other entrepreneurs. That such spillovers affect both those entrepreneurs adopting the new technology and those retaining the old one is not essential for the results but simplifies some of the formulas. When adopting the new technology requires some investment in human capital, the property that entrepreneurs retaining the old technology also benefit from a larger N reflects the idea that investments in human capital create 'know-how' that is also useful to those entrepreneurs retaining the old technology. More generally, the dependence of production on N captures a rich class of complementarities in production decisions that naturally arise when firms take advantage of the investments made by other firms (investment spillovers). Also note that the payoff differential

$$(\gamma - 1)\Theta\,(1 + \beta N)^{\alpha}\, l^{\psi}$$

between the two technologies is increasing in both N and the fundamental Θ.

The final good is produced by a competitive retail sector using the familiar CES technology

$$Y = \left(\int y_i^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}}. \tag{2}$$

The price of the final good is P and the profits of the competitive retail sector are given by

$$\Pi = PY - \int p_i y_i di,$$

where p_i is the price of the intermediate good produced monopolistically by firm i.

As anticipated above, the decisions of whether or not to adopt the new technology are made under incomplete information. We capture the (endogenous) uncertainty the firms face about Θ as follows. Let $\theta \equiv \log \Theta$. It is commonly believed that θ is drawn from a Normal distribution with mean 0 and precision π_θ. The realization of θ is unobserved by the entrepreneurs. Each entrepreneur i chooses the precision π_i^x of an additive private signal

$$x_i = \theta + \xi_i$$

about θ, with ξ_i drawn from a Normal distribution with mean zero and precision π_i^x, independently from θ, and independently across i. The cost of choosing information of precision π_i^x is equal to $\mathcal{I}(\pi_i^x)$, with $\mathcal{I}$ continuously differentiable and such that $\mathcal{I}'(\pi_i^x) > 0$ and $\mathcal{I}''(\pi_i^x) \geq 0$ for all $\pi_i^x > 0$. In order to guarantee interior solutions, we also assume that $\mathcal{I}'(0) = 0$. Such a cost can also be interpreted as disutility of effort.

Each entrepreneur maximizes her firm's profits, which are then used to finance the purchase of the final consumption good. Accordingly, each entrepreneur's objective function is given by

$$\Pi_i = \frac{p_i y_i - W l_i}{P} + T - k n_i - \mathcal{I}(\pi_i^x),$$

where W is the wage rate, and T is a transfer to the firm in terms of the final consumption good.

Each worker uses his labor income to purchase the final consumption good by maximizing

$$U = \frac{W}{P} l - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \Upsilon$$

where the function $l^{1+\varepsilon}/(1+\varepsilon)$ denotes the disutility of labor and Υ is a tax collected by the government.

Because labor is undifferentiated, in equilibrium, each worker provides the same amount of labor. The government's budget is balanced implying that $\int T_i di = \Upsilon$.

A benevolent planner maximizes the ex-ante sum of the firms' profits and of all workers' utilities

$$\mathcal{W} = \mathbb{E}\left[\int \Pi_i di + U\right]$$

Using the fact that the total labor demand must equal the total labor supply, the government's budget is balanced, all entrepreneurs choose the same precision of private information in equilibrium, firms' total revenues coincide with the total expenditure on the final good, and the total consumption of the final good C coincides with its production Y, we have that the government's objective can be expressed as

$$\mathcal{W} = \mathbb{E}\left[C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x).$$

In words, the planner maximizes aggregate consumption, net of the costs to upgrade the technology, the labor costs, and the information-acquisition costs.

The timing of events is the following.

1. Nature draws θ.
2. Each entrepreneur i chooses the precision π_i^x of her private information.
3. Each entrepreneur i receives a private signal x_i about θ.
4. Entrepreneurs simultaneously choose n_i.
5. After θ and N are publicly revealed, entrepreneurs simultaneously set prices p_i.
6. The competitive retail sector chooses how much of each intermediate good to purchase taking the prices of the intermediate goods and the price P of the final good as given.
7. Firms hire labor to meet the demand for their intermediate goods. That is, given the demand y_i for her intermediate good, entrepreneur i hires l_i units of labor to satisfy the demand for her intermediate product, taking N and θ as given.
8. A representative household comprising all workers and entrepreneurs chooses how much of the final good to buy taking the price of the final good P as given.

Because all firms set prices under complete information about θ, the money M in the economy used to finance the relevant transactions has only a nominal effect on prices and plays no other role. As a result, it is omitted.

The economy described above has two distinctive features: (a) the endogeneity of the firms' private information and (b) the investment spillovers.

3 Efficient allocation

The (decentralized) efficient allocation has two parts: the use of information (for given precision) and the acquisition of private information. Definition 1 and Proposition 1 define and characterize the efficient use of information, respectively. Definition 2 and Proposition 2, in turn, define and characterize the efficient acquisition of private information.[1]

Definition 1. Suppose that the precision of private information is equal to π^x for all i. The **efficient use of information** (for the economy with precision π^x) is given by a pair of functions $\hat{n}(x;\pi^x)$ and $\hat{l}(x,\theta;\pi^x)$ that jointly maximize the ex-ante expectation of $\mathcal{W}$ subject to the technology constraints (1) and (2).

Clearly, efficiency in the use of information requires that any two entrepreneurs investing in the new technology employ the same amount of undifferentiated labor, and likewise for any pair of entrepreneurs that decide to retain the old technology. Thus, let $\hat{l}_1(\theta;\pi^x)$ denote the efficient labor demand for the firms adopting the new technology and $\hat{l}_0(\theta;\pi^x)$ the corresponding demand for those retaining the old technology. Letting $\Phi(x|\theta;\pi^x)$ denote the cumulative distribution function of x given (θ,π^x), we have that the output produced by a firm with signal x in state θ under the efficient allocation is given by

$$\hat{y}(x,\theta;\pi^x)=\begin{cases}\gamma\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\hat{l}_1(\theta;\pi^x)^{\psi} \text{ if } \hat{n}(x;\pi^x)=1\\ \Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\hat{l}_0(\theta;\pi^x)^{\psi} \text{ if } \hat{n}(x;\pi^x)=0\end{cases},$$

where $\hat{N}(\theta;\pi^x)=\int_x \hat{n}(x;\pi^x)\,d\Phi(x|\theta;\pi^x)$ denotes the mass of firms adopting the new technology. Likewise, using Condition (2), we have that the amount of the final good produced in each state θ is given by $\hat{Y}(\theta;\pi^x)=\left(\int_x \hat{y}(x,\theta;\pi^x)^{\frac{v-1}{v}}d\Phi(x|\theta,\pi^x)\right)^{\frac{v}{v-1}}$. Finally, let $\hat{C}(\theta;\pi^x)=\hat{Y}(\theta;\pi^x)$ denote the amount of the final good consumed in each state θ under the efficient allocation.

The following proposition characterizes the efficient use of information.

Proposition 1. *Let $\varphi\equiv\frac{v-1}{v+\psi(1-v)}$. Suppose that $\gamma^{\varphi}\geq 1+\beta$ and $\alpha>\min\left\{0,1-\frac{1+\varepsilon+\varphi\psi}{\varphi(1+\varepsilon)}\right\}$. Then, for any precision of private information π^x, there exists a constant $\hat{x}(\pi^x)$ such that $\hat{n}(x;\pi^x)=\mathbb{I}(x\geq\hat{x}(\pi^x))$. The threshold $\hat{x}(\pi^x)$, along with the functions $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, and $\hat{l}_0(\theta;\pi^x)$ satisfy the following properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi}{1+\varepsilon-\psi}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{1+\varepsilon}{1+\varepsilon-\psi}}\times\right.$$
$$\left.\times\left(\frac{\gamma^{\varphi}-1}{\varphi\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)}+\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)}\right)\right|\hat{x}(\pi^x),\pi^x\right]=k, \quad (3)$$

[1]The definition of decentralized efficiency coincides with that in Vives (1988) and in Angeletos and Pavan (2007).

$$\hat{N}(\theta;\pi^x) = 1 - \Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x) = \psi^{\frac{1}{1+\varepsilon-\psi}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\right)^{\frac{1}{1+\varepsilon-\psi}} \times$$
$$\times\left(\gamma^{\varphi}\hat{N}(\theta;\pi^x) + 1 - \hat{N}(\theta;\pi^x)\right)^{\frac{1+\varepsilon-v\varepsilon}{(v-1)(1+\varepsilon-\psi)}}, \quad (4)$$

and

$$\hat{l}_1(\theta;\pi^x) = \gamma^{\varphi}\hat{l}_0(\theta;\pi^x). \quad (5)$$

Note that $\varphi > 0$. The role of the parameters' restrictions in the proposition is to guarantee that the social value of upgrading the technology (net of the disutility costs it involves) is increasing in the fundamental and in the mass of firms adopting the new technology. In fact, the parameter restrictions require that both γ, the return differential between the two technologies, and α, the returns to scale of the investment spillovers, are sufficiently large. Hence, under the efficient allocation, all entrepreneurs who receive a signal $x \geq \hat{x}(\pi^x)$ invest in the new technology, while all entrepreneurs who receive a signal $x < \hat{x}(\pi^x)$ do not invest. The threshold $\hat{x}(\pi^x)$ is given by the unique solution to (3). Given $\hat{x}(\pi^x)$, the labor demand for each firm retaining the old technology (alternatively, adopting the new technology) is given by (4) (alternatively, by (5)).

Definition 2. The **efficient acquisition of private information** is a precision π^{x*} that maximizes the ex-ante expectation of $\mathcal{W}$ when, for any π^x, the firms' decisions are determined by the functions $\hat{n}(x;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, and $\hat{l}_0(\theta;\pi^x)$ in Proposition 1.

Let

$$x^* \equiv \hat{x}(\pi^{x*}) \quad (6)$$

$$n^*(x) \equiv \hat{n}(x;\pi^{x*}) \quad (7)$$

$$l_1^*(\theta) \equiv \hat{l}_1(\theta;\pi^{x*}) \quad (8)$$

$$l_0^*(\theta) \equiv \hat{l}_0(\theta;\pi^{x*}) \quad (9)$$

$$N^*(\theta) \equiv \hat{N}(\theta;\pi^{x*}), \quad (10)$$

$$y_1^*(\theta) \equiv \gamma\Theta\left(1+\beta N^*(\theta)\right)^{\alpha} l_1^*(\theta)^{\psi} \quad (11)$$

$$y_0^*(\theta) \equiv \Theta\left(1+\beta N^*(\theta)\right)^{\alpha} l_0^*(\theta)^{\psi} \quad (12)$$

and

$$C^*(\theta) = Y^*(\theta) \equiv \left(y_1^*(\theta)^{\frac{v-1}{v}}N^*(\theta) + y_0^*(\theta)^{\frac{v-1}{v}}\left(1-N^*(\theta)\right)\right)^{\frac{v}{v-1}}. \quad (13)$$

The following proposition characterizes the efficient acquisition of private information.

Proposition 2. *The efficient acquisition of private information is implicitly defined by the solution to*

$$\mathbb{E}\left[C^{*}(\theta)\left(\frac{\alpha\beta}{1+\beta N^{*}(\theta)}+\frac{v}{v-1}\frac{(\gamma^{\varphi}-1)}{((\gamma^{\varphi}-1)N^{*}(\theta)+1)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]+$$
$$+\mathbb{E}\left[l_{0}^{*}(\theta)^{1+\varepsilon}\left[(\gamma^{\varphi}-1)N^{*}(\theta)+1\right]^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^{x}},$$

where x^{*}, $l_{0}^{*}(\theta)$, and $N^{*}(\theta)$ are given by (6), (9), and (10), respectively, whereas $\hat{N}(\theta;\pi^{x*}) = \int \hat{n}(x;\pi^{x*})\,d\Phi(x|\theta,\pi^{x*})$.

4 Equilibrium allocation

We start by characterizing the equilibrium allocations for given technology choices (n_i) and fundamentals θ. The assumption that the retail sector is competitive implies that, in equilibrium, $\Pi = 0$ and the price of the final good is equal to

$$P=\left(\int p_{i}^{1-v}di\right)^{\frac{1}{1-v}}, \tag{14}$$

while the demand for each intermediate good is given by

$$y_{i}=C\left(\frac{P}{p_{i}}\right)^{v}, \tag{15}$$

where C is the consumption of the final good and is determined by the interaction between the representative consumer and the competitive retail sector. Furthermore, because labor is undifferentiated and the labor market is competitive, the supply of labor is given by the familiar condition

$$\frac{W}{P}=l^{\varepsilon}$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal benefit of labor (the latter being equal to the real wage).

The demand for labor by each entrepreneur i when facing a demand for her intermediate good equal to y_i is given by

$$l_{1i}=\left(\frac{y_{i}}{\gamma\Theta(1+\beta N)^{\alpha}}\right)^{1/\psi}, \tag{16}$$

if entrepreneur i adopted the new technology, and by

$$l_{0i}=\left(\frac{y_{i}}{\Theta(1+\beta N)^{\alpha}}\right)^{1/\psi}, \tag{17}$$

if entrepreneur i retained the old technology. In both cases, the entrepreneur takes both N and θ as given. Market clearing then implies that

$$\frac{W}{P}=\left(\int l_{i}di\right)^{\varepsilon}.$$

Let $p_1(\theta;\pi^x)$ and $l_1(\theta;\pi^x)$ — alternatively, $p_0(\theta;\pi^x)$ and $l_0(\theta;\pi^x)$ — denote the equilibrium price and labor demand of each entrepreneur who invested in the new technology (alternatively, who retained the old technology), when the fundamental state is θ and the precision of private information is π^x. Note that the dependence of these functions on π^x originates in the fact that, in equilibrium, the fraction of firms adopting the new technology in state θ depends on π^x.

Definition 3. Given the fiscal policy $T(\cdot)$, a (decentralized) **equilibrium** is a precision π^x along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(\theta;\pi^x)$ and $p_1(\theta;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then chooses its technology following the rule $n(x;\pi^x)$ and its price following the rules $p_0(\theta;\pi^x)$ and $p_1(\theta;\pi^x)$, each entrepreneur i maximizes her payoff by doing the same.

Note that the above definition uses the property that, given θ and the technology choices, the prices of the intermediate goods along with the price and production level of the final good, uniquely pin down the demands y_i of the intermediate goods and hence the firms' labor demands. It also uses the property that the problem that each entrepreneur faces is the same irrespective of the variety of her product. As a result, the technology decisions of any two firms receiving the same signal x is the same. Likewise, any two firms with the same technology set the same price.

Given Definition 3, we define the optimal fiscal policy as follows.

Definition 4. The fiscal rule $T^*(\cdot)$ is **optimal** if it implements the efficient acquisition and usage of information as an equilibrium, that is if, given $T^*(\cdot)$, there exists an equilibrium such that the following hold: $\pi^x = \pi^{x*}$; $n(x;\pi^{x*}) = n^*(x)$; $p_0(\theta;\pi^{x*})$ and $p_1(\theta;\pi^{x*})$ are such that $l_0(\theta;\pi^{x*}) = l_0^*(\theta)$ and $l_1(\theta;\pi^{x*}) = l_1^*(\theta)$, where

$$l_0(\theta;\pi^{x*}) = \left(\frac{y_0(\theta;\pi^{x*})}{\Theta\left(1+\beta N^*(\theta)\right)^{\alpha}}\right)^{1/\psi} \quad \text{and} \quad l_1(\theta;\pi^{x*}) = \left(\frac{y_1(\theta;\pi^{x*})}{\gamma\Theta\left(1+\beta N^*(\theta)\right)^{\alpha}}\right)^{1/\psi}$$

are the labor demands by those firms that adopt the new and the old technology, respectively, and where $y_0(\theta;\pi^{x*})$ and $y_1(\theta;\pi^{x*})$ are the equilibrium demands when firms set prices $p_0(\theta;\pi^{x*})$ and $p_1(\theta;\pi^{x*})$, respectively.

To illustrate the role played by the endogeneity of the firms' private information, below we first characterize a simple fiscal policy implementing the efficient usage of information when the precision of private information π^x is exogenous. Next, we show that such a simple policy fails to induce the efficient acquisition of private information, but a certain amendment of such a policy guarantees efficiency in both the acquisition and the usage of information.

4.1 Exogenous Information

Let $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$ denote the price functions that induce demands for the intermediate goods that lead the firms to hire the efficient amount of labor $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$), respectively for those firms that adopted the new technology and those that retained the old one. Also let

$$\hat{P}(\theta;\pi_x) = \left(\hat{p}_1(\theta;\pi^x)^{1-v}\hat{N}(\theta;\pi^x) + \hat{p}_0(\theta;\pi^x)^{1-v}\left(1-\hat{N}(\theta;\pi^x)\right)\right)^{\frac{1}{1-v}} \tag{18}$$

denote the price of the final good when firms set prices according to $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$. Notice that $\hat{P}(\theta;\pi_x)$ is the price of the final good supporting an efficient allocation for given π^x.

We start by establishing the following result.

Lemma 1. *In any equilibrium implementing the efficient use of information, the price function for those firms adopting the new technology is given by*

$$\hat{p}_1(\theta;\pi^x) = \left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{v-1}}\gamma^{\frac{\varphi}{1-v}}\hat{P}(\theta;\pi_x),$$

whereas the price function for those firms retaining the old technology is given by

$$\hat{p}_0(\theta;\pi^x) = \left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{v-1}}\hat{P}(\theta;\pi_x),$$

with the function $\hat{P}(\theta;\pi_x)$ satisfying Condition (18).

Let $r_i = (p_i y_i)/P$ denote firm i's revenues in real terms (that is, in terms of the consumption of the final good) and denote by r the revenue of a generic firm in real terms.

Proposition 3. *Suppose that the conditions in Proposition 1 hold, and that the precision of private information is exogenously fixed at π^x. The following fiscal policy is optimal. In each state θ, the total transfer to each firm adopting the new technology is equal to*

$$\hat{T}_1(r) = \bar{s}_{\pi^x} + \frac{1}{v-1}r,$$

whereas the total transfer to each firm retaining the old technology is equal to

$$\hat{T}_0(r) = \frac{1}{v-1}r,$$

where the fixed subsidy $\bar{s}_{\pi^x}$ is given by

$$\bar{s}_{\pi^x} = \mathbb{E}\left[\hat{C}(\theta;\pi^x)\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)}\middle|\hat{x}(\pi^x),\pi^x\right].$$

When information is exogenous, efficiency in technology adoption and in production can be obtained with a simple fiscal policy that combines the familiar revenue subsidy $r/(1-v)$ designed to offset firms' market power, with an additional fixed subsidy $\bar{s}_{\pi^x}$ to the innovating firms. The familiar revenue subsidy makes each entrepreneur internalize the effect on consumer surplus of her pricing and hence her production decisions. The subsidy is inversely related to the elasticity of demand and proportional to the firm's revenue. Quite naturally, firms adopting the new technology receive a higher revenue subsidy as they produce more (one can in fact verify that $\hat{p}_1(\theta;\pi_x)\hat{y}_1(\theta;\pi_x) >$

$\hat{p}_0(\theta;\pi_x)\hat{y}_0(\theta;\pi_x))$. However, that the revenue subsidy is increasing does not guarantee that firms use their dispersed information efficiently and follow the rule $n(x;\pi^x)$. This is because firms do not internalize that, by adopting the new technology, they increase the output that other firms can produce, both when these other firms retain the old technology and when they adopt the new one. The additional subsidy $\bar{s}_{\pi^x}$ given to those firms adopting the new technology corrects for such an externality by guaranteeing that each firm with signal $\hat{x}(\pi^x)$ is indifferent between retaining the old technology and adopting the new one. When the economy satisfies the conditions in Proposition 1, efficiency requires that firms with a signal below $\hat{x}(\pi^x)$ retain the old technology whereas those with a signal above it adopt the new one. The same conditions guarantee that, when the fiscal rule takes the simple form in Proposition 3, following the efficient rule is optimal for each firm expecting the other firms to follow the same rule.

As we explain below, the term

$$\hat{C}(\theta;\pi^x)\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^x\right)}$$

in the formula for the additional subsidy $\bar{s}_{\pi^x}$ to the innovating firms coincides with the marginal externality created by the spillover. In other words, it represents the hypothetical increase in the production of the final good that would obtain if N increased by a small $\varepsilon > 0$ around the efficient level $\hat{N}\left(\theta;\pi^x\right)$, while holding all firms' technology and employment decisions fixed. Such a marginal effect naturally varies with the state θ. The subsidy $\bar{s}_{\pi^x}$ can then be interpreted as the marginal externality expected by each 'marginal innovator', i.e., by each firm with signal $\hat{x}(\pi^x)$.

4.2 Endogenous Information

We now turn to policies that induce firms not only to use information efficiently, but also to acquire the efficient amount of private information π^{x*}.[2]

Lemma 2. *Suppose that information is endogenous and that the economy satisfies the conditions in Proposition 1. Consider a fiscal rule that pays the firms retaining the old technology a transfer equal to*

$$T_0^*\left(r\right)=\frac{1}{\upsilon-1}r,$$

and the firms adopting the new technology a transfer equal to

$$T_1^*\left(\theta,r\right)=s(\theta)+\frac{1}{\upsilon-1}r.$$

Such a rule is optimal (that is, it induces efficiency in both information acquisition and information usage) only if the subsidy $s(\theta)$ to the innovating firms is non-decreasing and satisfies the following two conditions

$$\mathbb{E}\left[s\left(\theta\right)|x^*,\pi^{x*}\right]=\mathbb{E}\left[C^*\left(\theta\right)\frac{\alpha\beta}{1+\beta N^*\left(\theta\right)}|x^*,\pi^{x*}\right] \tag{19}$$

[2] Information acquisition and its welfare implications are the object of a broad literature. See, among others, Colombo *et al.* (2014), Myatt and Wallace (2012), Hellwig and Veldkamp (2009).

and

$$\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]=\mathbb{E}\left[C^{*}(\theta)\frac{\alpha\beta}{1+\beta N^{*}(\theta)}\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]. \tag{20}$$

Condition (19) is a restriction on the expected value of the subsidy $s(\theta)$, whereas Condition (20) is a restriction on the covariance between the subsidy $s(\theta)$ and the marginal effect of more precise private information on the aggregate investment $\hat{N}(\theta;\pi^{x*})$ in the new technology, under the efficient investment rule. Together with the condition that $s(\theta)$ be non-decreasing, the above two conditions guarantee that, in equilibrium, firms acquire information of precision π^{x*} and then use information efficiently, by adopting the new technology if and only if $x>x^{*}$ and by setting prices that induce them to employ labor $l_1^{*}(\theta)$ when they adopt the new technology and $l_0^{*}(\theta)$ when they retain the old one.

Note that the simple policy of Proposition 3, specialized to $\pi^{x}=\pi^{x*}$, i.e., with constant subsidy to the innovating firms equal to

$$\bar{s}_{\pi^{x*}}=\mathbb{E}\left[C^{*}(\theta)\left(\frac{\alpha\beta}{1+\beta N^{*}(\theta)}\right)\middle|\,x^{*},\pi^{x*}\right]$$

satisfies Condition (19) but not Condition (20), and hence fails to induce efficiency in information acquisition.

To see this, note that, when $s(\theta)=\bar{s}_{\pi^{x*}}$ for all θ,

$$\mathbb{E}\left[s(\theta)\tfrac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]=\mathbb{E}\left[C^{*}(\theta)\left(\tfrac{\alpha\beta}{1+\beta N^{*}(\theta)}\right)\middle|\,x^{*},\pi^{x*}\right]\mathbb{E}\left[\tfrac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]$$

$$\neq\mathbb{E}\left[C^{*}(\theta)\left(\tfrac{\alpha\beta}{1+\beta N^{*}(\theta)}\right)\tfrac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right].$$

In other words, a constant subsidy to the innovating firms equal to the externality expected by the marginal investor with signal x^{*} fails to induce the right covariance between the subsidy $s(\theta)$ and the (state-dependent) marginal effect of more precise information on aggregate investment $\partial\hat{N}(\theta;\pi^{x*})/\partial\pi^{x}$ necessary to realign the private benefit to information to its social counterpart.

The following result identifies a policy implementing efficiency in both the acquisition and usage of information.

Proposition 4. *Irrespective of whether the economy satisfies the conditions in Proposition 1, the fiscal policy of Lemma 2 with a state-contingent subsidy to the innovating firms equal to*

$$s(\theta)=C^{*}(\theta)\frac{\alpha\beta}{1+\beta N^{*}(\theta)} \tag{21}$$

induces efficiency in both the acquisition and usage of information.

As anticipated above, the state-contingent subsidy in (21) operates as a Pigouvian correction that induces each firm to internalize the effect of its technology adoption on the output produced by other firms, when such other firms acquire the efficient amount of information and then use it efficiently. To see this more formally, let Λ denote the distribution of firms' technology and employment decisions (n_i, l_i) in the cross-section of the population. Given θ and Λ, then let

$$\frac{\delta C(\theta, \Lambda)}{\delta N}$$

denote the marginal change in the production of the final good that obtains when, holding θ and Λ constant, one changes N in all firms' production functions, starting from $N = N_\Lambda$ where N_Λ is the aggregate investment in the new technology under the distribution Λ. Next, let $\Lambda^*(\theta, \pi^{x*})$ denote the cross-sectional distribution of firms' technology and employment decisions (n_i, l_i) that obtains at θ when the precision of all firms' private information is π^{x*} and all firms make efficient technology and employment decisions (that is, follow the policy $n^*(x)$ to determine their technology choice and the policies $l_0^*(\theta)$ and $l_1^*(\theta)$) to determine their employment decisions.

One can then verify that

$$C^*(\theta)\frac{\alpha\beta}{1+\beta N^*(\theta)} = \frac{\delta C(\theta, \Lambda^*(\theta, \pi^{x*}))}{\delta N}.$$

That is, the state-dependent subsidy in (21) coincides with the marginal change in the production and consumption of the final good that obtains as a result of a marginal change in N, evaluated at $N^*(\theta)$, holding all firms' technology choice and labor demand fixed at the efficient level. That such Pigouvian subsidies correct decisions under complete information is familiar. The contribution here is in showing that such a policy also induces efficiency in the acquisition and usage of information in economies in which information is dispersed and endogenous.

The Pigouvian policy of Proposition 4 is not the unique one implementing the efficient allocation. Other state-contingent policies also do the job. Furthermore, when information acquisition is verifiable, the planner can control separately the firms' incentives to use information efficiently (for example, through familiar revenue subsidies $T(r) = r/(1-v)$ that offset firms' market power) and the firms' incentives to acquire information efficiently (for example, through an additional subsidy that depends directly on firms' expenditure on information acquisition). However, one of the limitations of all the above policies (including the one in Proposition 4) is that they require that the planner knows the firms' information acquisition technology (formally, the type of signals that firms can acquire and their costs). Such a knowledge may not be available in many markets of interest. Importantly, efficiency in both information acquisition and usage can also be induced without such a knowledge by conditioning the transfer to the innovating firms directly on the cross-sectional distribution of firms' technology and employment decisions.

Proposition 5. *Suppose that the planner does not know the firms' information acquisition technology. Efficiency in information acquisition and usage can be induced through a policy that pays to the non-innovating firms a transfer equal to*

$$T_0^{\#}(r) = \frac{1}{v-1}r,$$

and to the innovating firms a transfer equal to

$$T_1^{\#}(\theta, r, \Lambda) = \frac{\delta C(\theta, \Lambda)}{\delta N} + \frac{1}{v-1} r,$$

where Λ *is the ex-post cross-sectional distribution of firms' technology and employment decisions.*

The result in Proposition 5 illustrates the power of the Pigouvian logic. When the planner announces that innovating firms will receive a subsidy equal to the ex-post externality that each firm's technology adoption exerts on the production of the final good it re-aligns firms' objective with total welfare non just at the interim stage but ex-post. Provided that firms themselves understand what efficiency entails, the planner can then leave it to the firms to figure out the efficient allocation, that is the choice of the information structure and of the subsequent technology-adoption rule that jointly maximize total welfare.

5 Richer Economies

We now extend the analysis to a richer family of economies in which agents are possibly risk averse and where firms may need to set prices under imperfect information about the underlying fundamentals. Such nominal rigidities introduce a role for monetary policy, in the spirit of Correia et al. (2008) and Angeletos and La'O (2020).

Consistently with the rest of the pertinent literature, we assume that each entrepreneur is a member of a representative household whose utility function is given by

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x) di,$$

where $R \geq 0$ is the coefficient of relative risk aversion in the consumption of the final good. This last assumption is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the entrepreneurs to fully insure against idiosyncratic consumption risk. The latter property in turn isolates the frictions (and associated inefficiencies) that originate in the endogenous dispersion of information at the time technology choices are made from the inefficiencies that may originate in the more familiar lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i = l$ for all i), which is a consequence of the assumption that labor is homogenous and exchanged in a competitive market. Being a member of the representative household, each entrepreneur maximizes her firm's *market valuation* taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each entrepreneur/firm maximizes

$$\mathbb{E}\left[C^{-R} \left(\frac{p_i y_i - W l_i}{P} + T \right) \middle| x_i, \pi_i^x \right] - k n_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consuming the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open. The household faces a '*cash-in-advance*' constraint according to which the maximal expenditure on the purchase of the final good cannot exceeds M, that is

$$PY \leq M.$$

The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The benevolent planner maximizes the ex-ante utility of the representative household

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi_i^x),$$

by means of a monetary policy rule $M(\theta)$ and a fiscal policy rule T, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in the baseline model, with the exception that prices are set under dispersed information about θ (that is, with each p_i based on x_i instead of θ) and that the supply of money is state-dependent and governed by the monetary rule $M(\theta)$. This richer economy is consistent with most of the assumptions typically made in the macroeconomic literature.

5.1 Efficient allocation

Definitions 1 and 2 apply also to this enriched economy. The following proposition extends the characterization of the efficient allocation in the previous section to the richer economy under consideration.

Proposition 6. *(1) Suppose that the conditions in Proposition 1 hold, and that* $0 \leq R \leq \bar{R}$, *with* $\bar{R} \equiv 1 - \frac{\varphi(1+\varepsilon)}{(1+\varepsilon)(1+\alpha\varphi)+\psi\varphi}$. *For any precision of private information* π^x, *there exists a constant* $\hat{x}(\pi^x)$ *such that efficiency in the use of information requires that* $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. *The threshold* $\hat{x}(\pi^x)$, *along with the functions* $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$ *satisfy the following properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \right.$$
$$\left. \times\left(\frac{\gamma^{\varphi}-1}{\varphi\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)} + \frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)}\right)\right|\hat{x}(\pi^x),\pi^x\right] = k, \quad (22)$$

$$\hat{N}(\theta;\pi^x) = 1 - \Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times\left(\gamma^{\varphi}\hat{N}(\theta;\pi^x)+1-\hat{N}(\theta;\pi^x)\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \quad (23)$$

and

$$\hat{l}_1(\theta;\pi^x) = \gamma^{\varphi}\hat{l}_0(\theta;\pi^x). \tag{24}$$

(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[C^*(\theta)^{1-R}\left(\frac{\alpha\beta}{1+\beta N^*(\theta)}+\frac{v}{v-1}\frac{(\gamma^{\varphi}-1)}{((\gamma^{\varphi}-1)N^*(\theta)+1)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[l_0^*(\theta)^{1+\varepsilon}\left[(\gamma^{\varphi}-1)N^*(\theta)+1\right]^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^x},$$

where x^*, $l_0^*(\theta)$, $C^*(\theta)$, and $N^*(\theta)$ are given by (6), (9), (13) and (10), respectively, whereas $\hat{N}(\theta;\pi^{x*}) = \int \hat{n}(x;\pi^{x*})\,d\Phi(x|\theta,\pi^{x*})$.

As in the baseline model, under the efficient allocation, all entrepreneurs who receive a signal $x \geq \hat{x}(\pi^x)$ invest in the new technology, while all entrepreneurs who receive a signal $x < \hat{x}(\pi^x)$ do not invest. The threshold $\hat{x}(\pi^x)$ is given by the unique solution to (22). Given $\hat{x}(\pi^x)$, the labor demand for each firm retaining the old technology (alternatively, adopting the new technology) is given by (23) (respectively, by (24)). The role of the various parameters' restrictions in the proposition is to guarantee that the social value of upgrading the technology is non-decreasing in the fundamental and in the mass of firms adopting the new technology, and that this is so even when the marginal utility of consuming the final good is decreasing. Notice that under the restriction $0 \leq R \leq \bar{R}$, the marginal utility of consumption does not decrease 'too quickly'. Indeed, were the restriction on R in the proposition not to be fulfilled, so that $R > \bar{R}$, a high value of θ would entail a low marginal utility of consumption. In this case, when taking into account the disutility of labor, it would be inefficient letting all firms that receive a signal $x \geq \hat{x}(\pi^x)$ to invest.

5.2 Equilibrium allocation

In the presence of sticky prices, firms' choices of which technology n_i to operate and of which price p_i to set are made under dispersed information about θ. Given these choices, firms then acquire labor l to meet their demands, after observing θ and the total investment N in the new technology.

In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by (14) and (15), respectively. Likewise, the labor demanded by each entrepreneur i when facing a demand y_i for her intermediate good continues to be given by (16) if the entrepreneur adopted the new technology, and by (17) if the entrepreneur retained the old technology. As explained above, in either case, the entrepreneur takes both N and θ as given when making her employment decisions. Because labor is undifferentiated and the labor market is competitive, the supply of labor is given by

$$\frac{W}{P}C^{-R} = l^{\varepsilon},$$

where the right-hand side continues to denote the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P starting from a level of consumption equal to C.

Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R} = \left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and labor demand, respectively, of each entrepreneur who invested in the new technology. The corresponding functions for the firms that retained the old technology are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$.[3]

Definition 5. Given the monetary rule $M(\theta)$ and the fiscal policy $T(\cdot)$, a **sticky-price equilibrium** is a precision π^x along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then chooses its technology according to the rule $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ or $p_1(x;\pi^x)$, each firm i maximizes its valuation by doing the same.

The following definition then extends the definition of optimal monetary and fiscal rules to the richer class of economies under consideration.

Definition 6. The monetary rule $M^*(\theta)$ along with the fiscal rule $T^*(\cdot)$ are **optimal** if they implement the efficient acquisition and usage of information as a sticky-price equilibrium: that is, if they induce all firms to choose the efficient precision of information π^{x*}, then follow the efficient rule $n^*(x)$ to determine whether or not to upgrade their technology, and finally set prices according to rules $p_0(x;\pi^{x*})$ and $p_1(x;\pi^{x*})$ such that, when all firms follow such rules, in each state θ, the equilibrium demands for the intermediate products $y_0(\theta;\pi^{x*})$ and $y_1(\theta;\pi^{x*})$ are invariant in x and the corresponding labor demands

$$l_0(\theta;\pi^{x*}) = \left(\frac{y_0(\theta;\pi^{x*})}{\Theta\left(1+\beta N^*(\theta)\right)^{\alpha}}\right)^{1/\psi} \text{ and } l_1(\theta;\pi^{x*}) = \left(\frac{y_1(\theta;\pi^{x*})}{\gamma\Theta\left(1+\beta N^*(\theta)\right)^{\alpha}}\right)^{1/\psi}$$

coincide withe the efficient levels, i.e., are such that $l_0(\theta;\pi^{x*}) = l_0^*(\theta)$ and $l_1(\theta;\pi^{x*}) = l_1^*(\theta)$.

Paralleling the analysis in the baseline model, for any precision of private information π^x (possibly different from π^{x*}), let $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$ denote a pair of pricing functions such that, when all firms acquire information of precision π^x, choose their technology according to $\hat{n}(x;\pi^x)$, and then set prices according to $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$, the induced employment decisions under the monetary rule $\hat{M}(\theta;\pi^x)$ coincides with the efficient levels $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$. Similarly, let $\hat{P}(\theta;\pi^x)$ denote the equilibrium price of the final good when all firms follow the aforementioned policies.

The following lemma characterizes the optimal monetary policy when the precision of private information is exogenous.

[3] As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ, the measure of firms N adopting the new technology depends on the precision π^x of the firms' signals.

Lemma 3. *Suppose that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\theta;\pi^x)$ that, together with some fiscal policy $\hat{T}$, implements the efficient use of information as a sticky-price equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}\left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^x)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}}, \tag{25}$$

where m is an arbitrary positive constant. The monetary rule $\hat{M}(\theta;\pi^x)$ induces all firms with the same technology to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\theta;\pi^x)$ implements the efficient allocations by inducing firms to disregard their information about the fundamentals and set prices based only on the adopted technology. That prices do not respond to the firms' information about θ is necessary to avoid allocative distortions in the induced employment and productions decisions. Relative prices must not vary with the firms' signals about θ when the latter signals are imprecise. The policy in Lemma 3 is designed so that, even if firms could condition their prices on θ, they would not find it optimal to do so. Under the proposed rule, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the amount of money supplied to the realized state θ in a way that replicates the same allocations sustained when the supply of money is constant and prices are flexible.

Lemma 3 along with arguments similar to those leading to Proposition 4 then permits us to establish the following result:

Proposition 7. *Irrespective of whether the economy satisfies the conditions in Proposition 6, the fiscal policy of Proposition 4 along with the monetary policy*

$$M^*(\theta) = ml_0^*(\theta)^{1+\varepsilon}\left((\gamma^{\varphi}-1)\,N^*(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}}$$

are optimal (i.e., implement the efficient acquisition and usage of information as a sticky-price equilibrium).

As we show in the online supplement, the monetary policy in the proposition (which belongs to the family in Lemma 3, specialized to $\pi^x = \pi^{x*}$) neutralizes the effects of price stickiness by replicating the same allocations as under flexible prices. When paired with the fiscal rule of Proposition 4, it guarantees that, if firms were constrained to acquire information of precision π^x, they would follow the efficient rule $n^*(x)$ to choose which technology to operate and then set prices that induce the efficient labor demands and hence the efficient production of the intermediate and final goods. This is accomplished through a fiscal policy that, in addition to offsetting the firms' market power with a familiar revenue subsidy $r/(1-v)$, it realigns the firms' private value of upgrading their technology to the social value through an additional subsidy

$$s(\theta) = C^*(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N^*(\theta)}$$

to the innovating firms that operates as a Pigouvian correction, by making each firm internalize the marginal effect of the investment in the new technology to the production of the final good, in each state θ. Once such a realignment is established, the value that firms assign to to their private information also coincides with the social value, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

6 Conclusions

We investigated firms' incentives to learn about the productivity of existing and new technologies when such technologies are affected by investment spillovers. We showed that efficiency in both information acquisition and information usage can be induced through a fiscal policy that, in addition to correcting for firms' market power, it provides those firms adopting the new technology a subsidy that makes firms internalize for the effects of the investment in the new technology on the production of intermediate and final goods.

The paper's main contribution is in showing that the power of Pigouvian corrections extends to economies with endogenous and dispersed information. The result may guide policy interventions not only in markets in which externalities originate in technological spillovers but also in pollution and/or inefficient investments in knowledge and other human capital investments.

In future work, it would be interesting to extend the analysis to richer economies in which firms, in addition to acquiring information about the profitability of existing and new technologies, also expand the set of available products over time and strategically time the replacement of existing products with new ones.

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Appendix

Proof of Propositions 1 and 2. The results follow directly from the proof of Proposition 6 below specialized to the case in which $R = 0$ — Proposition 6 establishes properties of efficient allocations for a broader class of economies in which entrepreneurs are weakly risk averse, i.e., $R \geq 0$.

Proof of Proposition 6. First, we prove part (1). Next, we prove part (2). The results below are for an economy in which the utility of the consumption of the final good is given by $C^{1-R}/(1-R)$. The baseline model in Section 2 corresponds to the case in which $R = 0$.

Part (1). We drop π^x from all expressions to ease the notation. The planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R} d\Omega(\theta) - k\int_\theta N(\theta) d\Omega(\theta) + \\ - \frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) + \\ - \int_\theta \mathcal{Q}(\theta)\left(N(\theta) - \int_x n(x)\Phi(x|\theta)\right) d\Omega(\theta),$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\mathcal{Q}(\theta)$ is the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}} \tag{A.1}$$

with

$$y_1(\theta) = \gamma\Theta(1+\beta N(\theta))^\alpha l_1(\theta)^\psi, \tag{A.2}$$

and

$$y_0(\theta) = \Theta(1+\beta N(\theta))^\alpha l_0(\theta)^\psi. \tag{A.3}$$

Using (A.1) and (A.2), the first-order condition with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R}\left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}} \left(\gamma\Theta(1+\beta N(\theta))^\alpha\right)^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}-1} \\ - \left(l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right)^\varepsilon = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta)), \tag{A.4}$$

and using (A.1) and (A.2), we have that the first order condition above reduces to

$$\psi C(\theta)^{\frac{1-vR}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta)L(\theta)^\varepsilon. \tag{A.5}$$

Following similar steps, the first order condition with respect to $l_0(\theta)$ boils down to

$$\psi C(\theta)^{\frac{1-vR}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta)L(\theta)^\varepsilon. \tag{A.6}$$

Using (A.2) and (A.3), the ratio between (A.5) and (A.6) can be written as

$$\gamma^{\frac{v-1}{v}}\left(\frac{l_1(\theta)}{l_0(\theta)}\right)^{\psi\frac{v-1}{v}} = \frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta) = \gamma^\varphi l_0(\theta). \tag{A.7}$$

Notice that (A.7) implies that, at the efficient allocation, the total labor demand (A.4) is equal to

$$L(\theta) = l_0(\theta)\left[(\gamma^{\varphi} - 1) N(\theta) + 1\right]. \tag{A.8}$$

Using (A.2) and (A.3), we can also write aggregate consumption as

$$C(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha} \left(\gamma^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}} N(\theta) + l_0(\theta)^{\psi\frac{v-1}{v}} (1 - N(\theta))\right)^{\frac{v}{v-1}}.$$

Using (A.7), we can rewrite the latter expression as

$$C(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha} l_0(\theta)^{\psi} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\frac{v}{v-1}}. \tag{A.9}$$

Next, use (A.9) and (A.7) to rewrite (A.6) as

$$\psi\left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{1-vR}{v}} l_0(\theta)^{\psi\frac{1-vR}{v}} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\frac{1-vR}{v-1}} \times$$
$$\times \left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}} l_0(\theta)^{\psi\frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon}.$$

Using (A.8), we can rewrite the latter condition as

$$\psi\left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\frac{1-vR}{v-1}} =$$
$$= l_0(\theta)^{1+\varepsilon} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\varepsilon}.$$

From the derivations above, we have that efficient labor demands are given by

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}} \left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \tag{A.10}$$

and by (A.7).

Note that $l_0(\theta) > 0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in l_0, for each θ.

Next, consider the derivative of the planner's problem with respect to N at θ. Ignoring that $N(\theta)$ must be restricted to be in $[0, 1]$, we have that

$$\mathcal{Q}(\theta) \equiv C(\theta)^{-R} \frac{dC(\theta)}{dN(\theta)} - k - L(\theta)^{\varepsilon} \left(l_1(\theta) - l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed and varying both the amounts that each firm produces for given technology choice and the proportion of firms investing into the new technology.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which – denoting by $\phi(x|\theta)$ the density of $\Phi(x|\theta)$ – is equal to

$$\Delta(x) \equiv \int_{\theta} \mathcal{Q}(\theta) \phi(x|\theta) \omega(\theta) d\theta.$$

Using the fact that $\phi(x|\theta)\,\omega(\theta) = f(\theta|x)\,g(x)$, where $f(\theta|x)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_\theta \mathcal{Q}(\theta) f(\theta|x)\, d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all entrepreneurs receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ invest, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$ do not invest.
Next, use (A.1) to observe that

$$C(\theta)^{-R}\tfrac{dC(\theta)}{dN(\theta)} = \tfrac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] +$$

$$+C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) + y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))\right],$$

and (A.2) and (A.3) to observe that

$$y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) + y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))$$

$$= \tfrac{\alpha\beta}{1+\beta N(\theta)}\left(y_1(\theta)^{\frac{v-1}{v}}N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)$$

$$= \tfrac{\alpha\beta}{1+\beta N(\theta)}C(\theta)^{\frac{v-1}{v}},$$

where the last equality uses (A.1).
Finally, use (A.5) and (A.6) to observe that

$$L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right) = \psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right).$$

We conclude that

$$\mathcal{Q}(\theta) = \left(\frac{v+\psi(1-v)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + C(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N(\theta)} - k. \tag{A.11}$$

Using (A.9), (A.2), (A.3), and (A.7), after some manipulations we have that

$$C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) =$$
$$= \left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right). \tag{A.12}$$

Using (A.9), we also have that

$$C(\theta)^{1-R} = \left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}.$$

We thus have that

$$\mathcal{Q}(\theta) = \left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)} \times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi[(\gamma^{\varphi}-1)N(\theta)+1]} + \frac{\alpha\beta}{1+\beta N(\theta)}\right) - k.$$

Next, recall that the optimal labor demand for the firms retaining the old technology is given by (A.10); replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\mathcal{Q}(\theta) = \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right) N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times \left(\frac{\gamma^{\varphi}-1}{\varphi} + \frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right) - k.$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. In turn, such a property implies that the first-best choice of N is either $N=0$ or $N=1$, for all θ. This property, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ.
Next, let

$$\bar{N}(\theta|\hat{x}) \equiv 1-\Phi(\hat{x}|\theta)$$

and

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right) \bar{N}(\theta|\hat{x})+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta \bar{N}(\theta|\hat{x})\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times \left(\frac{\gamma^{\varphi}-1}{\varphi} + \frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\bar{N}(\theta|\hat{x})+1\right)}{1+\beta \bar{N}(\theta|\hat{x})}\right) - k.$$

To highlight the dependence of the threshold $\hat{x}$ on the precision of private information, we re-introduce π^x in the arguments of the various functions, and we observe that, under the parameters' restriction in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x}(\pi^x);\pi^x)|\hat{x}(\pi^x)]$ is continuous in $\hat{x}(\pi^x)$ and is such that

$$\lim_{\hat{x}\to-\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x}(\pi^x);\pi^x)|\hat{x}(\pi^x)] < 0 < \lim_{\hat{x}\to+\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x}(\pi^x);\pi^x)|\hat{x}(\pi^x)].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x}(\pi^x);\pi^x)|\hat{x}(\pi^x)] = 0$ admits at least one solution. Furthermore, for any solution $\hat{x}(\pi^x)$ to the equation, the following is true: $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x}(\pi^x);\pi^x)|\hat{x}(\pi^x)] < 0$ for $x < \hat{x}(\pi^x)$ and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x}(\pi^x);\pi^x)|\hat{x}(\pi^x)] > 0$ for $x > \hat{x}(\pi^x)$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ that satisfies

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right) \hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta \hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \right.$$
$$\left. \times \left(\frac{\gamma^{\varphi}-1}{\varphi} + \frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)}{1+\beta \hat{N}(\theta;\pi^x)}\right) \right| \hat{x}(\pi^x)\right] = k,$$

when $\hat{N}(\theta;\pi^x) = 1-\Phi(\hat{x}(\pi^x)|\theta;\pi^x)$, and is such that the investment strategy $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition meet all the first-order conditions of the planner's problem.

Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}(x;\pi^x)$ is monotone), any solution of the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions (23) and (24) in the proposition and $\hat{n}(x;\pi^x) = \mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x] > 0)$, with

$$\hat{\mathcal{Q}}(\theta;\pi^x) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left((\gamma^\varphi - 1)\hat{N}(\theta;\pi^x) + 1 \right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))} - 1} \left(1 + \beta\hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times \left(\frac{\gamma^\varphi - 1}{\varphi} + \frac{\alpha\beta\left((\gamma^\varphi - 1)\hat{N}(\theta;\pi^x) + 1 \right)}{1 + \beta\hat{N}(\theta;\pi^x)} \right) - k$$

where $\hat{N}(\theta;\pi^x) = \int_\theta \hat{n}(x;\pi^x)\, d\Phi(x|\theta,\pi^x)$.

Part (2). For any precision of private information π^x, use Conditions (A.8) and (A.9) in Part (1) of the proof of Proposition 6, to observe that ex-ante welfare can be expressed as

$$\mathbb{E}[\mathcal{W}|\pi^x] =$$
$$= \frac{1}{1-R}\int_\theta \Theta^{1-R}\left(1 + \beta\hat{N}(\theta;\pi^x)\right)^{\alpha(1-R)} \hat{l}_0(\theta;\pi^x)^{\psi(1-R)} \left((\gamma^\varphi - 1)\hat{N}(\theta;\pi^x) + 1 \right)^{\frac{v}{v-1}(1-R)} d\Omega(\theta) +$$
$$- k\int_\theta \hat{N}(\theta;\pi^x)\, d\Omega(\theta) - \int_\theta \frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[(\gamma^\varphi - 1)\hat{N}(\theta;\pi^x) + 1 \right]^{1+\varepsilon} d\Omega(\theta) - \mathcal{I}(\pi^x).$$

Using the envelope theorem, the marginal effect of a variation in the precision of private information on welfare is given by

$$\frac{d\mathbb{E}[\mathcal{W}|\pi^x]}{d\pi^x} =$$
$$= \mathbb{E}\left[C^*(\theta)^{1-R} \left(\frac{\alpha\beta}{1+\beta N^*(\theta)} + \frac{v(\gamma^\varphi - 1)}{(v-1)((\gamma^\varphi - 1)N^*(\theta) + 1)} \right) \frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x} \right] +$$
$$- k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x} \right] + \mathbb{E}\left[l_0^*(\theta)^{1+\varepsilon} ((\gamma^\varphi - 1)N^*(\theta) + 1)^\varepsilon (\gamma^\varphi - 1) \frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x} \right] - \frac{d\mathcal{I}(\pi_x)}{d\pi_x}.$$

The result in part 2 follows from the above first-order condition. Q.E.D.

Proof of Lemma 1. From (1), in each state θ, given the measure $N(\theta;\pi^x)$ of firms that innovated, the output produced by each firm that adopted the new technology and that hires labor $l_1(\theta;\pi^x)$ is equal to

$$y_1(\theta;\pi^x) = \gamma\Theta\left(1 + \beta N(\theta;\pi^x)\right)^\alpha l_1(\theta;\pi^x)^\psi. \tag{A.13}$$

Likewise, the output produced by each firm that retained the old technology and that hires labor $l_0(\theta;\pi^x)$ is equal to

$$y_0(\theta;\pi^x) = \Theta\left(1 + \beta N(\theta;\pi^x)\right)^\alpha l_0(\theta;\pi^x)^\psi.$$

Using Conditions (2) and (A.7), in any equilibrium implementing the efficient allocation, we have that the amount of the final good produced in each state θ is equal to

$$\hat{Y}(\theta;\pi^x) = \left((y_1(\theta;\pi^x))^{\frac{v-1}{v}}\hat{N}(\theta;\pi^x) + (y_0(\theta;\pi^x))^{\frac{v-1}{v}}\left(1-\hat{N}(\theta;\pi^x)\right)\right)^{\frac{v}{v-1}} =$$
$$= \Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{v}{v-1}}\hat{l}_0(\theta;\pi^x)^{\psi}, \quad \text{(A.14)}$$

where we made use of the fact that $(1+\psi\varphi)\frac{v-1}{v} = \varphi$. Furthermore, market clearing implies that the demand of each firm given its price p, the price of the final good P, and the total production Y of the final good (and hence of its consumption, C) is equal to

$$y = Y\left(\frac{P}{p}\right)^{v}.$$

Hence, in any equilibrium implementing the efficient allocation, the demand of each firm in state θ must satisfy

$$\hat{y}_f(\theta;\pi^x) = \hat{Y}(\theta;\pi^x)\left(\frac{\hat{P}(\theta;\pi^x)}{\hat{p}_f(\theta;\pi^x)}\right)^{v}$$

both for $f=1$ (i.e., for innovating firms) and for $f=0$ (i.e., for firms retaining the old technology). Using (A.13) and (A.14) to substitute for $\hat{y}_1(\theta;\pi^x)$ and $\hat{Y}(\theta;\pi^x)$ into the above condition, we have that, for each firm that invested in the new technology it must be that

$$\gamma\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\hat{l}_1(\theta;\pi^x)^{\psi} =$$
$$= \Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{v}{v-1}}\hat{l}_0(\theta;\pi^x)^{\psi}\left(\frac{\hat{P}(\theta;\pi^x)}{\hat{p}_1(\theta;\pi^x)}\right)^{v}.$$

This means that the price that each such firm must set is given by

$$\hat{p}_1(\theta;\pi^x) = \frac{\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{v-1}}\hat{l}_0(\theta;\pi^x)^{\frac{\psi}{v}}}{\left(\gamma\hat{l}_1(\theta;\pi^x)^{\psi}\right)^{\frac{1}{v}}}\hat{P}(\theta;\pi^x).$$

Using (A.7) to express $\hat{l}_1(\theta;\pi^x)$ as a function of $\hat{l}_0(\theta;\pi^x)$ we then have that

$$\hat{p}_1(\theta;\pi^x) = \left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{v-1}}\gamma^{\frac{\varphi}{1-v}}\hat{P}(\theta;\pi^x).$$

Similar arguments imply that the price set by each firm that retained the old technology must satisfy

$$\hat{p}_0(\theta;\pi^x) = \left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{v-1}}\hat{P}(\theta;\pi^x).$$

Q.E.D.

Proof of Proposition 3. Because the precision of information is fixed, we drop it from the expressions below to ease the notation. When deciding which price to set, each firm conditions on θ.

Because the latter is fixed, we further simplify the notation by dropping θ from the arguments of the various functions when there is no risk of confusion.

Consider first the problem faced by a firm that has innovated. Each such firm chooses p_1 to maximize

$$\frac{p_1 y_1 - W l_1}{P} + T_1(r) \tag{A.15}$$

taking W and P as given, accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^{v}, \tag{A.16}$$

with C exogenous to the firm's problem, and accounting for the fact that, given y_1, the amount of labor that the firm needs to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta(1+\beta N)^{\alpha}}\right)^{\frac{1}{\psi}}. \tag{A.17}$$

The first-order condition with respect to p_1 is given by

$$(1-v)\,CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1(r)}{dr}\frac{d(p_1 y_1)}{dp_1} = 0. \tag{A.18}$$

Combining (A.16) with (A.17), we have that

$$l_1 = \left(\frac{CP^{v}}{p_1^{v}\gamma\Theta(1+\beta N)^{\alpha}}\right)^{1/\psi}, \tag{A.19}$$

from which we obtain that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}. \tag{A.20}$$

Using (A.16), we also have that

$$\frac{d(p_1 y_1)}{dp_1} = (1-v)\,CP^{v}p_1^{-v}. \tag{A.21}$$

Replacing these last formulas into the above first-order condition and using (A.16) to express y_1 as $y_1 = CP^{v}p_1^{-v}$, we obtain that

$$(1-v)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{\partial T_1(r)}{\partial r}\frac{(1-v)\,y_1}{P} = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\frac{1-v}{v}\frac{y_1 p_1}{P} + \frac{1}{\psi}\frac{W}{P}l_1 + \frac{1-v}{v}\frac{dT_1(r)}{dr}\frac{y_1 p_1}{P} = 0. \tag{A.22}$$

Next, suppose that all other firms follow policies that induce the efficient allocations. Hereafter, we use 'hats' to denote the efficient choices by such firms as well as the corresponding aggregate variables. Observe that market-clearing in the labor market requires that

$$\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon},$$

with $\hat{L}$ as defined in (A.8). Recall that, by virtue of Condition (A.5) (specialized to $R = 0$), efficiency requires that

$$-\psi \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon} \hat{l}_1 = 0.$$

Accordingly, from (A.22), we have that

$$\frac{1-v}{v} \frac{y_1 p_1}{\hat{P}} + \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} \frac{l_1}{\hat{l}_1} + \frac{1-v}{v} \frac{dT_1(r)}{dr} \frac{y_1 p_1}{\hat{P}} = 0. \tag{A.23}$$

From of (A.16) we obtain that

$$\hat{y}_1^{-\frac{1}{v}} = \hat{C}^{-\frac{1}{v}} \frac{\hat{p}_1}{\hat{P}},$$

so that the first-order condition (A.23) becomes

$$\frac{1-v}{v} \frac{y_1 p_1}{\hat{P}} + \frac{\hat{y}_1 \hat{p}_1}{\hat{P}} \frac{l_1}{\hat{l}_1} + \frac{1-v}{v} \frac{dT_1(r)}{dr} \frac{y_1 p_1}{\hat{P}} = 0.$$

Multiplying all the addenda in the last condition by $\hat{P}/(y_1 p_1)$, we obtain that

$$\frac{1-v}{v} + \frac{\hat{y}_1 \hat{p}_1}{y_1 p_1} \frac{l_1}{\hat{l}_1} + \frac{1-v}{v} \frac{dT_1(r)}{dr} = 0. \tag{A.24}$$

Notice that (A.19) allows us to express the ratio between the amount of labor that the firm needs to hire and the efficient one in terms of the ratio between the firm's own price and the efficient one

$$\frac{l_1}{\hat{l}_1} = \left(\frac{\hat{p}_1}{p_1}\right)^{\frac{v}{\psi}}.$$

The ratio between the efficient revenue and the one obtained by the firm choosing p_1 can also be expressed in terms of the ratio between the firm's price and the efficient one. In fact, using (A.16) we have that

$$\frac{\hat{y}_1 \hat{p}_1}{y_1 p_1} = \left(\frac{\hat{p}_1}{p_1}\right)^{1-v}. \tag{A.25}$$

The first-order condition (A.24) thus becomes

$$\frac{1-v}{v} + \left(\frac{\hat{p}_1}{p_1}\right)^{1-v+\frac{v}{\psi}} + \frac{1-v}{v} \frac{dT_1(r)}{dr} = 0.$$

For the fiscal rule T to implement the efficient allocation, it must be that the price choice $p_1 = \hat{p}_1$ solves the above first-order condition. This is the case if and only if

$$\frac{1}{v} = \frac{v-1}{v} \frac{dT_1(\hat{r})}{dr}.$$

We thus have that the above first-order condition is satisfied when the fiscal rule satisfies

$$T_1(r) = s + \frac{1}{v-1} r, \tag{A.26}$$

with s invariant in r. Furthermore, one can verify that, under the proposed fiscal rule, the payoff of each firm that adopted the new technology is quasi-concave in p_i which implies that the above first-order condition is also sufficient for the firm to optimally choose $p_1 = \hat{p}_1$. In fact, when all other firms follow policies that induce the efficient allocations, under the fiscal rule (A.26), the firm's objective (A.15) becomes

$$\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}} l_1 + \frac{s}{\hat{P}}.$$

Using (A.16) and (A.20), the derivative of the firm's objective with respect to p_1 is equal to

$$-v\frac{y_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}.$$

We then obtain that the second derivative of the firm's objective with respect to p_1 is equal to

$$\frac{1}{p_1}\left(v^2\frac{y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right),$$

which is negative when $p_1 = \hat{p}_1$ (in which case $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$, so that the first derivative cancels out). Because the firm's objective has a unique stationary point at $p_1 = \hat{p}_1$, the above result implies that the firm's objective is quasi-concave in p_1.

Applying similar arguments to those firms that retained the old technology, we have that a fiscal policy that provides a transfer equal to

$$T_0(r) = \frac{1}{v-1} r \tag{A.27}$$

to those firms retaining the old technology induces such firms to set a price equal to $\hat{p}_0$ in each state θ (equivalently, to hire the efficient labor $\hat{l}_0$).

Next, consider the firms' technology adoption. Since firms do not know θ when they choose their technology, we reintroduce θ in the notation to highlight the uncertainty that they face. When the fiscal rule T satisfies conditions (A.26) and (A.27), each firm anticipates that, if it innovates, in each state θ it will then set a price $\hat{p}_1(\theta)$, hire $\hat{l}_1(\theta)$ and produce $\hat{y}_1(\theta)$, whereas, if it retains the old technology, it will then set a price $\hat{p}_0(\theta)$, hire $\hat{l}_0(\theta)$ and produce $\hat{y}_0(\theta)$. As a result of these observations, each firm receiving a signal x finds it optimal to adopt the new technology if

$$\mathbb{E}\left[\hat{r}_1(\theta) - \frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{l}_1(\theta) + T_1(\theta, \hat{r}_1(\theta)) \middle| x\right] - k > \mathbb{E}\left[\hat{r}_0(\theta) - \frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{l}_0(\theta) + T_0(\hat{r}_1(\theta)) \middle| x\right]$$

and retain the old one if the above inequality is reversed.

Equivalently, each firm invests if

$$\mathbb{E}\left[\mathcal{R}(\theta)|x\right] > 0$$

where

$$\mathcal{R}(\theta) \equiv \hat{r}_1(\theta) - \hat{r}_0(\theta) - \frac{\hat{W}(\theta)}{\hat{P}(\theta)}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_1(\theta)) - k$$

and does not invest if the above inequality is reversed.

Recall that the Dixit and Stiglitz demand system implies that $\hat{p}_f(\theta) = \hat{P}(\theta)\,\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{-\frac{1}{v}}$ so that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}}$ for $f = 0, 1$. Also recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)} = \hat{L}(\theta)^{\varepsilon}. \tag{A.28}$$

Hence, $\mathcal{R}(\theta)$ can be rewritten as follows

$$\mathcal{R}(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_1(\theta)) - k,$$

which, using (A.5) and (A.6) (specialized to $R = 0$) becomes

$$\mathcal{R}(\theta) = (1-\psi)\,\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_1(\theta)) - k.$$

Recall again that (A.16) implies that

$$\hat{r}_f(\theta) \equiv \frac{\hat{p}_f(\theta)\,\hat{y}_f(\theta)}{\hat{P}(\theta)} = \hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}} \text{ for } f = 0, 1.$$

Hence, when the policy takes the form in (A.26) and (A.27), with $s(\theta)$ possibly depending on θ, we have that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_1(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\mathcal{R}(\theta)$ can be written as

$$\mathcal{R}(\theta) = \left(\frac{v + \psi(1-v)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + s(\theta) - k. \tag{A.29}$$

Now recall that efficiency requires that each entrepreneur invests if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$, and does not invest if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$, where, as shown in (A.11) – specialized to$R = 0$,

$$\mathcal{Q}(\theta) = \left(\tfrac{v+\psi(1-v)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)\tfrac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

We conclude that, for the proposed policy to induce efficiency in information usage, it suffices that, for all θ,

$$s(\theta) = \hat{C}(\theta)\frac{\alpha\beta}{1+\beta\hat{N}(\theta)}.$$

The above condition guarantees that, state by state, the net private return to innovation coincides with the social return, i.e., $\mathcal{R}(\theta) = \mathcal{Q}(\theta)$ for all θ. However, realigning the private value to the social value state by state is not necessary. It suffices that $\mathbb{E}[\mathcal{R}(\theta)|x] > 0$ whenever $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and $\mathbb{E}[\mathcal{R}(\theta)|x] < 0$ whenever $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$. When the economy satisfies the properties of Proposition 1, $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ has the single-crossing property, turning from negative to positive at $x = \hat{x}$. In this case, it suffices that $\mathbb{E}[\mathcal{R}(\theta)|\hat{x}] = 0$, and that $\mathbb{E}[\mathcal{R}(\theta)|x]$ has the single-crossing property, turning from negative to positive at $x = \hat{x}$. When the policy takes the form in (A.26) and (A.27), the above last two properties hold if $s(\theta)$ is non-decreasing in θ and satisfies

$$\mathbb{E}\left[s(\theta)|\,\hat{x}\right] = \mathbb{E}\left[\hat{C}(\theta)\frac{\alpha\beta}{1+\beta\hat{N}(\theta)}\right|\left.\hat{x}\right]. \tag{A.30}$$

To see this, use (A.12) and (A.10) (specialized to $R = 0$) to rewrite the first addendum in (A.29) as

$$\left(\frac{v+\psi(1-v)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) =$$

$$= \psi^{\frac{\psi}{1+\varepsilon-\psi}}\Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}}\left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right),$$

and note that this expression is increasing in N (for given θ) and increasing in θ (for given N). Hence, when the second addendum in (A.29), which is equal to $s(\theta)$, is also non-decreasing in θ, $\mathcal{R}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}[\mathcal{R}(\theta)|x]$ is non-decreasing in x. Because condition (A.30) implies that $\mathbb{E}[\mathcal{R}(\theta)|\hat{x}] = 0$, we then have that $\mathbb{E}[\mathcal{R}(\theta)|x] > 0$ for $x > \hat{x}$ and $\mathbb{E}[\mathcal{R}(\theta)|x] < 0$ for $x < \hat{x}$.

Given the properties discussed above, it is then clear that, when the economy satisfies the properties of Proposition 1, the simple policy of Proposition 3 implements the efficient use of information, and hence is optimal. Q.E.D.

Proof of Lemma 2. In the proof of Proposition 3, we already established that, when the economy satisfies the conditions of Proposition 1 and the policy takes the form in Lemma 2 with $s(\theta)$ non-decreasing and satisfying Condition (19) (specialized to $\pi^x = \pi^{x*}$), if the precision of information was exogenously fixed at $\pi^x = \pi^{x*}$, all firms would use the information efficiently. Here we establish that, when information is endogenous, for the firms to acquire information of precision π^{x*}, in addition to the above two properties, $s(\theta)$ must satisfy Condition (20) in the lemma.

To see this, suppose that all firms other than i acquire information of precision π^{x*} and consider firm i's problem. Under the policy T^*, in each state θ, the price that maximizes firm i's profit coincides with the one that induces the efficient allocation for precision π^{x*}, irrespective of firm i's choice of π_i^x. This price is equal to p_1^* if the firm adopted the new technology and p_0^* if the firm retained the old technology, where p_1^* and p_0^* are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, specialized to a precision of private information equal to π^{x*}.

Now let $W^*(\theta) \equiv \hat{W}(\theta; \pi^{x*})$ and

$$P^*(\theta) \equiv \left(p_1^{*1-v}N^*(\theta) + p_0^{*1-v}(1-N^*(\theta)\right)^{\frac{1}{1-v}}. \tag{A.31}$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal rule, so as to ease the exposition, we have that firm i's value function, for any choice π_i^x of its private information, is equal to

$$\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma; \pi_i^x),$$

where

$$\Pi_i(\varsigma;\pi_i^x) \equiv \mathbb{E}\left[r_1^*\bar{n}(\pi_i^x) + r_0^*\left(1-\bar{n}(\pi_i^x)\right)\right] - \mathbb{E}\left[\frac{W^*}{P^*}\left(l_1^*\bar{n}(\pi_i^x) + l_0^*\left(1-\bar{n}(\pi_i^x)\right)\right)\right] +$$
$$+\,\mathbb{E}\left[T_1^*\bar{n}(\pi_i^x) + T_0^*\left(1-\bar{n}(\pi_i^x)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x)\right] - \mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x) \equiv \tilde{n}(\theta;\zeta,\pi_i^x) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i's invests in the new technology at state θ when the precision of its private information is π_i^x and the firm uses the strategy $\varsigma : \mathbb{R} \to [0,1]$ to map its signal x_i into the probability of adopting the new technology.
Given that $r_f^* = C^{*\frac{1}{v}} y_f^{*\frac{v-1}{v}}$ for $f = \{0,1\}$ by (A.16), we have that

$$\Pi_i(\varsigma;\pi_i^x) =\mathbb{E}\left[C^{*\frac{1}{v}}\left(y_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x) + y_0^{*\frac{v-1}{v}}\left(1-\bar{n}(\pi_i^x)\right)\right)\right] - \mathbb{E}\left[\frac{W^*}{P^*}\left(l_1^*\bar{n}(\pi_i^x) + l_0^*\left(1-\bar{n}(\pi_i^x)\right)\right)\right] +$$
$$+\,\mathbb{E}\left[T_1^*\bar{n}(\pi_i^x) + T_0^*\left(1-\bar{n}(\pi_i^x)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x)\right] - \mathcal{I}(\pi_i^x),$$

which, using (11), (12) and (A.7), can be rewritten as

$$\Pi_i(\varsigma;\pi_i^x) =\mathbb{E}\left[C^{*\frac{1}{v}}\left(\Theta\left(1+\beta N^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x)+1\right) l_0^{*\psi\frac{v-1}{v}}\right] +$$
$$-\,\mathbb{E}\left[\frac{W^*}{P^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x)+1\right) l_0^*\right] +$$
$$+\,\mathbb{E}\left[T_1^*\bar{n}(\pi_i^x) + T_0^*\left(1-\bar{n}(\pi_i^x)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x)\right] - \mathcal{I}(\pi_i^x).$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} = \mathbb{E}\left[C^{*\frac{1}{v}}\left(\Theta\left(1+\beta N^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x)}{\partial\pi_i^x}\right) l_0^{*\psi\frac{v-1}{v}}\right] +$$
$$-\,\mathbb{E}\left[\frac{W^*}{P^*}\left(\left(\gamma^{\varphi}-1\right) l_0^*\frac{\partial\bar{n}(\pi_i^x)}{\partial\pi_i^x}\right)\right] +$$
$$+\,\mathbb{E}\left[\left(\frac{T_1^*-T_0^*}{P^*}\right)\frac{\partial\bar{n}(\pi_i^x)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}, \quad \text{(A.32)}$$

where

$$\frac{\partial\bar{n}(\pi_i^x)}{\partial\pi_i^x} = \frac{\partial\tilde{n}(\theta;\zeta,\pi_i^x)}{\partial\pi_i^x}$$

is the marginal effect of varying π_i^x on the probability of adopting the new technology at θ, holding fixed the rule ζ.
Next, recall again that, for $f = \{0,1\}$,

$$r_f^* \equiv \frac{p_f^* y_f^*}{P^*} = C^{*\frac{1}{v}} y_f^{*\frac{v-1}{v}}.$$

Using (11) and (12), we obtain that

$$r_1^* - r_0^* = C^{*\frac{1}{v}}\Theta^{\frac{v-1}{v}}\left(1+\beta N^*\right)^{\alpha\frac{v-1}{v}}\gamma^{\frac{v-1}{v}}\left(l_1^{*\psi\frac{v-1}{v}} - l_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (A.7) and the structure of the proposed fiscal policy, we have that

$$T_1^* - T_0^* = s + \frac{1}{v-1} C^{*\frac{1}{v}} \left(\Theta \left(1 + \beta N^*\right)^\alpha\right)^{\frac{v-1}{v}} \left(\gamma^\varphi - 1\right) l_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (A.32), we obtain that

$$\frac{\partial \Pi_i(\varsigma; \pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[C^{*\frac{1}{v}} \left(\Theta \left(1 + \beta N^*\right)^\alpha\right)^{\frac{v-1}{v}} \left(\gamma^\varphi - 1\right) \frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x} l_0^{*\psi\frac{v-1}{v}}\right] + \\ - \mathbb{E}\left[\frac{W^*}{P^*}\left(\left(\gamma^\varphi - 1\right) l_0^* \frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right)\right] + \mathbb{E}\left[s\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Now, recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $n_i(x) = n^*(x)$. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(n^*; \pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[C^{*\frac{1}{v}} \left(\Theta \left(1 + \beta N^*\right)^\alpha\right)^{\frac{v-1}{v}} \left(\gamma^\varphi - 1\right) \frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x} l_0^{*\psi\frac{v-1}{v}}\right] + \\ - \mathbb{E}\left[\frac{W^*}{P^*}\left(\left(\gamma^\varphi - 1\right) l_0^* \frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}\right)\right] + \mathbb{E}\left[s\frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Note that in writing the expression above, we use the fact that, when $n_i(x) = n^*(x)$, $\tilde{n}(\theta; n^*, \pi^{x*}) = \hat{N}(\theta; \pi^{x*})$ for any θ, which implies that

$$\frac{\partial \bar{n}(\pi^{x*})}{\partial \pi_i^x} = \frac{\partial \tilde{n}(\theta; n^*(\cdot), \pi_i^x)}{\partial \pi_i^x} = \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}.$$

For the proposed policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[C^*(\theta)^{\frac{1}{v}} \left(\Theta \left(1 + \beta N^*(\theta)\right)^\alpha\right)^{\frac{v-1}{v}} \left(\gamma^\varphi - 1\right) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x} l_0^*(\theta)^{\psi\frac{v-1}{v}}\right] + \\ - \mathbb{E}\left[\frac{W^*(\theta)}{P^*(\theta)}\left(\left(\gamma^\varphi - 1\right) l_0^*(\theta) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right)\right] + \\ + \mathbb{E}\left[s(\theta)\frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] = \frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}, \quad \text{(A.33)}$$

where we reintroduced the arguments of the various functions to ease the comparison with the corresponding condition defining the efficient acquisition of information.

Next, we use the fact that the equilibrium wage satisfies

$$\frac{W^*(\theta)}{P^*(\theta)} = L^*(\theta)^\varepsilon$$

and (A.7) to note that

$$\frac{W^*(\theta)}{P^*(\theta)} = \left(l_1^*(\theta) N^*(\theta) + l_0^*(\theta) \left(1 - N^*(\theta)\right)\right)^\varepsilon = l_0^*(\theta)^\varepsilon \left(\left(\gamma^\varphi - 1\right) N^*(\theta) + 1\right)^\varepsilon.$$

Hence, using the fact that $C^*(\theta)^{\frac{1}{v}} = C^*(\theta)C^*(\theta)^{\frac{1-v}{v}}$, along with Condition (A.9) (computed at π^{x*}), we have that

$$C^*(\theta)^{\frac{1}{v}} = C^*(\theta) \left(\Theta \left(1 + \beta N^*(\theta)\right)^{\alpha}\right)^{\frac{1-v}{v}} l_0^*(\theta)^{\psi \frac{1-v}{v}} \frac{1}{(\gamma^{\varphi} - 1) N^*(\theta) + 1}.$$

It follows that (A.33) is equivalent to

$$\begin{aligned}
&\mathbb{E}\left[C^*(\theta) \frac{v(\gamma^{\varphi} - 1)}{(v-1)((\gamma^{\varphi} - 1) N^*(\theta) + 1)} \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] + \\
&\qquad - \mathbb{E}\left[l_0^*(\theta)^{1+\varepsilon} \left((\gamma^{\varphi} - 1) N^*(\theta) + 1\right)^{\varepsilon} (\gamma^{\varphi} - 1) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] + \\
&\qquad\qquad + \mathbb{E}\left[s(\theta) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] = \frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(A.34)}
\end{aligned}$$

Finally, recall that the efficient precision of private information π^{x*} solves

$$\begin{aligned}
&\mathbb{E}\left[C^*(\theta) \left(\frac{\alpha\beta}{1 + \beta N^*(\theta)} + \frac{v(\gamma^{\varphi} - 1)}{(v-1)((\gamma^{\varphi} - 1) N^*(\theta) + 1)}\right) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] \\
&+ \mathbb{E}\left[l_0^*(\theta)^{1+\varepsilon} \left((\gamma^{\varphi} - 1) N^*(\theta) + 1\right)^{\varepsilon} (\gamma^{\varphi} - 1) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] = \frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \quad \text{(A.35)}
\end{aligned}$$

Comparing (A.34) with (A.35), we thus have that the policy in Lemma 2 induces the firms to acquire the efficient precision of private information only if, in addition to $s(\theta)$ being non-decreasing and satisfying Condition (19), it also satisfies the following condition

$$\mathbb{E}\left[s(\theta) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right] = \mathbb{E}\left[C^*(\theta) \left(\frac{\alpha\beta}{1 + \beta N^*(\theta)}\right) \frac{\partial \hat{N}(\theta; \pi^{x*})}{\partial \pi^x}\right],$$

which is equivalent to Condition (20) in the lemma. Q.E.D.

Proof of Proposition 4. In the proof of Proposition 3, we showed that, when the fiscal policy has the structure of Lemma 2, with the subsidy $s(\theta)$ satisfying Condition (21), in each state θ, each firm who expects all other firms to (a) acquire information of precision π^{x*} and follow the efficient investing strategy $n^*(x)$ and (b) use the policies $p_0^*(\theta) \equiv \hat{p}_0(\theta; \pi^{x*})$ and $p_1^*(\theta) \equiv \hat{p}_1(\theta; \pi^{x*})$ to set the prices that induce the efficient allocations, finds it optimal to follow the same price policies, irrespective of its choices of precision π^x. We also showed that, in each state θ, a firm with the above expectations, assigns a value $\mathcal{R}^*(\theta) = \mathcal{Q}^*(\theta)$ to upgrading its technology that coincides with the planner's value, where the superscript '*' is meant to highlight that these functions are those corresponding to the efficient precision of private information π^{x*}. These properties hold irrespective of whether or not the efficient investment strategy $n^*(x)$ is monotone or, equivalently, of whether or not the economy satisfies the conditions in Proposition 1. The same properties also imply that the value that the firm assigns to acquiring information of quality π^x coincides with the planner's value. Because the private cost of information acquisition also coincides with the social cost, the above results imply that acquiring information of precision π^{x*} and then using the information efficiently (both when it comes to choosing the technology and when it comes to setting the prices) is optimal for each firm expecting the others firms to do the same. We conclude that the fiscal policy in the proposition implements the efficient allocation (i.e., it induces the efficient acquisition and usage of information) irrespective of whether the economy satisfies the conditions in Proposition 1. Q.E.D.

Proof of Proposition 5. The result follows from arguments similar to those establishing Proposition 4. Under the proposed policy, a firm that expects all other firms to (a) acquire the efficient amount of private information (with the latter taking the form of a generic experiment mapping θ into a distribution over firms' hierarchies of beliefs over θ), (b) use the available information efficiently when it comes to the technology choice, and (c) finally set prices in each state θ so as to induce the efficient employment (and hence production) choices, has incentives to do the same. This is because each firm has enough knowledge about the economy to compute the efficient allocation. Once the latter is computed, the revenue subsidy $r/(1-v)$ guarantees that each firm, no matter its technology, after learning θ has the right incentives to set the price for its intermediate good at a level that induces the efficient demand for its product (and hence the efficient employment decisions). This property follows from the same arguments as in the proof of Proposition 3 where the result is established without using the specific properties of the firms' information structure. Furthermore, one can verify that the same arguments as in the proof of Proposition 3 imply that, when, in each state θ, the subsidy to the innovating firms takes the form of the marginal externality exerted by N on the production of the final good, holding all firms' information acquisition, technology adoption, and pricing rules fixed, then the marginal value that each firm assigns to upgrading its technology coincides with the planner's value in each state. The above properties in turn imply that the private value to information coincides with the social one and hence that all firms have the right incentives to acquire the efficient amount of information (and use it efficiently) when expecting the other firms to do the same. Q.E.D.

Proof of Lemma 3. We drop π^x from all formulas to ease the notation. Using (A.5) and (A.6), we have that efficiency requires that

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{Y}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$
$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{Y}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}.$$

The Dixit and Stiglitz demand system implies that $y_i = Y(P/p_i)^v$. Hence, the prices set by any two firms adopting the same technology coincide, so that they are independent from the signal x. Let $\hat{p}_1$ be the price set by the firms investing in the new technology and $\hat{p}_0$ that set by firms retaining the old technology. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{Y}(\theta)^{1-R}\left(\hat{P}(\theta)/\hat{p}_1\right)^{v-1}, \tag{A.36}$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{Y}(\theta)^{1-R}\left(\hat{P}(\theta)/\hat{p}_0\right)^{v-1}, \tag{A.37}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (A.7), implies that

$$\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0.$$

The price of the final good is then equal to

$$\hat{P}(\theta) = \left((\gamma^{\varphi}-1)\hat{N}(\theta)+1\right)^{\frac{1}{1-v}}\hat{p}_0. \tag{A.38}$$

Combining the cash-in-advance constraint $M = PY$ with (A.37) and (A.38), we then have that

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\hat{P}(\theta)^{v+R-2}\hat{p}_0^{1-v}$$

and

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\left((\gamma^{\varphi}-1)\hat{N}(\theta)+1\right)^{\frac{v+R-2}{1-v}}\hat{p}_0^{R-1},$$

respectively. Finally, using Eq. (A.8) for $\hat{L}(\theta)$, we obtain

$$\hat{M}(\theta)^{1-R} = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left((\gamma^{\varphi}-1)\hat{N}(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}}\hat{p}_0^{1-R}.$$

It is immediate to verify that the same conclusion can be obtained starting from (A.36). Because $\hat{p}_0^{1-R}$ can be taken to be arbitrary, the result in the lemma obtains by setting $m = \frac{1}{\psi}\hat{p}_0^{1-R}$. Q.E.D.

Proof of Proposition 7. The formal proof is in the online appendix. It shows that, under the assumed monetary and fiscal policy, firms find it optimal to set prices that depend only on the chosen technology and that, when combined with the state-varying money supply, induce the efficient employment and production decisions in each state θ. Furthermore, the value that firms assign to upgrading their technology coincides with the planner's value in each state θ. These properties in turn imply that the value that firms assign to their private information coincides with the efficient level, and hence that acquiring the efficient amount of private information and then using it efficiently are equilibrium strategies. Q.E.D.

Subsidies to Innovation with Endogenous Uncertainty

Supplementary Material

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Abstract

This document contains proofs omitted in the main text. All numbered items in this document contain the prefix 'S'. Any numbered reference without the prefix 'S' refers to an item in the main text. Please refer to the main text for notation and definitions. The notation and definitions here are the same as in the main text.

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Proof of Proposition 7. The proof is in two steps and establishes a slightly more general result than the corresponding one in the main text. Step 1 fixes the precision of information and identifies a condition on the fiscal policy T that guarantees that, when T is paired with the monetary policy of Lemma 3, firms have incentives to use information efficiently when the latter is exogenous and the economy satisfies the properties of Proposition 6 in the main text. Step 2 identifies an additional restriction on the fiscal policy that, when combined with the condition in Step 1, guarantees that, when the economy satisfies the properties of Proposition 6 in the main text, agents have also incentives to acquire the efficient amount of private information when information is endogenous. The analysis in Steps 1 and 2 also permits us to verify that, irrespective of whether or not the economy satisfies the properties of Proposition 6 in the main text, when the fiscal policy of Proposition 4 in the main text is paired with the monetary policy in Proposition 7 in the main text, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same, thus establishing the result in Proposition 7.

Step 1. We fix the precision of information π^x and drop it to ease the notation. We further simplify the notation by dropping θ from the arguments of the various functions whenever this is not confusing. Consider first the pricing decision of a firm that adopted the new technology. The firm sets p_1 to maximize

$$\mathbb{E}\left[C^{-R}\left(\frac{p_1 y_1 - W l_1}{P} + T_1(r_1)\right)\middle| x\right], \tag{S1}$$

taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^v \tag{S2}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta(1+\beta N)^\alpha}\right)^{\frac{1}{\psi}}. \tag{S3}$$

The first-order condition for the maximization of (S1) with respect to p_1 is given by

$$\mathbb{E}\left[C^{-R}\left((1-v)CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{dT_1(r_1)}{dr_1}\frac{dr_1}{dp_1}\right)\middle| x\right] = 0. \tag{S4}$$

Using the fact that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{S5}$$

$$\frac{d(p_1 y_1)}{dp_1} = (1-v)CP^v p_1^{-v}, \tag{S6}$$

and that $y_1 = CP^v p_1^{-v}$, we have that (S4) can be rewritten as

$$\mathbb{E}\left[C^{-R}\left((1-v)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{\partial T_1(r_1)}{\partial r_1}\frac{(1-v)y_1}{P}\right)\middle| x\right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}C^{-R}\frac{y_1p_1}{P}+\frac{1}{\psi}C^{-R}\frac{W}{P}l_1+\frac{1-v}{v}C^{-R}\frac{dT_1\left(r_1\right)}{dr_1}\frac{y_1p_1}{P}\right|x\bigg]=0. \tag{S7}$$

Now suppose that all other firms follow policies that induce the efficient allocations (meaning that they follow the rule $\hat{n}(x)$ to determine which technology to use and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the technology they adopted but not on their signal x, as in the proof of Lemma 3 in the main text). Hereafter, we add 'hats' to all relevant variables to highlight that these are computed under the efficient policies.

Observe that market clearing in the labor market requires that

$$\hat{C}^{-R}\frac{\hat{W}}{\hat{P}}=\hat{L}^{\varepsilon}, \tag{S8}$$

and recall that, as established in the Proof of Proposition 6 in the main text,

$$\hat{L}=\hat{l}_0\left[\left(\gamma^{\varphi}-1\right)\hat{N}+1\right]. \tag{S9}$$

Also recall that efficiency requires that

$$-\psi\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1=0. \tag{S10}$$

Accordingly, using Condition (S7), we have that for each firm adopting the new technology to find it optimal to set the price $\hat{p}_1$ that induces the efficient allocation, it must be that

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}C^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr_1}\hat{r}_1\right|x\bigg]=0, \tag{S11}$$

where $\hat{r}_1=\left(\hat{p}_1\hat{y}_1\right)/\hat{P}$. Using again (S2), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which permits us to rewrite Condition (S11) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr_1}\hat{r}_1\right|x\bigg]=0, \tag{S12}$$

or, equivalently, as

$$\mathbb{E}\left[\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr_1}\right)\right|x\bigg]=0. \tag{S13}$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr_1=1/(v-1)$, the firm's first-order condition is satisfied. Furthermore, one can verify that, under the proposed fiscal rule, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1=\hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r)=r/(v-1)+s=(p_1y_1)/(v-1)+s, \tag{S14}$$

where s may depend on θ but is invariant in r, the firm's objective (S1) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}l_1+s\left(\theta\right)\right)\right|x\bigg]. \tag{S15}$$

Using (S2) and (S5), the first derivative of the firm's objective with respect to p_1 is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\bigg| x\right], \tag{S16}$$

whereas the second derivative is equal to

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\bigg| x\right]. \tag{S17}$$

From the analysis above, when $p_1=\hat{p}_1$, in each state θ, $y_1=\hat{y}_1$ and $l_1=\hat{l}_1$. Furthermore, no matter x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1=\hat{p}_1$, is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\bigg| x\right]=0. \tag{S18}$$

Using (S18), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1=\hat{p}_1$, is negative. Because the firm's objective has a unique stationary point at $p_1=\hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1.

Applying similar arguments to the firms retaining the old technology, we have that a fiscal policy that pays each firm retaining the old technology a transfer $T_0(r)=r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' technology choice. Hereafter, we reintroduce θ in the notation. When the fiscal rule T has the structure in Lemma 2 in the main text, i.e.,

$$T_0(r)=\frac{1}{v-1}r \tag{S13}$$

and

$$T_1(\theta,r)=s(\theta)+\frac{1}{v-1}r, \tag{S14}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by innovating, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by retaining the old technology, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Hence, each firm receiving signal x finds it optimal to adopt the new technology if

$$\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]\geq 0,$$

where

$$\hat{\mathcal{R}}(\theta)\equiv\hat{C}(\theta)^{-R}\left(\hat{r}_1(\theta)-\hat{r}_0(\theta)-\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\left(\hat{l}_1(\theta)-\hat{l}_0(\theta)\right)+T_1(\theta,\hat{r}_1(\theta))-T_0(\hat{r}_0(\theta))\right)-k,$$

and it does not invest if the above inequality is reversed.

Recall from (S2) that the Dixit and Stiglitz demand system implies that $\hat{p}_f(\theta)=\hat{P}(\theta)\,\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f(\theta)=\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}}$ for $f=0,1$. Also recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{C}(\theta)^{-R}=\hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}\left(\theta\right)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + \\ + \hat{C}\left(\theta\right)^{-R}\left(T_1\left(\theta, \hat{r}_1(\theta)\right) - T_0\left(\hat{r}_0(\theta)\right)\right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition 6 in the main text)

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon}, \tag{S15}$$

and

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon}, \tag{S16}$$

we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) = \left(1-\psi\right)\hat{C}\left(\theta\right)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \\ + \hat{C}\left(\theta\right)^{-R}\left(T_1\left(\theta, \hat{r}_1(\theta)\right) - T_0\left(\hat{r}_0(\theta)\right)\right) - k.$$

Next, use (S2) to note that

$$\hat{r}_f = \hat{C}^{\frac{1}{v}}\hat{y}_f^{\frac{v-1}{v}},$$

$f = 0, 1$. It follows that

$$T_1\left(\theta, \hat{r}_1(\theta)\right) - T_0\left(\hat{r}_0(\theta)\right) = s(\theta) + \frac{1}{v-1}\hat{C}\left(\theta\right)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v+\psi\left(1-v\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}\left(\theta\right)^{-R}s(\theta) - k. \tag{S17}$$

Now recall from the proof of Proposition 6 in the main text that efficiency requires that each entrepreneur invests if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and does not invest if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v+\psi\left(1-v\right)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta\right)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient technology adoption rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ 0 whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition 6 in the main text,

$$\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) = \\ = \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{1-R}\hat{l}_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right). \tag{S18}$$

Furthermore,

$$\hat{l}_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}} \left(\Theta\left(1+\beta\hat{N}(\theta)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}. \tag{S19}$$

Using the last two expressions, the first addendum in (S17) can be rewritten as

$$\left(\frac{v+\psi(1-v)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right) =$$
$$= \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right).$$

When the economy satisfies the conditions in Proposition 6 in the main text, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum

$$\hat{C}(\theta)^{-R}s(\theta)$$

in (S17) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the conditions in Proposition 6 in the main text, a subsidy $s(\theta)$ to the innovating firms that satisfies (a) $\hat{C}(\theta)^{-R}s(\theta)$ non-decreasing in θ and (b)

$$\mathbb{E}\left[\hat{C}(\theta)^{-R}s(\theta)\middle|\hat{x}\right] = \mathbb{E}\left[\hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}(\theta)}\middle|\hat{x}\right]$$

guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$.
The analysis above also reveals that, when the fiscal policy takes the form in (S13) and (S14) with

$$s(\theta) = \hat{C}(\theta)\frac{\alpha\beta}{1+\beta\hat{N}(\theta)}$$

for all θ, and the monetary policy takes the form in Lemma 3 in the main text, then irrespective of whether or not the economy satisfies the conditions in Proposition 6 in the main text, each firm that expects all other firms to follow the efficient technology rule $\hat{n}(x)$, and then set prices according to $\hat{p}_0$ and $\hat{p}_1$ irrespective of its signal (thus inducing the efficient employment decisions), finds it optimal to do the same.

Step 2. We now show that, when the economy satisfies the conditions in Proposition 6 in the main text, for the fiscal policy in (S13) and (S14) to implement the efficient acquisition and usage of information when paired with the monetary policy

$$M^*(\theta) = ml_0^*(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}}$$

of Proposition 7 in the main text, the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) identified in Step 1, must also be such that

$$\mathbb{E}\left[C^*(\theta)^{-R}s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] = \mathbb{E}\left[C^*(\theta)^{1-R}\left(\frac{\alpha\beta}{1+\beta N^*(\theta)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and then follow the efficient technology and pricing rule. Then consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the assumed fiscal and monetary rule, the firm finds it optimal to set a price equal to p_1^* after adopting the new technology and equal to p_0^* after retaining the old one, where p_1^* and p_0^* are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.
Let $W^*(\theta) \equiv \hat{W}(\theta; \pi^{x*})$ and

$$P^*(\theta) \equiv \left(p_1^{*1-v} N^*(\theta) + p_0^{*1-v}(1 - N^*(\theta))\right)^{\frac{1}{1-v}}. \tag{S20}$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal rule, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to

$$\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma; \pi_i^x),$$

where

$$\Pi_i(\varsigma;\pi_i^x) \equiv \mathbb{E}\left[C^{*-R}\left(r_1^*\bar{n}(\pi_i^x) + r_0^*\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] - \mathbb{E}\left[\frac{C^{*-R}}{P^*} W^*\left(l_1^*\bar{n}(\pi_i^x) + l_0^*\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] +$$
$$+ \mathbb{E}\left[C^{*-R}\left(T_1\bar{n}(\pi_i^x) + T_0\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x)\right] - \mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x) = \tilde{n}(\theta; \zeta, \pi_i^x) \equiv \int \varsigma(x) d\Phi(x|\theta, \pi_i^x)$ denoting the probability that firm i's invests in the new technology at state θ when the precision of its information is π_i^x and the firm uses the strategy $\varsigma : \mathbb{R} \to [0,1]$ to map its signal x_i into the probability of adopting the new technology.
Using (S2), we have that $r_f^* = C^{*\frac{1}{v}} y_f^{*\frac{v-1}{v}}$ for $f = 0, 1$. Hence

$$\Pi_i(\varsigma;\pi_i^x) = \mathbb{E}\left[C^{*\frac{1-vR}{v}}\left(y_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x) + y_0^{*\frac{v-1}{v}}\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] - \mathbb{E}\left[C^{*-R}\frac{W^*}{P^*}\left(l_1^*\bar{n}(\pi_i^x) + l_0^*\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] +$$
$$+ \mathbb{E}\left[C^{*-R}\left(T_1\bar{n}(\pi_i^x) + T_0\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x)\right] - \mathcal{I}(\pi_i^x),$$

which, using

$$y_1^* \equiv \gamma\Theta\left(1 + \beta N^*\right)^\alpha l_1^{*\psi}, \tag{S21}$$

$$y_0^* \equiv \Theta\left(1 + \beta N^*\right)^\alpha l_1^{*\psi}, \tag{S22}$$

and

$$l_1^* = \gamma^\varphi l_0^*, \tag{S23}$$

can be rewritten as

$$\Pi_i(\varsigma;\pi_i^x) = \mathbb{E}\left[C^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta N^*\right)^\alpha\right)^{\frac{v-1}{v}}\left(\left(\gamma^\varphi - 1\right)\bar{n}(\pi_i^x) + 1\right) l_0^{*\psi\frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[C^{*-R}\frac{W^*}{P^*}\left(\left(\gamma^\varphi - 1\right)\bar{n}(\pi_i^x) + 1\right) l_0^*\right] +$$
$$+ \mathbb{E}\left[C^{*-R}\left(T_1\bar{n}(\pi_i^x) + T_0\left(1 - \bar{n}(\pi_i^x)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x)\right] - \mathcal{I}(\pi_i^x).$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \mathbb{E}\left[C^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta N^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right)l_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\frac{C^{*-R}}{P^*}W^*\left(\left(\gamma^{\varphi}-1\right)l_0^*\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right)\right]+$$
$$+\mathbb{E}\left[C^{*-R}\left(\frac{T_1-T_0}{P^*}\right)\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}, \quad \text{(S24)}$$

where

$$\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x} = \frac{\partial \tilde{n}(\theta;\zeta,\pi_i^x)}{\partial \pi_i^x}$$

is the marginal effect of varying π_i^x on the probability that the firm adopts the new technology at θ, holding fixed the rule ζ.

Next, recall again that, for $f=0,1$,

$$r_f^* \equiv \frac{p_f^* y_f^*}{P^*} = C^{*\frac{1}{v}} y_f^{*\frac{v-1}{v}}.$$

Using (S21) and (S22), we have that

$$r_1^* - r_0^* = C^{*\frac{1}{v}}\Theta^{\frac{v-1}{v}}\left(1+\beta N^*\right)^{\alpha\frac{v-1}{v}}\gamma^{\frac{v-1}{v}}\left(l_1^{*\psi\frac{v-1}{v}} - l_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (S23) and the structure of the assumed fiscal policy, we have that

$$T_1 - T_0 = s + \frac{1}{v-1}C^{*\frac{1}{v}}\left(\Theta\left(1+\beta N^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)l_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (S24), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[C^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta N^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}l_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\frac{C^{*-R}}{P^*}W^*\left(\left(\gamma^{\varphi}-1\right)l_0^*\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right)\right]+\mathbb{E}\left[C^{*-R}s\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Now, recall that, when $\pi_i^x = \pi^{x*}$, the optimal technology rule is $n^*(x)$. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(n^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[C^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta N^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}l_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\frac{C^{*-R}}{P^*}W^*\left(\left(\gamma^{\varphi}-1\right)l_0^*\frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}\right)\right]+\mathbb{E}\left[C^{*-R}s\frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}(\pi^{x*})}{\partial \pi^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Note that, in writing the expression above, we used the fact that, when $\varsigma = n^*$, $\tilde{n}(\theta;n^*,\pi^{x*}) = \hat{N}(\theta;\pi^{x*})$ for any θ, which implies that

$$\frac{\partial \bar{n}(\pi^{x*})}{\partial \pi_i^x} = \frac{\partial \tilde{n}(\theta;n^*,\pi_i^x)}{\partial \pi_i^x} = \frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}.$$

For the fiscal rule to induce efficiency in information acquisition (when paired with the monetary rule in Proposition 7 in the main text), it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[C^*(\theta)^{\frac{1-vR}{v}}\left(\Theta\left(1+\beta N^*(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}l_0^*(\theta)^{\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\frac{C^*(\theta)^{-R}}{P^*(\theta)}W^*(\theta)\left(\left(\gamma^{\varphi}-1\right)l_0^*(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right)\right]+$$
$$+\mathbb{E}\left[C^*(\theta)^{-R}s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x},\quad\text{(S25)}$$

where we reintroduced the arguments of the various functions to ease the comparison with the corresponding condition defining the efficient acquisition of information.
Next, use (S8) and (S23) to note that

$$\frac{C^*(\theta)^{-R}}{P^*(\theta)}W^*(\theta)=\left(l_1^*(\theta)N^*(\theta)+l_0^*(\theta)\left(1-N^*(\theta)\right)\right)^{\varepsilon}=l_0^*(\theta)^{\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*\left(\theta\right)+1\right)^{\varepsilon}.$$

Hence, using the fact that $C^*(\theta)^{\frac{1-vR}{v}}=C^*(\theta)^{1-R}C^*(\theta)^{\frac{1-v}{v}}$, along with the fact that, as shown in the proof of Proposition 6 in the main text,

$$C^*(\theta)=\Theta\left(1+\beta N^*\left(\theta\right)\right)^{\alpha}l_0^*(\theta)^{\psi}\left(\left(\gamma^{\varphi}-1\right)N^*\left(\theta\right)+1\right)^{\frac{v}{v-1}},\quad\text{(S26)}$$

we have that

$$C^*(\theta)^{\frac{1-vR}{v}}=C^*(\theta)^{1-R}\left(\Theta\left(1+\beta N^*(\theta)\right)^{\alpha}\right)^{\frac{1-v}{v}}l_0^*(\theta)^{\psi\frac{1-v}{v}}\frac{1}{\left(\gamma^{\varphi}-1\right)N^*(\theta)+1}.$$

It follows that (S25) is equivalent to

$$\mathbb{E}\left[C^*(\theta)^{1-R}\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)N^*\left(\theta\right)+1\right)}\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]+$$
$$-\mathbb{E}\left[l_0^*(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*\left(\theta\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[C^*(\theta)^{-R}s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}.\quad\text{(S27)}$$

Finally, recall that the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[C^*\left(\theta\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta N^*\left(\theta\right)}+\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)N^*\left(\theta\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]$$
$$+\mathbb{E}\left[l_0^*(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*\left(\theta\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}.\quad\text{(S28)}$$

Comparing (S27) with (S28), we thus have that, for the fiscal rule T to implement the efficient acquisition and usage of information (when paired with the monetary rule in Proposition 7, which, by virtue of Lemma 3 in the main text is the only monetary rule that can induce efficiency in both information usage and information acquisition), the subsidy $s(\theta)$ to the innovating firms must satisfy the following condition

$$\mathbb{E}\left[C^*(\theta)^{-R}s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]=\mathbb{E}\left[C^*(\theta)^{1-R}\left(\frac{\alpha\beta}{1+\beta N^*(\theta)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right].$$

Finally, note that, independently of whether the economy satisfies the conditions in Proposition 6 in the main text, when the subsidy to the innovating firms is equal to

$$s(\theta)=C^*(\theta)\left(\frac{\alpha\beta}{1+\beta N^*(\theta)}\right)$$

in each state, then, as shown in Step 1, the private value $\mathcal{R}$ that each firm assigns to adopting the new technology coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting other firms to do the same, which establishes the claim in Proposition 7 in the main text. Q.E.D.