The optimal quantity of CBDC in a bank-based economy*

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Abstract

Recent studies suggest that the risk of bank disintermediation through deposit substitution could undermine the potential benefits of issuing a central bank digital currency (CBDC); a technologically superior means of payment issued by a central bank. First, we provide evidence on the estimated impact of digital euro news on euro area banks' stock prices. The expected impact of CBDC on banks' valuations and lending supply crucially depends on the design features aimed at calibrating the amount of CBDC in circulation. Then, we develop a quantitative DSGE model that incorporates these trade-offs and a selection of mechanisms through which the issuance of a CBDC could affect bank intermediation and the real economy. The sign and magnitude of the impact depend on the design of a CBDC as well as on the response of the central bank's balance sheet and its collateral framework. Welfare-maximizing CBDC policy rules are effective in mitigating the risk of bank disintermediation and induce significant welfare gains. The model suggests that the welfare-maximizing amount of CBDC in circulation for the case of the euro area lies between 15% and 45% of quarterly real GDP in equilibrium.

Keywords: central bank digital currency, bank intermediation, DSGE models.

JEL classification: E42, E58, G21.

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Non-technical Summary

In recent years, the use of cash for transactions has significantly declined while the demand for digital means of payment for retail purposes has steadily increased. In response, central banks have started to investigate the implications of issuing central bank digital currencies (CBDCs). Among the potential benefits of CBDCs, satisfying the demand for a safe, digital means of payment stands out. One of the most discussed challenges of issuing a CBDC is the risk of bank disintermediation through deposit substitution. Much of the current policy debate focuses on how to calibrate the amount of CBDC in circulation so as to ensure that potential benefits of CBDC materialize without harming monetary and financial stability through bank disintermediation. One challenge in this regard is that advanced economies have no experience with CBDCs and, hence, there is no available data on which empirical analysis can be performed. For this reason, the literature has focused on studying the implications of CBDCs based on theoretical models.

This paper provides novel empirical evidence on the impact of digital euro news on bank stock prices and bank lending behavior. The study finds that such impacts depend on the CBDC design features aimed at calibrating the amount of central bank digital currency in circulation. The magnitude of these effects is sensitive to the reliance of banks on deposit funding.

Then, the paper develops a quantitative macro-banking DSGE model for CBDC analysis that: (i) is calibrated to quarterly data of the euro area for the period 2000:I - 2021:II so as to match a number of first and second moments from banking and macroeconomic aggregates; (ii) replicates the empirical evidence provided in the paper; (iii) incorporates the main trade-offs and key selected transmission channels through which CBDC could affect the banking sector and the macroeconomy; and (iv) allows for a careful quantitative analysis of the macroeconomic and welfare implications of issuing CBDC for both, hypothetical winners (i.e., CBDC holders) and potential losers (i.e., households who do not hold CBDC and rely on bank lending).

The analysis concludes the following. First, the sign and magnitude of introducing CBDC depends on the design of this central bank liability as well as on the response of the central bank's balance sheet and its collateral framework. Second, the sign of the net steady state impact of issuing CBDC on bank lending and real GDP tends to be negative due to the dominance of a bank disintermediation effect whereas net cyclical effects tend to be positive due to the dominance of a fiscal expansion effect. Third, welfare-maximizing CBDC (quantity and interest rate) policy rules are effective in mitigating the risk of bank disintermediation and induce significant welfare gains. Welfare gains attained under optimal interest rate rules are comparatively larger due to an improved stabilization/bank disintermediation trade-off. Fourth, the optimal amount of CBDC in circulation for the case of the euro area lies between 15% and 45% of quarterly real GDP in equilibrium.

1 Introduction

In recent years, the demand for digital means of payment for retail purposes has steadily increased while the use of cash for transactions has gradually declined (Auer et al. 2020). In response to this shift in individual preferences, central banks all over the world have started to investigate the potential benefits and implications of issuing central bank digital currencies (CBDCs). The ultimate goal of introducing a CBDC is to ensure that individuals operating in an increasingly digitalized economy keep having access to the safest form of money, central bank money. Among the many potential benefits CBDCs entail, satisfying the demand for a safe, digital means of payment stands out. The most discussed challenge of issuing a CBDC is the risk of bank disintermediation through deposit substitution (Carapella and Flemming 2020).

Against this background, the current debate focuses on how to calibrate the amount of CBDC in circulation so as to ensure that potential benefits of CBDC materialize without harming monetary and financial stability through bank disintermediation. One challenge in this regard is that advanced economies have no experience with CBDCs and, hence, there is no available data on which empirical analysis can be performed. For this reason, the literature has focused on studying the implications of CBDCs based on theoretical models which can be grouped into three main categories: (i) models of payments and modern monetarist models in the spirit of Lagos and Wright (2005), useful to explore design choices of a CBDC as a means of payment; (ii) models of bank runs in the tradition of Diamond and Dybvig (1983), relevant to study the potential implications of CBDC for the severity of bank runs; and (iii) quantitative DSGE models, important to evaluate the general equilibrium and macroeconomic effects of issuing CBDCs.

The main contributions of this paper are threefold. First, we provide empirical evidence on the impact of news on the development of a digital euro on bank stock prices and bank lending behavior. Second, we develop and calibrate a quantitative euro area DSGE model to study the transmission of issuing a CBDC on the bank intermediation and the economy in the euro area. This model incorporates some of the key mechanisms and trade-offs discussed above and accounts for the empirical findings that we provide. Finally, we use the model to analyze welfare-maximizing CBDC policy rules.

The response of bank valuations to news about the digital euro project may provide insights as to what market participants think the effect of a digital euro on bank profitability may be. In section 2, we isolate, by means of Fama-French factors, the abnormal returns on euro area banks' stocks around events related to digital euro news, and look at which bank characteristics are more related to these returns. Moreover, we check whether credit supply was affected by exposure to these events. We find that the impact depends on the bank's business model, in particular on whether it relies on deposits as an important source of funding, but also on the CBDC design features aimed at calibrating the amount of central bank digital currency in circulation.

In section 3, we develop a quantitative DSGE model with a banking sector calibrated to the euro area economy. We model a monetary economy populated by two types of households: patient households who are net savers and hold a variety of financial and monetary instruments, three of which provide them with liquidity services (i.e. bank deposits, cash and CBDC), and impatient households who borrow funds from banks against housing collateral (Iacoviello 2005).[1]Patient households own all firms operating in the economy: capital and final goods producing firms, entrepreneurial firms, and banks. Each entrepreneurial firm is run by a manager, who obtains bank lending against eligible collateral (Kiyotaki and Moore, 1997), and a retailer (intermediate good producer) who operates under monopolistic competition in the market of her own variety and sets prices a la Calvo (1983). Banks intermediate funds by borrowing from patient households (in the form of one-period deposits) to lending to impatient households and entrepreneurs (in the form of one-period loans). The banks' assets (i.e., loans, government bonds and reserves) are funded by equity, deposits and central bank borrowing. The banks operate subject to a capital adequacy constraint (Iacoviello, 2015) and a liquidity (reserves) requirement (Brunnermeier and Koby, 2019), and obtain complementary funding from the central bank against eligible collateral (government bonds). All borrowing and regulatory constraints are binding in a neighborhood of the steady state.

The model is completed with a policy block. Government spending is a constant fraction of steady-state real output. The government finances its deficit by issuing one-period government bonds. Tax revenues, collected in a lump-sum fashion from households, are adjusted in response to changes in the holdings of government debt by banks and patient households. The central bank sets the lending facility rate according to a simple Taylor-type rule and the interest rate on reserves so as to maintain a constant corridor between these two policy rates. Central bank assets (i.e., loans to private banks) are financed by issuing reserves, banknotes and CBDC (central bank's balance sheet identity) and its profits are transferred to the government. CBDC supply is set by means of simple quantity or interest rate policy rules.

We then calibrate the model to quarterly data of the euro area for the period 2000:I - 2021:II, and match a number of first and second moments from banking and macroeconomic aggregates .

The model captures the following transmission channels of the issuance of CBDC to the economy. Due to the imperfect substitutability between the three assets that provide liquidity services, an increase in the amount of CBDC in circulation is associated with a decrease in savers' holdings of cash and deposits. In response, banks reduce their holdings of reserves in line with the reserve

[1]The idea that these monetary instruments provide liquidity services is captured by allowing for money in the utility function (Sidrauski 1967). The substitutability across these means of payment is accounted for by defining liquidity services as a CES aggregator of the three monetary instruments with an elasticity of substitution larger than 1.

requirement. As a result, the issuance of CBDC is not fully offset by the aggregate decrease in reserves and cash and the central bank's balance sheet expands. This in turn leads to an increased demand for collateral (i.e. government bonds) by banks and a reduction in bank lending and real output: The issuance of CBDC induces a rebalancing effect on both sides of banks' balance sheets. The weight of central bank financing in banks' total liabilities increases at the expense of deposits and that of government bonds expands at the expense of reserves and loans (bank disintermediation effect). In addition, there is a channel through which issuing CBDC tends to stimulate the real economy and bank lending activity. The shift in the size and composition of the central bank balance sheet leads to an increase in central bank profits. In order for the government budget constraint to hold, a fiscal expansion occurs. Taxes collected from households decline and private consumption soars (fiscal expansion effect).

In line with the empirical evidence presented above, our analysis shows that, under the baseline calibration, the net steady state impact of issuing a CBDC on banks' valuations and lending is negative, as the bank disintermediation effect dominates the fiscal expansion effect. By inspecting the impulse responses of key aggregates to a positive CBDC supply shock, we find that the opposite applies to the net cyclical impact of issuing CBDC. A dominance of the fiscal expansion effect yields a net positive impact on bank lending and economic activity.

In Section 4 we then use the calibrated bank-based DSGE model to analyze the quantitative effects and welfare implications of six different CBDC rules. We compare the results with the baseline case under which there is no CBDC. We consider both quantity and interest rate rules. Since patient households own all financial and non-financial firms in the economy, the welfare analysis can be restricted to households. The optimal CBDC policy rule is obtained by maximizing a measure of social welfare - defined as a weighted average of the expected lifetime utility of the two types of households - with respect to the relevant policy parameter vector. We study the steady state and cyclical effects of optimal CBDC policy rules.

CBDC-induced welfare implications and trade-offs are fundamentally driven by three main effects: (i) a liquidity services effect according to which patient households benefit from the availability of a monetary instrument that provides them with liquidity services and for which there is no perfect substitute in the economy; (ii) a bank disintermediation effect by which the relative increase in the weight of government bond holdings and central bank funding - triggered by the CBDC-induced deposit substitution - leads to a compression in banks' net interest margins which adversely affects lending supply and, thus, borrowers' welfare; and (iii) a stabilization effect according to which as CBDC supply increases, the fraction of the adjustment in the face of exogenous shocks (that hit liquidity services) which is borne by this central bank liability rises, thereby inducing a smoothing effect on the rest of monetary instruments including bank deposits and, hence, bank lending and aggregates of the real economy.[2,3]

Regardless of the specification of the CBDC (quantity or interest rate) rule under consideration, welfare-maximizing CBDC policy rules are effective in mitigating the risk of bank disintermediation and induce significant welfare gains for both savers and borrowers. However, under optimal interest rate rules, the stabilization/bank disintermediation trade-off faced by borrowers improves and, thus, welfare gains are more sizable despite the fact that the amount of CBDC in circulation and, thus, the magnitude of the bank disintermediation effect, are also larger.[4]

Overall, our findings suggest that the optimal amount of CBDC in circulation for the case of the euro area lies between 15% and 45% of quarterly real GDP in equilibrium, with the steady state impact of CBDC on bank lending and valuations likely to be moderate under this range of values. By way of contrast, if CBDC were to be supplied under no quantity limits and no remuneration, the amount of CBDC in circulation would be around 65% of quarterly real GDP and the steady state effects on banks' valuations and lending would be more sizable. These findings are consistent with the novel empirical evidence on bank stock price's reactions to CBDC news that we present in this paper. It also underscores the importance of adequately calibrating the amount of CBDC in circulation in order to mitigate the impact of introducing a CBDC on the banking sector.

Related Literature This paper contributes to a recent and growing literature that studies the consequences of issuing CBDCs. Much of this literature focuses on the trade-off between the potential benefits of CBDC as a safe and innovative means of payment and the risk of bank disintermediation through deposit substitution (see, e.g., Piazessi and Schneider 2020; Keister and Sanches 2021). Brunnermeier and Niepelt (2019) and Fernández-Villaverde et al. (2020) prove that, under certain conditions, there are no allocative and macroeconomic consequences of CBDC-induced bank disintermediation as society is implicitly indifferent between obtaining lending through bank deposit funding and via central bank financing. A key common feature of these models is that they abstract from modelling the central bank's collateral requirement, a financial friction that has been shown to play a key role in the potential (non-neutral) effects

[2] With regards to the bank disintermediation effect, it is worth noting that - according to the baseline calibration of the model - the steady state weighted average interest rate on loans is larger than the interest rate on government bonds, whereas the interest rate on central bank funding is larger than that on household deposits. That is, the compression in banks' net interest margins tends to occur from both the revenues and the funding costs side.

[3] The magnitude of the stabilization effect is particularly sensitive to the capacity the CBDC policy rule grants the market of this monetary instrument to make the adjustment via quantities. In general, under CBDC interest rate rules (and as opposed to CBDC quantity rules), the bulk of the adjustment is made via quantities and, hence, the size of the stabilization effect and of CBDC-induced welfare gains is larger. The same argument applies to the comparison between dynamic CBDC quantity rules - which allow for a certain adjustment in the quantity of CBDC in response to changes in macroeconomic conditions - and static CBDC quantity rules, under which the adjustment is made via prices.

[4] Note that welfare gains under optimal interest rate rules are also larger for savers (i.e., CBDC holders). A higher quantity of CBDC in circulation also leads to a more prominent liquidity services effect.

the introduction of a CBDC may trigger on the banking sector and the real economy (see, e.g., Williamson 2019; Assenmacher et al. 2021; Ahnert, Muñoz and Soons 2022). These papers, however, rely on a framework other than a DSGE. Our paper contributes to the strand of the literature that highlights the importance of this transmission channel by showing that if these requirements are binding and the cost of central bank funding relative to that of deposits differs, issuing CBDC has a non-neutral effect on bank lending and the real economy.

Our analysis is complementary to other papers in the literature that emphasize different market imperfections to show the macroeconomic non-neutrality of issuing CBDC. If banks operate in an imperfectly competitive environment, the introduction of a central bank digital currency may actually lead to an expansion in deposits, bank lending and real output (see Andolfatto 2018; Chiu et al. 2019). In our model, the introduction of a CBDC can also have a positive net impact on economic activity, but this is due to a CBDC-induced increase in central bank profits that triggers a fiscal expansion through the government budget constraint.

In particular, this paper relates to recent work that follows a DSGE or business cycle modelling approach to capture the above mentioned trade-off and other potential implications of issuing CBDC. Fundamental key distinctive features of our set up when compared with others in this strand of the literature, are that ours not only models the central bank balance sheet (and central bank assets in the form of credit to the bankig sector) but also explicitly and simultaneously accounts for the two most quantitatively important central bank liabilities in advanced economies: reserves and banknotes. The impact of incorporating reserves into the analysis is that - subject to liquidity and reserve requirements - banks may opt for responding to the issuance of CBDC by drawing down their stock of reserves. The motivation for modelling cash is that this central bank liability is a close substitute for CBDC. We show that the substitutability between CBDC and cash has an impact on banks and the real economy by affecting the response of the central bank balance sheet. The only fraction of CBDC that is issued against eligible collateral (through central bank lending) is the one that is not compensated by an adjustment in other central bank liabilities (i.e., in our model; cash and reserves).

Ferrari et al. (2020) focus on the international dimension of CBDC as a means of payment and thus, abstract from modelling the substitutability between CBDC and bank deposits and other important mechanisms through which CBDC can affect bank intermediation and the real economy. Barrdear and Kumhof (2021) captures the impact that introducing a CBDC has on the issuance of public debt by incorporating the net cash flows generated by this liability for the public sector in the government intertemporal budget constraint. Our model goes one step further by accounting for the interactions between CBDC and public debt through an intertemporal government budget constraint that also incorporates other central bank liabilities which play a relevant role in the transmission mechanism (i.e., reserves and banknotes). Gross and Schiller (2021) explicitly model central bank funding but omit the central bank balance sheet identity and its two main liabilities (i.e., cash and reserves) in the analysis. Niepelt (2020) identifies the conditions under which the macro irrelevance (Brunnermeier and Niepelt 2019) also holds in a business cycle model with reserves and an imperfectly competitive market for bank deposits and omits the role of the central bank's balance sheet and collateral framework.

Overall, our paper differs from those in this strand of the literature as: (i) it simultaneously accounts for the substitutability between CBDC, cash and bank deposits, (ii) it captures the interactions among three mechanisms that play a key role in understanding how the banking sector and the macroeconomy may adjust when a CBDC is issued (i.e., the central bank's balance sheet, its collateral framework and liquidity (reserve) requirements), (iii) it is calibrated to quarterly data of the euro area so as to match a large number of relevant steady sate ratios, spreads and second moments, which makes it useful for policy analysis, and (iv) it carries out a welfare analysis of CBDC that investigates the welfare implications of issuing CBDC for both, hypothetical winners (i.e., CBDC holders) and potential losers (i.e., households who do not hold CBDC and rely on bank lending) and provides a sensible range of values for the optimal amount of CBDC in circulation based on a welfare-maximizing CBDC policy rules approach.

The paper is organized as follows. Section 2 presents novel empirical evidence on the estimated impact of CBDC news on euro area banks' stock prices and lending. Section 3 describes the macro-banking DSGE model and the transmission mechanisms of issuing CBDC. Section 4 develops a quantitative exercise to assess the effects of welfare-maximizing CBDC rules under different policy options. Section 5 concludes.

2 Empirical Evidence

Bank valuations provide some insight as to what investors currently think a digital euro might entail for banks' business models. To the extent that banks' stock prices reflect also the present discounted value of the future stream of profitability, their changes around events that define agents' expectations about the digital euro, net of a potential change in discount factors, can be a measure of the impact of the digital euro on bank profitability. Since the digital euro project is still under development, stock market developments might be one of the few, however partial, sources of evidence one can look at to gain insights over expectations over the digital euro and its consequences for the euro area bank lending conditions.

2.1 Stock Market Reactions to Digital Euro News

We run a cross-sectional event study to analyze banks' stock price reactions to news related to the digital euro.[5] Following Sefcik and Thompson (1986), we start by estimating banks' abnormal returns associated with digital euro news using a Fama and French (1993) three-factor model. We fit the model to stock market returns of euro area banks, and we classify returns as abnormal to the extent that they deviate from the returns explained by the regularities captured by Fama-French factors. The sample with data from Iboxx is based on 134 banks from 1 January 2007 to 31 May 2021. For each bank, we estimate the following model:

$$R_{b,t} = \alpha_b + \beta_{m,b} R_{m,t} + \beta_{HML,b} R_{HML,t} + \beta_{SMB,b} R_{SMB,t} + \sum_{e=1}^{E} \gamma_b^e D_t^e + \varepsilon_{b,t}, \tag{1}$$

where $R_{b,t}$ is the return on the stock of bank b between the day before and the day after t, $R_{m,t}$, $R_{HML,t}$ and $R_{SMB,t}$ are the excess return on the market portfolio, the value vs. growth factor (i.e., the return on a portfolio long high market-to-book firms and short low market-to-book firms), and the size factor (i.e., the return on a portfolio long small firms and short large firms), respectively. The abnormal daily returns are computed by using the estimated coefficients γ_b^e of the dummy variables D_t^e for each event $e = 1, ..., E$, which take value 1 if the event e takes place in day t.

The series of daily events related to digital euro are distributed over 2020 and 2021. Table A.1 reports the full list of events considered. They all relate to public interventions by ECB board members, and range from official press releases to interviews and speeches, to entries in ECB's official blog and the publication of a VoxEU column. All these events have a precise date of publication, which is used as date to identify the event.

Figure 1 reports the results of the analysis. We compute the average cumulated abnormal return up to day t as $1/B \sum_b \sum_{e=1}^{e(t)} \hat{\gamma}_b^e$, where $e(t)$ is the latest event up to day t, B being the total number of banks, and $\hat{\gamma}_b^e$ is the abnormal return of bank b in event e estimated with model (1). The average cumulated abnormal return has remained relatively stable until 2 October 2020, date in which the ECB stated its intention to intensify work on a digital euro by means of a press release. After that date, every additional communication on the subject has led to a marginal negative return on bank stocks, stabilizing between end October 2020 and the early February 2021 at around 1% below the level prevailing since the beginning of 2020. The trend was inverted with the publication of an interview on 9 February 2021, when the potential limit on individual holdings of EUR 3,000 was floated again among other aspects. After that date, events were associated with positive or neutral reactions of stock market valuations, ending by May 2021 on average at around 1% above the level at the beginning of 2020.

[5] For overviews of the event study methodology, see MacKinlay (1997) and Binder (1998).

The aggregate picture hides important heterogeneity in the cross-section. Stocks of banks with different business models have been reacting in a systematically different way to digital euro news. Stocks reflected the cleavage between banks perceived to be more exposed to the new technological threat, like those relying on deposit funding, and other banks. In particular, banks with a ratio of deposits over total liabilities above the median have experienced larger drops in valuations in response to digital euro events, summing up to a cumulated drop of over 2% be end-2020 on average. At the same time, they have also experienced a rebound after 9 February 2021, ending the year at the same valuation that they had in early 2020. This reaction is consistent with market participants either discounting a potentially large disintermediation effect or needing several months to absorb the information flow on this subject. The pattern was different for banks less reliant on deposit funding, which instead experienced an increase in valuations since October 2020, followed by a plateau over 2021. This is in line with the considerations on the potential positive impact on bank profitability, related to the potential new business opportunities created by the digital euro like innovative payment services as well as the levelling of the playing field with the digital payment and financial services offered by global tech giants.[6]

Table 1 illustrates further that reliance on deposit funding is the variable that most consistently helps to explain abnormal returns around digital euro events. The model estimated is as follows:

$$\hat{\gamma}_b^e = \delta \text{Deposit ratio}_{b,e} + \zeta_e + \zeta_b + X_{b,e} + \varepsilon_{b,e}, \tag{2}$$

where the observation is a given bank b in an event e, $\hat{\gamma}_b^e$ are the abnormal returns estimated with model (1) for each bank b and each event e, and $X_{b,e}$ is a set of (pre-existing) bank characteristics. The fixed effects ζ_e and ζ_b capture event- and bank-specific unobserved heterogeneity in abnormal returns. Deposit $\text{ratio}_{b,e}$ is the ratio of deposit from the non-financial private sector over main liabilities registered by the end of the month before event e. The controls $X_{b,e}$ cover several other bank characteristics that may in principle contribute in explaining bank stocks' abnormal returns, especially if the estimation strategy of model (1) was not successful in ruling out confounding factors. The controls include a proxy for size like bank assets, the ratio of TLTRO over assets to measure reliance on central bank funding, securities holdings over assets to measure exposure to asset purchases by the central bank, excess liquidity over assets to measure exposure to the negative interest rate policy, ROA to proxy for general profitability, the NPL ratio to measure the quality of the loan portfolio and the sensitivity to potential deterioration in the economic outlook, and the CDS spread to measure markets' assessment of the bank creditworthiness. In the last column we also offer a robustness check based on Fama-French factors for the aggregate European

[6]See, e.g., Panetta, F., (2020). "Preparing for the euro's digital future." The ECB Blog, 14 July 2021 (table A.1).

economy instead of those computed using stocks of euro area banks.[7] The results show that one standard deviation of difference in the deposit ratio (18 percentage points) is associated with over 1 percentage point of difference in abnormal return in each event.

These early considerations on the perceived impact of the digital euro on banks' future profitability are subject to some uncertainty. First, it might still be difficult for market participants to gauge the potential relevance that a digital euro might have on banks' business model. Second, the model used to isolate abnormal returns from otherwise normal fluctuations of banks' stock prices, however standard, may be misspecified. Third, the period under consideration for quantifying the abnormal returns might also be special, in light of the chronically low price-to-book ratios over the past 5-10 years and the extraordinary environment emerged from the pandemic. Fourth, there may be concomitant events that increase the measurement error of single events. The current approach partially addresses these concerns with a long time period spanning since 2007, considering a wide set of events referring to digital euro that should average out the potential misrepresentation of single events, with both positive and negative news in terms of their likely impact on stock market valuations.

2.2 Impact on Lending Conditions

The reaction of stock prices may have conveyed information to banks as to the impact that the digital euro project may have on their business model. Moreover, an adverse assessment by market participants as to the prospects of a given bank in a world with a digital euro may have also directly translated into more expensive market-based funding options for that bank. Hence, there may be scope for the stock market developments in late 2020 to have had a bearing on banks' lending conditions in the following months. To understand whether that was the case, we look at the developments in loan markets using transaction level data from AnaCredit (the European credit register). We perform a diff-in-diff exercise where the continuous treatment is the bank-level exposure to abnormal returns up to end-October 2020 and the dependent variable is the growth in lending volume since October 2020. In order to control for demand factors, we follow a Khwaja and Mian (2008) strategy, that is, we saturate the model with firm effects, relying on the cross-sectional variation in exposure at the bank level for firms with multiple lenders to achieve identification. For robustness, we also implement industry-location-size (ILS) fixed effects à la Degryse et al. (2019) to include single-lender firms, with no difference in results. The sample is constituted by the banks for which we can isolate the abnormal returns in model (1), which have around 1.6 million outstanding credit relations with 1.3 million firms distributed in 14 euro area countries over 2020

[7] The data for this robustness exercise were retrieved from French's webpage: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_3developed.html

and 2021. We estimate the following model:

$$\Delta^h \log(\text{Volume})_{b,f} = \alpha^h_{s,g,d} + \xi^h \hat{\Gamma}_b^{\text{October 2020}} + X_b + X_f + \varepsilon^h_{b,f}, \tag{3}$$

where $\Delta^h \log(\text{Volume})_{b,f}$ is the percentage change of outstanding amounts of loans between bank b and firm f occurred in the months after October 2020 up to horizon h, $\hat{\Gamma}_b^{\text{October 2020}}$ is our treatment variable defined as the (cumulated) abnormal returns in October 2020, X_b are bank controls and X_f are firm controls, and $\alpha^h_{s,g,d}$ is the ILS fixed effects, which in some specifications we substitute with firm fixed effects α^h_f for robustness. Since our treatment is at the bank level, we control for the spurious correlation in errors introduced in this way by clustering standard errors at the bank level.

In Table 2 we look at changes in loan volumes occurred in the three months following October 2020, that is, until January 2020. The results show a consistently significant impact across specifications, ranging between 0.1 and 0.4% of ex-ante volumes for each percentage point of additional stock market returns attributable to digital euro news. The impact is also economically meaningful, as one standard deviation in abnormal returns (almost 10 percentage points in our sample) can explain over 7% of the standard deviation of changes in loan volumes (using the coefficient from column 3 as a benchmark). The relation is quite robust to the inclusion of bank-level observables capturing banks' exposure to confounding factors such as monetary policy, and to a high level of saturation of the model.

In Figure 2 we apply the benchmark specification of column 3 to other horizons. The changes in loan volumes in the months leading up to October 2020 show that there was no differential trend in lending before the actual drop in stock returns of October 2020. Moreover, and consistent with the retrenchment in different patterns of stock market returns due to digital euro news observed since early February 2020, the impact on lending seems to be partially transitory, at least up to the horizon covered in the analysis. The reaction of lending volumes by May 2021 is almost half of the trough reached in January 2021, with progressively widening uncertainty surrounding the coefficient.

3 The Model

Consider a monetary, closed, decentralized and time-discrete economy populated by two types of households. Patient households (net savers) and impatient households (net borrowers). Both of them work, consume and accumulate housing. However, impatient households discount the future more heavily than patient ones (i.e., $\beta_i < \beta_p$) implying that, in the aggregate, patient households are net savers whereas impatient ones are net borrowers. Impatient households obtain funds from banks against housing collateral. Patient households hold a variety of assets, some of which are forms of money that serve as means of payments and provide them with liquidity services (i.e., bank deposits, cash and central bank digital currency). Net savers own all different types of firms operating in the economy, including banks, entrepreneurial firms, capital goods producers and final goods producers. For each type of agent and firm in the economy, there is a continuum of individuals in the $[0, 1]$ interval.

Banks intermediate financial resources by borrowing from patient households and the central bank and lending to impatient households and non-financial corporations (i.e., entrepreneurial firms). Financial intermediaries have to comply with certain capital and liquidity (reserve) requirements whose modelling is similar to the one proposed in Iacoviello (2015) and Brunnermeier and Koby (2019), respectively. The borrowing capacity of banks with the central bank is tied to the value of government bonds, which serve as the eligible asset within the collateral framework of the monetary authority.[8] For each entrepreneurial firm, there is a manager who obtains bank lending to acquire physical capital and commercial real estate and a retailer who rents such inputs and combines them with labor to produce intermediate goods under monopolistic competition and by setting prices a la Calvo (1983).

The government issues one-period bonds to finance its deficit. Tax revenues respond to changes in government bonds held by patient households and banks whereas government spending is assumed to be a constant fraction of steady state real GDP. The central bank sets two policy rates: the rate that is charged to banks when providing them with funds, which is set according to a simple Taylor-type policy rule, and the rate at which bank reserves are remunerated, which is set to maintain a constant corridor between the two policy rates. The monetary authority issues reserves, cash and CBDC and provides lending to the banking sector.

3.1 Main Features

3.1.1 Patient Households: net savers and CBDC holders

Let $c_{p,t}$, $n_{p,t}$, $h_{p,t}$ and z_t represent consumption, hours worked, housing demand and liquidity services demand by patient households in period t. The representative patient household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta_p^t \left\{ \frac{1}{1-\sigma_h} \left[c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)} \right]^{1-\sigma_h} + j_{p,t} \log h_{p,t} + \chi_{z,t} \log z_t \right\}, \tag{4}$$

[8] The modelling of banks extends the one proposed in Muñoz (2021) by: (i) allowing for government debt and reserve holdings on the asset side of the balance sheet, as well as central bank funding on the liabilities side; and (ii) incorporating a liquidity (reserves) requirement and a central bank's collateral requirement.

where $\beta_p \in (0,1)$ is the patient households' subjective discount factor, σ_h stands for the risk parameter, and ϕ refers to the inverse of the Frisch elasticity.[9] $j_{p,t}$ and $\chi_{z,t}$ denote possibly time-varying preference parameters for housing and liquidity services, respectively. More precisely, $j_{p,t} = j_p \varepsilon_{h,t}$ is the exogenously time-varying patient households' preference parameter for housing services, where $j_p > 0$ and $\varepsilon_{h,t}$ captures exogenous housing preference shocks. Similarly, $\chi_{z,t} = \chi_z \varepsilon_{cbdc,t}$ is the time-varying preference parameter for liquidity services, where $\chi_z > 0$ and $\varepsilon_{z,t}$ captures exogenous liquidity preference shocks.

Liquidity services are derived from holding cash, $m_{p,t}$, central bank digital currency, $cbdc_{p,t}$,and bank deposits, d_t, according to the following CES aggregator:

$$z_t\left(m_t, cbdc_t, d_t\right) = \left[m_t^{\left(\eta_{z,t}-1\right)/\eta_{z,t}} + \vartheta_t cbdc_t^{\left(\eta_{z,t}-1\right)/\eta_{z,t}} + \omega_d d_t^{\left(\eta_{z,t}-1\right)/\eta_{z,t}}\right]^{\eta_{z,t}/\left(\eta_{z,t}-1\right)}, \tag{5}$$

where ω_d measures the liquidity of bank deposits relative to central bank money (i.e., cash and central bank digital currency), ϑ_t captures exogenous CBDC preference shocks and $\eta_{z,t} = \eta_z \varepsilon_{\eta_t,t}$ is the possibly time-varying elasticity of substitution across different forms of money.[10] Cash, CBDC and deposits provide liquidity and, thus, are substitutes, implying $\eta_z > 1$.Finally, $\varepsilon_{\eta,t}$ captures exogenous shocks to the degree of substitutability between forms of money.

The maximization of (4) is subject to the sequence of budget constraints

$$\begin{aligned} c_{p,t} + q_t(h_{p,t} - h_{p,t-1}) + m_t + f(m_t) + cbdc_t + d_t + b_{p,t} + R_{g,t} b_{p,t-1} + \omega_T T_t \\ = \frac{m_{t-1}}{\pi_t} + R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} + R_{d,t-1}\frac{d_{t-1}}{\pi_t} + R_{g,t-1}\frac{b_{p,t-1}}{\pi_t} + w_t n_{p,t} + \Omega_t, \end{aligned} \tag{6}$$

where $b_{p,t}$ are government bond holdings, $\omega_T T_t$ is the fraction, $\omega_T \in [0,1]$, of total lump-sum taxes, T_t, paid by this type of households and $\Omega_t = \Omega_{e,t} + \Omega_{b,t}$ are dividends obtained from their ownership of non-financial corporations (i.e., entrepreneurial firms) and banks. $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, q_t the real price of housing and w_t the real wage rate. $R_{cbdc,t}$, $R_{d,t}$ and $R_{g,t}$ denote the real gross interest rates on CBDC, deposits and government bonds, respectively. The technological superiority of CBDC (relative to cash) is captured by the existence of cash storage

[9] Households are assumed to have GHH preferences in consumption and hours worked (see Greenwood et al. 1988). This type of preferences - under which wealth effects on labor supply are arbitrarily close to zero - has been extensively used in the business cycle literature as a useful device to match several empirical regularities. As in this paper, GHH preferences have been formulated by other authors, when evaluating macroeconomic policies, in order to prevent a counterfactual increase in labor supply during crises (see, e.g., Bianchi and Mendoza 2018).

[10] The specification of the CES aggregator for liquidity services, z_t, resembles that of Drechsler et al. (2017): the weighting parameters with which the different forms of central bank money enter the CES aggregator (in this case, cash and CBDC) are normalized to unity and the weighting parameter of bank deposits, ω_d, is allowed to differ and can be calibrated in order to capture the difference in liquidity preferences between public money and private one.

costs, $f(m_t)$, with $f_m > 0$ and $f_{mm} > 0$.[11] In particular, we assume that $f(m_t) = \left(\frac{\psi_m}{2} m_t^2\right)$.[12]

3.1.2 Impatient Households: net borrowers

Let $c_{i,t}$, $n_{i,t}$, and $h_{i,t}$ represent consumption, hours worked and housing demand by impatient households in period t. Then, the representative impatient household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta_i^t \left\{ \frac{1}{1-\sigma_h} \left[c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right]^{1-\sigma_h} + j_{i,t} \log h_{i,t} \right\}, \tag{7}$$

subject to a sequence of budget constraints and a borrowing limit

$$c_{i,t} + q_t \left(h_{i,t} - h_{i,t-1}\right) + R_{i,t-1} \frac{l_{i,t-1}}{\pi_t} + \left(1 - \omega_T\right) T_t = l_{i,t} + w_t n_{i,t} \tag{8}$$

$$l_{i,t} \leq m_{H,t} E_t \left(\frac{q_{t+1}}{R_{i,t}} h_{i,t} \pi_{t+1} \right). \tag{9}$$

where $\beta_i \in (0,1)$ is the impatient households' subjective discount factor $\left(\beta_i < \beta_p\right)$ and $j_{i,t} = j_i \varepsilon_{h,t}$ denotes a possibly time-varying preference parameter for housing, with $j_i > 0$. Bank loans obtained by impatient households are denoted by $l_{i,t}$ and the gross interest rate on loans to impatient households by $R_{i,t}$. According to (8), in each period, impatient households devote their available resources in terms of wage earnings and bank loans to consume, demand housing, repay their debt and pay lump-sum taxes. Expression (9) dictates that the borrowing capacity of impatient households is tied to the value of their collateral. In particular, they cannot borrow more than a possibly time-varying fraction $m_{H,t}$ of the expected value of their real estate stock. More precisely, $m_{H,t} = m_H \varepsilon_{mh,t}$ is the exogenously time-varying loan-to-value ratio, where $m_H \in [0,1]$ and $\varepsilon_{mh,t}$ captures exogenous shocks to constrained households' collateral.

[11] f_m and f_{mm} denote the first and second derivate of $f(m_t)$ with respect to cash holdings, m_t, respectively.

[12] Alternatively, we could have accounted for the technological superiority of CBDC, relative to cash, by allowing for cash and CBDC to weigh differently in the CES aggregator and in the utility function (see, e.g., Ferrari et al. 2020). Feenstra (1986) shows that there is a broad range of specifications for which assuming a money-in-utility function is equivalent to having liquidity costs in the budget constraint. The motivation for our modelling choice is twofold. First, given the uncertainty about many of the design features that CBDCs will have in advanced economies we remain agnostic about them, to the extent possible, by assigning the same weight in the CES aggregator to cash and CBDC. Second, the assumption of cash storage costs is based on a well documented evidence on which other models also rely (see, e.g., Ahnert, Muñoz and Soons 2022) and captures the idea that costs of holding cash may increase non-linearly (e.g., costs of storage, insurance, security).

3.1.3 Banks

Let $\Lambda_{t,t+1} = \beta_p \frac{\lambda_{t+1}^p}{\lambda_t^p}$ be the stochastic discount factor (with λ_t^p being the Lagrange multiplier of the patient households' optimization problem), $\Omega_{b,t}$ earnings distributed by banks and σ the elasticity of intertemporal substitution in dividends. Then, the representative bank manager maximizes

$$E_0 \sum_{t=0}^{\infty} \Lambda_{t,t+1} \quad \frac{1}{(1-\frac{1}{\sigma})} \Omega_{b,t}^{\left(1-\frac{1}{\sigma}\right)} \tag{10}$$

subject to a balance sheet identity, a sequence of cash flow restrictions, a borrowing constraint, a liquidity (reserves) requirement and a central bank collateral requirement, respectively:

$$L_{i,t} + L_{e,t} + b_{b,t} + \widetilde{R}_{b,t} = e_t + D_t + f_t, \tag{11}$$

$$\begin{aligned} &\Omega_{b,t} + e_t - (1-\delta^e)\frac{e_{t-1}}{\pi_t} \\ &\quad = \frac{\left(r_{i,t-1}L_{i,t-1} + r_{e,t}L_{e,t-1} + r_{g,t-1}b_{b,,t-1} + r_{\widetilde{R},t-1}\widetilde{R}_{b,t-1} - r_{d,t-1}D_{t-1} - r_{f,t-1}f_{t-1}\right)}{\pi_t}, \end{aligned} \tag{12}$$

$$D_t + f_t \leq \gamma_i L_{i,t} + \gamma_e L_{e,t} + \gamma_b b_{b,t} + \gamma_{\widetilde{R}} \widetilde{R}_{b,t}, \tag{13}$$

$$\theta_{R,t} D_t \leq \widetilde{R}_{b,t}, \tag{14}$$

$$f_t \leq \theta_{b,t} E_t \left(\frac{b_{b,t}}{R_{f,t}} \pi_{t+1} \right). \tag{15}$$

Bank assets comprise loans extended to impatient households, $L_{i,t}$, and entrepreneurial firms, $L_{e,t}$, government bonds, $b_{b,t}$, and reserves held at the central bank, $\widetilde{R}_{b,t}$.Formally, $A_{b,t} = L_{i,t} + L_{e,t} + b_{b,t} + \widetilde{R}_{b,t}$. Identity (11) states that total bank assets are financed by the sum of bank equity, e_t (also referred to as bank capital), deposits held by patient households, D_t, and central bank funding, f_t.

The model assumes full inside equity financing, in the sense that bank equity is solely accumulated out of retained earnings. Formally, the law of motion for bank capital reads[13]

$$e_t = J_{b,t} - \ \Omega_{b,t} + \ (1-\delta^e)e_{t-1}/\pi_t, \tag{16}$$

[13]Expression (16) for the law of motion for bank capital is identical to the one assumed in Muñoz (2021) and only differs from the one proposed in Gerali et al. (2010) in that these authors assume net profits are fully retained, period by period (i.e., there is no bank payout policy whatsoever).

where $J_{b,t}$ stands for bank net profits. Rearranging in expression (16), bank net profits can be decomposed into three terms:

$$J_{b,t} = \underbrace{\underbrace{(e_t - e_{t-1}/\pi_t)}_{\text{reinvested profits}} + \underbrace{(\delta^e e_{t-1}/\pi_t)}_{\text{eroded equity}}}_{\text{retained earnings}} + \underbrace{\Omega_{b,t},}_{\text{distributed earnings}} \tag{17}$$

where the term $(e_t - e_{t-1}/\pi_t)$ refers to the part of profits made in period t which are reinvested in the financial intermediation business, and $(\delta^e e_{t-1}/\pi_t)$ is the fraction of bank own resources which, due to exogenous factors, cannot be further accumulated as bank capital into the next period. The term $(\delta^e e_{t-1}/\pi_t)$ can be interpreted in several manners: (i) own resources the banker devotes to manage bank capital and to play its role as financial intermediary, or (ii) equity that erodes due to a variety of factors which are not explicitly accounted for in the model and which may relate to specific characteristics of bank capital such as its quality.

Equation (12) is a flow of funds constraint which states that in each period the banker has to distribute net profits $J_{b,t}$ between dividend payouts, $\Omega_{b,t}$, and retained earnings. In the model, bank net profits are defined as the net interest income (i.e., right hand side of equation 12). Note that $r_{i,t}$, $r_{e,t}$ $r_{g,t}$, $r_{\widetilde{R},t}$, $r_{d,t}$ and $r_{f,t}$ denote the net interest rates on loans to households, loans to non-financial corporations, government bonds, reserves, household deposits and central bank funding, respectively.

Expression (13) stipulates that bankers are constrained in their ability to issue liabilities (i.e., deposits and central bank funding). For a given period t, deposits and central bank financing cannot exceed total risk-weighted assets. γ_i, γ_e, γ_b and γ_R denote the proportions of loans to households, loans to firms, government bonds and reserves that can be financed with debt. Given that this expression is binding in a neighborhood of the steady state, $(1 - \gamma_{h,t})$ can be interpreted as the sectorial capital requirement on holdings of asset class h ($\forall h = e,\ i,\ b,\ R$) and equation (13) as a capital adequacy constraint.

Equation (14) dictates that reserves held by the representative bank in the central bank cannot fall below a certain threshold specified as a possibly time-varying fraction,$\theta_{R,t}$, of deposits, where $\theta_R \in (0, 1)$ and $\varepsilon_{\theta_r,t}$ captures exogenous shocks to banks' relative reserve holdings. This expression can be interpreted as a liquidity or reserves requirement faced by banks (see, eg., Brunnermeier and Koby, 2019) and it is relevant due to various quantitative and empirically -related reasons. First, an important fraction of total central bank liabilities is represented by reserves and, thus, modelling them allows to improve the model fit (see section 3.3). Second, outside periods of unconventional monetary policy and/or extraordinary uncertainty, the reserves-to-deposits ratio has been notably stable over time in the euro area. Third, expression (14) captures the idea that part of the adjustment banks would have to make in the event of a CBDC-induced deposit outflow could take the form of a shift in their stock of reserves.

Expression (15) dictates that the capacity of the representative bank to obtain funding from the monetary authority is tied to the value of the asset holdings that are eligible as collateral according to the collateral framework of the central bank. In this version of the model, we assume that government bonds are the only eligible asset under such framework. In this model economy, banks cannot borrow from the central bank more than a possibly time-varying fraction, $\theta_{b,t}$, of the expected value of their government bond holdings. More precisely, $\theta_{b,t} = \theta_b \varepsilon_t^{\theta_b}$ can be interpreted as the complementary of the exogenously time-varying haircut on government bonds, where $\theta_b \in [0, 1]$ and $\varepsilon_{\theta_b,t}$ captures exogenous shocks to banks' collateral (for central bank operations).

3.1.4 Entrepreneurial Firms

The entrepreneurial firm industry is populated by two types of agents. For each entrepreneurial firm, there is a manager who obtains bank lending to acquire new housing in the form of commercial real estate and a retailer who rents such input and combines it with physical capital and labor (through a Cobb-Douglas technology) to produce intermediate goods under monopolistic competition.

Entrepreneurial Managers Let $\Omega_{e,t}$ be earnings distributed by entrepreneurs. Then, entrepreneurial managers seek to maximize

$$E_0 \sum_{t=0}^{\infty} \Lambda_{t,t+1} \quad \frac{1}{(1-\frac{1}{\sigma})} \Omega_{e,t}^{(1-\frac{1}{\sigma})}$$

subject to a sequence of budget constraints and the corresponding borrowing limit

$$\Omega_{e,t} + R_{e,t} \frac{l_{e,t-1}}{\pi_t} + q_{k,t} \left[k_{e,t} - (1-\delta_t^k) k_{e,t-1} \right] + q_t (h_{e,t} - h_{e,t-1}) = r_{h,t} h_{e,t-1} + r_{k,t} u_t k_{e,t-1} + l_{e,t} + J_{er,t}, \quad (18)$$

$$l_{e,t} \leq m_{K,t} E_t \left[\frac{q_{k,t+1}}{R_{e,t+1}} (1-\delta_{t+1}^k) k_{e,t} \pi_{t+1} \right], \quad (19)$$

where $l_{e,t}$ is bank loans extended to entrepreneurial firms, $k_{e,t}$ refers to physical capital, u_t is its utilization rate and $J_{er,t}$ are distributed profits obtained from the ownership of intermediate good producing firms (i.e., entrepreneurial retailers). $R_{e,t}$ denotes the real gross interest rate on bank loans to firms and $q_{k,t}$ is the real price of physical capital. $r_{h,t}$ and $r_{k,t}$ denote the real net interest rates entrepreneurial managers charge when renting physical capital and commercial real estate to entrepreneurial retailers. The depreciation rate of capital is an increasing and convex function of the rate of capacity utilization

$$\delta_t^k(u_t) = \delta_0^k + \delta_1^k(u_t - 1) + \frac{\delta_2^k}{2}(u_t - 1)^2. \tag{20}$$

According to (18), in each period, entrepreneurial managers devote their available resources in terms of loans and rents to distribute earnings, repay their debt, accumulate physical capital, $k_{e,t}$, and commercial real estate, $h_{e,t}$. Expression (19) dictates that the borrowing capacity of entrepreneurial firms is tied to the value of their physical capital collateral. In particular, they cannot borrow more than a possibly time-varying fraction $m_{K,t} = m_K \varepsilon_{mk,t}$ of the expected value of their capital stock, where $m_K \in [0,1]$ and $\varepsilon_{mk,t}$ captures exogenous shocks to entrepreneurial firms' collateral.

Entrepreneurial Retailers There is a continuum of entrepreneurial retailers (also referred to as intermediate good producers). Each intermediate good producer j operates the following Cobb-Douglas production function:

$$Y_t(j) = A_t \left[u_t(j) k_{e,t-1}(j)\right]^{\alpha} h_{e,t-1}(j)^{\nu} N_t(j)^{(1-\alpha-\nu)}, \tag{21}$$

where $k_{e,t}(j)$ and $h_{e,t}(j)$ denote the quantities of physical capital and commercial real estate and labor rented by firm j, $N_t(j)$ refers to labor demand by the same firm and A_t captures technology shocks in the intermediate good production sector. Intermediate good producers solve a two-stage problem. In the first stage they choose the trajectories of $k_{e,t}(j)$, $h_{e,t}(j)$, and $N_t(j)$ that minimize total real costs, $r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_t N_t(j)$, subject to the available technology, represented by (21). Assuming Calvo (1983) price-setting, in the second stage intermediate good producers choose the price, $P_t(j)$, that maximizes discounted real profits:

$$E_t \sum_{s=0}^{\infty} \left[(\beta_p \theta)^s \frac{\lambda_{t+s}^p}{\lambda_t^p} \right] \left\{ \left[\prod_{\tau=1}^{s} \pi_{t+\tau-1}^{\chi} \frac{P_t(j)}{P_{t+s}} - mc_{t+s} \right] Y_{t+s}(j) \right\}, \tag{22}$$

where θ is the probability of not adjusting the price, $\chi_\pi \in [0,1]$ is the indexation parameter, and mc_t denotes the real marginal cost of the intermediate good producer. In each period, a fraction θ of firms reoptimize their prices. All other firms can only index their prices by past inflation, with $\chi_\pi = 0$ and $\chi_\pi = 1$ referring to the particular cases of no indexation and total indexation, respectively. The first-order condition is standard (see Appendix C), with all time-t price setters choosing a common optimal price P_t^*.

3.1.5 Capital and Final Goods Producers

The representative, perfectly competitive, final goods producer chooses the trajectory of intermediate good $Y_t(j)$ that maximizes $P_tY_t - \int_0^1 P_t(j)\,Y_t(j)\,di$, where Y_t denotes final production and P_t is the aggregate price level. $Y_t(j)$ denotes demand for intermediate good j and $P_t(j)$ is the corresponding price. The homogeneous final good is produced by means of a Dixit-Stiglitz technology, $Y_t = \left[\int_0^1 Y_t(j)^{(\varepsilon-1)/\varepsilon}\,dj\right]^{\varepsilon/(\varepsilon-1)}$,where $\varepsilon > 1$ is the elasticity of substitution across intermediate goods. Profit maximization yields demand functions for intermediate good j: $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$, $\forall j$. From the zero profit condition, $P_tY_{c,t} = \int_0^1 P_{c,t}(j)\,Y_{c,t}(j)\,di$, it follows that P_t can be interpreted as the price index: $P_t \equiv \left[\int_0^1 P_{c,t}(j)^{(1-\varepsilon)}\,dj\right]^{1/(1-\varepsilon)}$.

At the beginning of each period, capital producers demand an amount I_t of final good from entrepreneurs, which combined with the available stock of capital, allows them to produce new capital goods which are then sold back to entrepreneurial firms. Capital producers choose the trajectory of net investment in physical capital, I_t, that maximizes $E_0 \sum_{t=0}^{\infty} \Lambda_{t,t+1}\left(q_{k,t}\Delta x_{k,t} - I_t\right)$, subject to $x_{k,t} = x_{k,t-1} + I_t\left[1 - \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]$, where $\Delta x_{k,t} = K_t - (1-\delta_t^k)K_{t-1}$ is flow output and $S\left(\dfrac{I_t}{I_{t-1}}\right) = \dfrac{\psi_I}{2}\left(\dfrac{I_t}{I_{t-1}} - 1\right)^2$ an investment adjustment cost function.

3.1.6 Government

The government collects tax revenues from households in a lump-sum fashion. Such revenues are determined according to a fiscal rule

$$T_t = \phi_p b_{p,t-1} + \phi_b b_{b,t-1}, \tag{23}$$

where $\phi_p > 0$ and $\phi_b > 0$ determine the response of tax revenues to changes in government bond holdings of patient households and banks, respectively.

Government spending is assumed to be equal to a constant fraction, $\varrho > 0$, of steady state output

$$G_t = \varrho Y^{ss}. \tag{24}$$

Consequently, the issuance of short-term government bonds, $B_{g,t}$, is endogenously determined by the intertemporal budget constraint of the government

$$R_{g,t-1}\frac{B_{g,t-1}}{\pi_t} + G_t = T_t + B_{g,t} + \Omega_{cb,t}. \tag{25}$$

According to expression (25), in each period, the government devotes its available resources in terms of tax revenues, T_t, central bank profits, $\Omega_{cb,t}$, and funds obtained from the issuance of bonds, $B_{g,t}$, to consume, G_t, and to repay its debt, $R_{g,t-1}\frac{B_{g,t-1}}{\pi_t}$.

3.1.7 Central bank

The central bank sets two nominal short-term policy rates: the lending policy rate (also referred to as the lending facility rate), $r_{f,t}$, and the interest rate on reserves (also referred to as the deposit facility rate), $r_{\widetilde{R},t}$. The former is the interest rate the central bank charges when providing the banking sector with funding and is set according to a Taylor-type policy rule:

$$r_{f,t} = \rho_r r_{f,t-1} + (1-\rho_r)\left(r_f^{ss} + \alpha_\pi \tilde{\pi}_t + \alpha_Y \tilde{y}_t\right) + e_{rf,t}, \tag{26}$$

where ρ_r is the interest rate smoothing parameter, r_f^{ss} is the steady-state lending policy rate, $\alpha_\pi > 1$ determines the response of the lending policy rate to inflation deviations from the target $\tilde{\pi}_t = \log(\pi_t/\bar{\pi})$, $\alpha_Y \geq 0$ measures the degree of responsiveness of the same policy rate to output growth $\tilde{y}_t = \log(Y_t/Y_{t-t})$, and $e_{rf,t}$ is a white noise shock to the lending facility rate.

The deposit facility rate is the interest rate at which bank reserves are remunerated. This policy rate is assumed to be set such that a constant corridor of width $\alpha > 0$ is maintained between the lending facility rate and the deposit facility rate,

$$r_{\widetilde{R},t} = r_{f,t} - \alpha. \tag{27}$$

Central bank assets consist of lending to banks and are financed by the sum of reserves, cash and central bank digital currency. Formally:

$$F_t = \widetilde{R}_t + M_t + CBDC_t. \tag{28}$$

Central bank net profits are transferred to the government in each period and evolve as

$$\Omega_{cb,t} = \widetilde{R}_t + M_t + CBDC_t + R_{f,t-1}\frac{F_{t-1}}{\pi_t} - R_{\widetilde{R},t-1}\frac{\widetilde{R}_{t-1}}{\pi_t} - \frac{M_{t-1}}{\pi_t} - R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} - F_t. \tag{29}$$

Finally, the central bank issues central bank digital currency according to a policy rule, which - for the most general case - stipulates that CBDC supply in period t is equal to a constant fraction, $\phi_Y \geq 0$, of steady state real output.[14]Formally

[14]The choice of this specification for the CBDC policy rule under the most general case is motivated by the wide academic and policy discussion on the desirability of counting with a constant limit on individual CBDC holdings as a tool to calibrate the quantity of central bank digital currency in circulation. See, e.g., Bindseil, U., and F. Panetta (2020), "CBDC remuneration in a world with low or negative nominal interest rates." VoxEU column, 5 October 2020, and Panetta, F., (2021). Interview with Der Spiegel, 9 February 2021 (see table A.1).

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$$CBDC_t = \phi_Y Y^{ss}. \tag{30}$$

As discussed in section 4, under the baseline (counterfactual) scenario, the central bank does not issue CBDC (i.e., $\phi_Y = 0$). The quantitative analysis presented in that section considers various alternative CBDC policy scenarios which differ from one another in the specification and/or calibration of the CBDC policy rule in order to carry out a counterfactual analysis and assess the main implications for bank intermediation, the real economy and welfare of various CBDC quantity and interest rate type of policy rules.

3.1.8 Aggregation and market clearing

In equilibrium, all markets clear. In the case of the final goods market, the aggregate resource constraint dictates that the income generated in the production process is fully spent in the form of aggregate final private consumption, C_t, final public consumption, G_t, investment, $q_{k,t}I_t$, and resources to do both; manage the capital position of the bank, $\delta^e e_{t-1}$ (also interpretable as eroded equity), and hold cash $f(m_t)$

$$Y_t = C_t + q_{k,t}I_t + G_t + \delta^e e_{t-1} + f(m_t).$$

The supply in all markets is endogenous with the exception of housing supply, which is specified as a fixed endowment that is normalized to unity

$$\overline{H} = h_{p,t} + h_{i,t} + h_{e,t}.$$

3.1.9 Shocks

There are nine different types of zero-mean, AR(1), shocks that hit this model economy under the baseline (no CBDC) scenario: Housing preference shocks, $\varepsilon_{h,t}$; liquidity preference shocks, $\varepsilon_{z,t}$; shocks to the elasticity of substitution across monetary instruments, $\varepsilon_{\eta,t}$; shocks to housing collateral, $\varepsilon_{mh,t}$; shocks to physical capital collateral, $\varepsilon_{mk,t}$; technology shocks, A_t; reserves requirement shocks, $\varepsilon_{\theta_R,t}$; and central bank collateral shocks, $\varepsilon_{\theta_R,t}$. Two additional shocks are also considered in very concrete sections of the paper and for particular purposes: (i) a CBDC supply shock, $\varepsilon_{cbdc,t}$, is modelled in section 3.3 to investigate the transmission of CBDC issuance cyclical effects, and (ii) a CBDC preference shock, ϑ_t, is assumed in section 4.3 to study the stabilization capacity of alternative CBDC policy rules in the event of an exogenous shift in the demand for CBDC.

3.2 Calibration

We follow a three-stage strategy in order to calibrate the model to quarterly euro area data for the period 2000:I-2021:II.[15] First, several parameters are set following convention (table 3). The inverse of the Frisch elasticity of labor is set to a value of 1, whereas the risk aversion parameter of household preferences is fixed to a standard value of 2. Parameter ω_T is set to a value of 0.5 so that each group of households accounts for 50% of collected taxes. Regarding the dynamic depreciation rate of physical capital δ_t^k; δ_0^k is fixed to a standard value of 0.025 while, following convention, δ_1^k and δ_2^k are defined as specific fractions of the steady state interest rate on physical capital. Based on the evidence for the euro area and on the literature, the loan-to-value on residential mortgages, m_H, is set to a value of 0.7 (see, e.g., Gerali et al. 2010; Muñoz 2021). Since the risk weights of reserves and government bonds are both equal to 0 under the Basel III accord, the fraction of bank reserves and government bonds that can be financed with bank debt is assumed to be equal to one (i.e., $\gamma_{\widetilde{R}} = 1$, $\gamma_b = 1$). The elasticity of substitution between intermediate goods is fixed to a value of 6. The Calvo parameter, the inflation indexation parameter and the three parameters of the Taylor rule (i.e., ρ_r, α_Y, and α_π) are fixed to values of 0.82, 0.23, 0.9, 0.1 and 2.5, within the range of values typically obtained when calibrating or estimating a DSGE model to quarterly data of the euro area (see, e.g., Smets and Wouters 2003, Gerali et al. 2010, Coenen et al. 2018). The autoregressive coefficients in the AR(1) processes followed by all shocks are set equal to 0.90.

Second, another group of parameters is calibrated by using steady state targets (tables 4, 5 and 6). Some of these targets are intended to ensure that the size of asset holdings relative to the size of the Eurosystem´s balance sheet and to that of the euro area banking sector' s balance sheet is taken into account for all key financial assets and monetary instruments considered in the baseline model.[16]In this regard, the size of the central bank balance sheet is proxied by the sum of cash (i.e., banknotes in circulation) and reserves (i.e., liabilities to euro area credit institutions related to monetary policy operations denominated in euro), which are the two central bank liabilities available under the baseline scenario.[17] Similarly, the size of the euro area banking sector's consolidated balance sheet is proxied by the sum of total bank loans to households and firms, government debt held by credit institutions and bank reserves.

The patient households' discount factor, $\beta_p = 0.993$, is chosen such that the annual interest

[15] All time series expressed in Euros are seasonally adjusted and deflated. With regards to the matching of second moments, the log value of deflated time series has been linearly detrended before computing standard deviation targets. All details on the dataset constructed for calibration purposes are available in Appendix B.

[16] Of course, many of these model-based steady state ratio targets are larger than what they are in reality, as there are various assets held by banks and the central bank in practice whose modelling has been omitted. However, since the weight of each asset from each other is respected and the bulk of key assets participating in the transmission process are modelled, this simplification should not affect the main findings of the quantitative analysis.

[17] From a quantitative perspective, the sum of these two central bank liabilities have represented the bulk of total central bank liabilities over the entire sample (i.e., 2000:I-2021:II).

rate equals 2.3%. The impatient households' discount factor is set to 0.980, in order to generate an annualized lending-deposit spread of 3.05%. Household weights on housing utility, j_p and j_i, have been calibrated to match the private consumption-to-GDP ratio and the household loans-to-GDP ratio, respectively.

The weight of liquidity services in the utility function of patient households is set to 0.0541, which is consistent with a cash-to-GDP ratio of 0.3443. The weight of deposits and the elasticity of substitution across monetary instruments - both entering the liquidity services aggregator - have been calibrated to match the bank deposits-to-assets ratio and the annualized reserves-deposit spread. The cash storage cost parameter is fixed to a value of 0.002, in order to have a cash-to-central bank assets ratio of 0.51. The loan-to-value ratio on loans to entrepreneurial firms, m_K, is set to 0.214, which is consistent with a weight of loans to firms in total bank assets of 0.37. The shares in production of physical capital, α, and commercial real estate, η, are set to match an investment-to-GDP ratio of 0.21 and a corporate loans-to-GDP ratio of 1.78.

With regards to bank parameters, we proceed as follows. The fractions of residential mortgages and corporate loans that can be financed with bank debt are fixed to 0.92 and 0.895, which are consistent with a household lending-to-bank assets ratio of 0.43 and a bank equity-to-loans ratio of roughly 0.105.[18] The depreciation rate of bank capital, δ^e, is set to 0.071 in order to allow for a bank reserves-to-assets ratio of 0.068. Reserves and central bank collateral requirements, θ_R and θ_b, are set to 0.0874 and 0.995 to match a reserves-to-GDP ratio of 0.33 and a central bank funding-to-assets ratio of 0.086.

As far as policy parameters are concerned, the response parameters of the fiscal rules, ϕ_{Bp} and ϕ_{Bb}, are chosen to generate a bank government bonds-to-GDP ratio of 0.647 and a bank government bonds-to-assets ratio of 0.13, respectively. The parameter of the government spending equation, ϱ, is fixed to 0.207 in line with the data target for the steady state consumption-to-GDP ratio. The gross inflation target, $\bar{\pi}$, is set to 1.005 so to generate an annualized inflation rate of 2%, in line with the Eurosystem' s quantitative objective of price stability. The parameter that determines the constant corridor between the lending policy rate and the deposit facility rate, μ, is set to match an annualized spread between the two policy rates of 1.39%. Finally the parameter of the CBD quantity rule, ϕ_{YSS}, is fixed to 0 to ensure that, under the baseline scenario, there is no CBDC in circulation and to allow for a reserves-to-central bank assets ratio of 0.49.

Third, the size of shocks and other parameters affecting the dispersion of key aggregates are calibrated to improve the fit of the model to the data in terms of relative volatilities (see tables 7 and 8). The investment adjustment cost parameter ψ_I is set to target a relative standard deviation of investment of 2.02 % while the relative volatility of bank dividends is matched by calibrating

[18] These values and the fact that, in the baseline calibration model, sectorial capital requirements on corporate loans are relatively higher than those imposed on residential mortgages is consistent with existing (Basel III) capital regulation.

the elasticity of intertemporal substitution (EIS) of banks. The size of the nine different types of shocks that hit this model economy under the baseline scenario have been calibrated to match the second moment (in terms of relative standard deviations) of GDP, total consumption, cash, reserves, central bank assets, bank loans, bank equity, bank deposits and the interest rate on bank deposits.

3.3 Transmission

We explore the mechanisms through which aggregate effects of CBDC issuances are transmitted to the banking sector and the real economy for the case of policy rule (30). In order to do so, we consider three CBDC scenarios that differ from one another in their associated CBDC issuance levels. Let $\Psi = \left(\phi_{Y,1},\ \phi_{Y,2},\ \phi_{Y,3}\right)$ be a vector containing information on the value that CBDC policy parameter $\phi_{Y,h}$ takes under scenario h, for $h = 1,\ 2,\ 3$. For the purpose of this exercise, we assume that $\widetilde{\Psi} = (0.25,\ 0.45,\ 0.632)$.

Each of these scenarios CBDC policy scenarios is compared against the baseline scenario of no CBDC supply (i.e., $\phi_Y = 0.00$).[19]

First, we study the transmission of CBDC-induced steady state effects. Figure 3 plots the percentage change in the steady state level of selected aggregates that emerges when comparing each of the three CBDC policy scenarios with the baseline (no CBDC) scenario.[20] This gives us information on how the steady state levels of relevant macro and banking aggregates evolve as the steady state CBDC-to-GDP ratio increases. Due to the imperfect substitutability between the three monetary instruments, as the amount of CBDC in circulation increases, cash and deposit holdings decline.[21] Expression (14) stipulates that, for every unit of deposits that are withdrawn, bank reserves are going to decrease by θ_R units.[22]

These considerations have two main implications for the accounting of the central bank. First, its balance sheet expands since the issuance of CBDC (i.e., a central bank liability) is never fully compensated by the joint decline in cash and reserves.[23] Second, its profits also increase for two

[19] Under the baseline calibration, the steady state CBDC quantity implied by $\phi_Y = 0.632$ is consistent with a steady state CBDC interest rate qual to zero, $r^{ss}_{cbdc} = 0$. For all $\phi_Y < 0.632$, it holds that $r^{ss}_{cbdc} < 0$.

[20] Note that these percentage changes in levels are only attributed to shifts from one steady state (i.e., baseline - no CBDC - scenario) to another (i.e., CBDC policy scenario) and disregard any possible impacts that may occur during the transition.

[21] Note two important considerations. First, as CBDC supply increases, the substitution for cash and deposits becomes more pronounced since the equilibrium rate at which CBDC holdings are remunerated also increases. Second, the fall in steady state cash holdings is more severe than that in steady state bank deposits mainly due to the spread between the two: Recall the presence of cash storage costs, and the comparatively higher gross return on bank deposits.

[22] Recall that, in this model economy, reserve requirements are binding in a neighbourhood of the steady state.

[23] Note that this is always going to be the case precisely due to the imperfect substitutability between CBDC and the other two forms of money and the range of values that the reserves requirement parameter, $\theta_R \in (0, 1)$, can take.

reasons: (i) the expansion of the central bank balance sheet is profitable from both, the asset and the liabilities side, since for the three considered policy scenarios it holds that $r_f^{ss} > 0$ and $r_{cbdc}^{ss} \leq 0$; and (ii) there is a change in the composition of central bank liabilities entailing a partial shift from costly liabilities (i.e., reserves) to profitable or costless liabilities (i.e., CBDC).[24]

These implications for the accounts of the monetary authority translate into two effects with consequences for banks' valuations, lending and economic activity. On the one hand, there is a fiscal expansion effect. As public revenues in the form of central bank profits increase, the government decides to collect less taxes from households (see expression 25), thereby fostering private consumption, real economic activity and demand for bank lending. On the other hand, there is a bank disintermediation effect. To the extent that the central bank collateral requirement is binding in a neighborhood of the steady state, an expansion of central bank assets automatically translates into an increased demand for central bank collateral (i.e., government bonds) through expression (15): $\frac{\partial b_b}{\partial F} > 0$.[25] The weight of government bond holdings and central bank funding in the balance sheet of the representative bank increases at the expense of lending to the private sector and deposits, respectively. Given that the weighted average return on loans to households and firms is larger than the return on government bond holdings and the cost of central bank funding is higher than that of household deposits, it follows that banks face a compression in their net interest margins through this channel.[26] Such negative impact on bank profitability has an adverse effect on bank valuations, lending and real GDP (bank disintermediation effect). Although moderate, the net steady state impact of CBDC on bank valuations, lending to NFCs and real GDP is negative as the bank disintermediation effect dominates the fiscal expansion effect.[27]

These findings resemble three important conclusions that can be drawn from the empirical evidence presented in section 2. First, the magnitude of the CBDC structural impact (or net steady state effect) on banks' valuations and lending crucially depends on the amount of CBDC in circulation. Second, the dominance of the bank disintermediation effect underscores the importance

[24] Recall that, under the baseline calibration and for the considered CBDC policy scenarios, $r_{\widetilde{R}}^{ss} > 0$ whereas $r_{cbdc}^{ss} \leq 0$.

[25] Note that for tax revenues to fall and banks' government bond holdings to soar as the amount of CBDC in circulation increases it must hold that government debt held by patient households declines.

[26] Recall from table 4 how do the main spreads look like under the baseline calibration. Also note that, given the central bank collateral requirement and the baseline parameterization of θ_b, the interest rate on government bonds is almost identical to the lending facility rate, in equilibrium.

[27] These two effects directly follow from the CBDC-induced expansion in central bank profits and balance sheet and highlight the relevance of the fiscal rule's specification (see equation 23) to understand the transmission. In order for the government budget constraint to hold (expression 25), the increase in funds available to the government via central bank profits and banks' government bond holdings requires collected taxes to decrease. Since banks' public debt holdings must increase as the central bank balance sheet expands, it follows that the downward adjustment in tax revenues can only occur through a decline in the supply of government bonds to patient households. From the demand side, as CBDC holdings soar, savers' holdings of other assets - including government bonds - also tend to decrease (see the first order conditions of the representative saver's problem in Appendix C).

of the deposit ratio as a key factor to understand the transmission of CBDC-induced net steady state effects on the banking sector. Third, at the aggregate level, the net steady state impact of CBDC on bank valuations, lending to NFCs and real GDP is contained.

Finally, we inspect the transmission of CBDC-induced cyclical effects by studying the impulse responses of key selected aggregates to a CBDC supply shock (see figure 4). In order to do so, we slightly modify equation (30):

$$CBDC_t = \phi_{Y,t} Y^{ss}, \tag{31}$$

where CBDC in period t is now assumed to be a possibly time-varying fraction, $\phi_{Y,t} = \phi_Y \varepsilon_{cbdc,t}$, of steady state real GDP, with $\phi_Y \geq 0$ and $\varepsilon_{cbdc,t}$ capturing exogenous CBDC supply shocks. The size of these shocks, σ_{cbdc}, is set equal to 0.1. The transmission channels are analogous to those through which steady state effects are transmitted although in this case the fiscal expansion effect dominates the bank disintermediation effect. As the central bank issues CBDC, savers benefit from increased liquidity services, $z_t \left(m_t, cbdc_t, d_t\right)$.[28] Although the increase in CBDC holdings exerts a downward pressure on cash and deposit holdings, the net impact on the latter is positive due to the large fiscal expansion effect. As taxes collected from households decline and disposable income increases, private consumption and real GDP soar, ultimately leading to a net positive effect on bank profits (and valuations), aggregate lending and real GDP. Note that the sign of these net cyclical effects is positive due to a sufficiently large expansion of bank's balance sheets and despite the fact that the usual rebalancing effects on the assets and liabilities side still apply (i.e., the weights of government debt and central bank funding increase at the expense of those of loans and deposits, respectively). As real GDP and inflation increase, monetary conditions tighten through adjustments in the lending policy rate (see expression 26).

4 Welfare Analysis

This section evaluates the welfare effects and trade-offs of issuing CBDC to, then, derive the optimal CBDC policy rule under different welfare criteria and for various types of rules. This quantitative exercise permits us to obtain a sensible range of values for the optimal amount of CBDC in circulation and to study the main steady state and cyclical consequences of supplying CBDC under optimal CBDC policy rules.

[28] Due to the specification of the liquidity services aggregator (see expression 5), the calibration of its parameter values, and the distinctive features of each of the three monetary instruments, the decline in cash and deposit holdings does not fully compensate for the increase in CBDC holdings.

4.1 CBDC Policy Regimes

First, we construct various CBDC policy scenarios that are compared with the baseline scenario of no CBDC supply (i.e., expression 30 with $\phi_Y = 0.00$). Each alternative CBDC policy scenario differs from one another in the specification of the CBDC policy rule (i.e., equation 30 for the case of the baseline scenario). Our analysis considers both, CBDC quantity rules and CBDC interest rate rules.

4.1.1 CBDC quantity rules

Quantity rule (i) CBDC in period t is specified as a constant fraction, $\phi_Y > 0$, of quarterly real GDP in period t :

$$CBDC_t = \phi_Y Y_t. \tag{32}$$

Quantity rule (ii) Under this scenario, CBDC in period t is specified as a constant fraction, $\phi_Y > 0$, of steady state quarterly real GDP for $t = 0, 1, 2, ..$

$$CBDC_t = \phi_Y Y^{ss}. \tag{33}$$

While, under quantity rule (i), CBDC supply is time-varying and comoves with real GDP, under quantity rule (ii) CBDC issuance is constant over time. As mentioned in section 3.1, the latter case is particularly relevant from a policy perspective, since this policy option would be similar to adopting a constant limit on individual CBDC holdings, a proposal that has been discussed by policymakers in the recent past.

Quantity rule (iii) In this case, the central bank is assumed to set CBDC supply according to the rule:

$$CBDC_t = \rho_{cbdc} CBDC_{t-1} + (1 - \rho_{cbdc}) \left[\phi_Y Y^{ss} + \phi_X \widetilde{X}_t \right], \tag{34}$$

where ρ_{cbdc} is the CBDC supply smoothing parameter, $\phi_Y Y^{ss}$ is the steady-state CBDC quantity (expressed as a proportion of steady state real output), and ϕ_X determines the response of CBDC supply to deviations of a macroeconomic indicator of the choice of the regulator, X_t, from its steady state level, $\bar{X}$; with $\widetilde{X}_t = \log(X_t/\bar{X})$.

4.1.2 CBDC interest rate rules

Interest rate rule (i) Under this scenario, the interest rate at which CBDC holdings are remunerated is constant and equal to zero. Formally:

$$r_{cbdc,t} = 0. \tag{35}$$

The choice of this scenario is motivated by the fact that the existing version of central bank money (i.e., cash) is not remunerated. In what follows, we will also refer to this case as the unconstrained CBDC supply scenario and it will also be taken as a reference when assessing certain effects of optimal CBDC policy rules.

Interest rate rule (ii) The central bank sets the interest rate on CBDC holdings in period t as a constant fraction, $\phi_r > 0$, of the steady state interest rate on reserves, for $t = 0, 1, 2, ..$:

$$r_{cbdc,t} = \phi_r r_{\widetilde{R}}^{ss}. \tag{36}$$

Interest rate rule (iii) The monetary authority sets the CBDC interest rate according to the following rule:

$$r_{cbdc,t} = \phi_r r_{\widetilde{R},t}, \tag{37}$$

where $\phi_r > 0$ determines the response of the CBDC interest rate to changes in the deposit facility rate.[29] While the rate at which CBDC holdings are remunerated under interest rate rules (i) and (ii) is constant over time, under interest rate rule (iii) such rate commoves with the interest rate on reserves. Since the central bank sets $r_{\widetilde{R},t}$ so to maintain a constant corridor between the lending policy rate and the deposit facility rate (i.e., expression 27), it follows that - under interest rate rule (iii) - the interest rate on CBDC holdings commoves with the lending facility rate and, thus, is indirectly set according to a Taylor-type policy rule (i.e., expression 26).

4.2 Welfare Effects and Optimal CBDC Policy Rules

Then, we adopt a normative approach to investigate the welfare consequences of issuing central bank digital currency and the main implications of doing so under welfare-maximizing CBDC policy rules. In order to do so, a measure of social welfare - specified as a weighted average of the

[29] See Bindseil, U., and F. Panetta (2020), "CBDC remuneration in a world with low or negative nominal interest rates." VoxEU column, 5 October 2020 for a policy proposal on a CBDC remuneration scheme that takes the deposit facility rate as a reference (see table A.1).

expected life-time utility of savers and borrowers - is maximized with respect to the corresponding policy parameter/s. Formally:

$$\arg\max_{\Theta} V_0 = \zeta_p V_0^p + \zeta_i V_0^i, \tag{38}$$

where $V_0^p = E_0 \sum_{t=0}^{\infty} \beta_p^t u\left(c_{p,t}, h_{p,t}, n_{p,t}, z_t\right)$ and $V_0^i = E_0 \sum_{t=0}^{\infty} \beta_i^t u\left(c_{i,t}, h_{i,t}, n_{i,t}\right)$ are the expected life-time utility functions of patient and impatient households, respectively. ζ_p and ζ_i denote the utility weights of each household; and Θ refers to the vector of policy parameters with respect to which the objective function is maximized. Problem (38) is subject to all the competitive equilibrium conditions of the extended model. As in Schmitt-Grohe and Uribe (2007), welfare gains of each agent type are defined as the implied permanent differences in consumption between two different scenarios. Formally, and for the case of patient households, consumption equivalent gains can be specified as a constant λ_p, that satisfies:

$$E_0 \sum_{t=0}^{\infty} \beta_p^t u\left(c_{p,t}^a, h_{p,t}^a, n_{p,t}^a, z_t^a\right) = E_0 \sum_{t=0}^{\infty} \beta_p^t u\left[\left(1+\lambda_{\varkappa}\right) c_{p,t}^b, h_{p,t}^b, n_{p,t}^b, z_t^b\right], \tag{39}$$

where superscripts a and b refer to the alternative CBDC policy scenario and the baseline case, respectively.

In order to assign values to ζ_p and ζ_i, we rely on two alternative but complementary criteria that are typically used in the literature. Welfare weighting criterion "A" solves problem (38) by further assuming that $\zeta_p = 0.5$ and $\zeta_i = 0.5$. That is, this criterion assigns the same weight to each of the two agent types.[30] Welfare criterion "B" goes one step further in treating both types of agents equally and solves (38) by further assuming that $\zeta_p = (1-\beta_p)$ and $\zeta_i = (1-\beta_i)$. That ensures the same utility weights across households discounting future utility at different rates.[31]

Figure 5 plots the individual and social welfare effects of changing the value of parameter ϕ_Y for quantity rules (i), (ii) and (iii), and welfare criteria "A" and "B", with $X_t = Y_t$. While there is a considerable range of positive ϕ_Y values for which both agent types are better off than under the baseline (no CBDC) scenario, figure 5 also shows that each type of household faces a different trade-off when being exposed to changes in ϕ_Y. The issuance of CBDC induces three key effects that allow for understanding these welfare implications. First, it permits to satisfy the demand for a monetary instrument that provides patient households with liquidity services and for which there is no perfect substitute in the economy (i.e., liquidity services effect). Second, it partially replaces

[30] Since the population weights of savers and borrowers are implicitly assumed to be identical, this criterion is equivalent to assuming a utilitarian social welfare function. For references proposing this welfare criterion in models with the same type of individual heterogeneity see, e.g., Antunes and Cavalcanti (2013) and Elenev et al. (2016).

[31] This is a welfare weighting criterion typically considered in the macro-banking literature to prevent an over-weight of savers' welfare related to a higher discount factor (see, e.g., Lambertini et al. 2013; Alpanda and Zubairy 2017).

cash and bank deposit holdings, produces an upward pressure on deposit and lending interest rates, and ultimately exerts a negative level effect on lending to households and firms (i.e., bank intermediation effect). Third, it induces a stabilizing effect on cash and bank deposits, ultimately leading to a smoothing effect on lending supply and, hence, on variables of the real economy such as consumption and hours worked (i.e., stabilization effect).

The stabilization effect has two dimensions. First, and regardless of the type of CBDC policy rule, the proportion of the adjustment to exogenous shocks that hit liquidity services, z_t, which is borne by CBDC holdings increases with the amount of central bank digital currency in circulation.[32] Second, the relationship between the quantity of CBDC and the fraction of the adjustment to exogenous shocks that is borne by the central bank digital liability varies with the type and specification of the CBDC policy rule. In particular, the proportion of such adjustment for a given amount of central bank digital currency in circulation increases with the capacity the policy rule grants the market of this monetary instrument to adjust via quantities. Figure D.1 reports the main level and volatility effects that are behind the welfare effects reported in figure 5 and clearly illustrates the workings of the stabilization effect's two dimensions.

In the case of savers (i.e., CBDC holders), the liquidity services (level) effect clearly dominates and, thus, welfare increases with the level of CBDC supply. In the case of borrowers (i.e., impatient households), up to a certain level, the stabilization effect dominates and issuing CBDC is welfare-improving also for non-holders of CBDC. Nonetheless, beyond a certain threshold - which depends on the specification of the CBDC quantity rule - the bank disintermediation effect starts to weigh comparatively more and higher values of ϕ_Y translate into lower levels of borrowers' welfare.

For the particular case of quantity rule (iii), ϕ_X has been fixed to a value of -5.00. Under this calibration of quantity rule (iii), CBDC supply expands when real output falls below its steady state level, thereby granting the quantity of CBDC a more prominent capacity to adjust in response to varying macroeconomic conditions. Panel B of figure 5 shows that, indeed, such countercyclical response allows for borrowers to attain comparatively higher welfare levels under this policy scenario due to a comparatively larger stabilization effect. Consequently, social welfare levels attainable under welfare criteria A and B are also higher (panels C and D). By displaying the individual and social welfare effects of simultaneous ceteris paribus changes in ϕ_Y and ϕ_X for the case of quantity rule (iii), figure 6 confirms that allowing for CBDC supply to adjust in a countercyclical fashion permits borrowers - and the society as a whole - to reach comparatively higher welfare levels.

Based on the information provided by these welfare trade-offs, we numerically solve problem (38) for the two proposed welfare criteria by searching over the relevant grid of parameter

[32] Note that, as the proportion of the adjustment to exogenous shocks that hit liquidity services, z_t, which is borne by CBDC holdings increases, the one borne by the other two monetary instruments decreases. Such smoothing effect on deposits stabilizes lending supply and aggregates of the real economy.

values. For the cases of quantity rules (i) and (ii), the considered grid of parameter values is $\phi_Y \{0.00 - 0.40\}$; whereas for the case of quantity rule (iii) it is $\phi_Y \{0.00 - 0.40\}$; ϕ_x $\{(-5.00) - 0.00\}$. Table 9 reports the corresponding optimized parameter values and the welfare gains.[33] Since the liquidity services effect is quantitatively the most important one, welfare gains attained by savers (i.e., CBDC holders) under optimal CBDC quantity rules are significantly larger than those attained by borrowers. Not surprisingly, under welfare criterion B, optimal quantity rules are associated to comparatively lower amounts of CBDC in circulation; By preventing an overweight of savers' welfare related to a higher discount factor, this welfare criterion implicitly weighs the bank disintermediation effect more heavily.

The same analysis is carried out for the interest rate rules. Figure 7 plots the individual and social welfare effects of changing the value of parameter ϕ_r for interest rate rules (i), (ii) and (iii) under welfare criteria "A" and "B", whereas table 10 reports the corresponding optimized parameter values and the welfare gains.[34] Figure D.2 provides complementary information by displaying the main level and volatility effects that are behind the welfare effects induced by the three types of interest rate rules. Given the policy relevant grid of parameter values over which it has been searched, ϕ_r $\{(-2.00) - 0.25\}$, for steady state CBDC price-quantity vector, $\Xi = (r_{cbdc},\ CBDC)$, welfare gains that borrowers can attain under interest rate rules are larger than those attainable under quantity rules. This is the case due to the comparatively larger stabilization effect of the former.[35] This larger stabilization effect, in turn, implies that - under optimal interest rate rules - the quantity of CBDC is larger. That is, borrowers optimally tolerate a larger bank disintermediation effect due to a more beneficial stabilization/bank disintermediation trade-off whereas savers (i.e., CBDC holders) benefit from a comparatively more prominent liquidity services effect.

This social preference for interest rate rules is reminiscent of the conclusions reached in Poole (1970). CBDC interest rate rules are prefered as they have a stronger capacity to stabilize the other components of money demand (i.e., cash and bank deposits). It is worth noting, however, that this result is particularly robust across different types of shocks. For each of the nine shocks that hit this model economy under the baseline calibration, figure D.3 plots the individual and

[33] In each case, the model is solved by using second-order perturbation techniques in Dynare. Unconditional lifetime utility is computed as the theoretical mean based on first order terms of the second-order approximation to the nonlinear model, resulting in a second-order accurate welfare measure. This approach ensures that the effects of aggregate uncertainty are taken into account.

[34] Note that, under interest rate rule (i), there is no policy parameter with respect to which it can be optimized and welfare gains attained by each type of household are independent from the welfare criterion.

[35] To clearly understand the difference between CBDC interest rate rules and quantity rules in terms of the capacity they grant CBDC holdings to adjust, one may wish to consider the two "extreme" cases of quantity rule (ii) and interest rate rule (ii). Under the former, the quantity of CBDC cannot adjust at all. The adjustment in response to exogenous shocks is fully made via prices (i.e., via changes in the CBDC interest rate). In the latter, adjustments in the market for CBDC fully take place via quantities, thereby exerting a comparatively larger smoothing effect on deposits, lending an aggregates of the real economy.

social welfare effects of changing the value of parameter ϕ_Y when only one type of shock hits the economy. Regardless of the type of shock under consideration, the stabilization effect and the welfare trade-off faced by borrowers is present. That is, regardless of the source that originates fluctuations in liquidity services, z_t, issuing CBDC under a policy rule triggers a stabilization effect that makes interest rate-type rules comparatively more attractive.[36]

4.2.1 Steady State Effects

This section offers an overview of the main steady state effects triggered by the issuance of CBDC under the six CBDC policy rules.[37] Panel A of figure 8 displays the steady state CBDC interest rate-quantity vector, $\Xi = (r_{cbdc},\ CBDC)$, associated to each of the six different optimal CBDC policy rules.[38] Panel B shows the steady state impact the introduction of a CBDC has - under each CBDC policy rule - on the present value of banks as well as on bank lending to firms.[39] Three conclusions stand out as they are relevant for the current policy debate and are consistent with the empirical evidence presented in section 2. First, there is a high and positive correlation between the amount of CBDC in circulation and the structural impact of issuing a central bank digital currency on banks' valuations and lending to firms. Second, by adequately calibrating the amount of CBDC in circulation (through an optimal policy rule), these effects can be significantly mitigated (see the difference between the magnitude of steady state effects on banks' valuations and lending under the unconstrained CBDC supply scenario - i.e., interest rate rule (i) - and those under optimal CBDC policy rules). Third, regardless of the CBDC policy rule we look at, the optimal quantity of CBDC in equilibrium lies between 15% and 45% of quarterly real GDP. On 9 February 2021, ECB Board member Panetta made a statement on the possibility of adopting a limit on individual CBDC holdings of EUR 3,000 which led to a trend reversal in the estimated impact of digital euro news on bank valuations and lending to firms (see section 2). If all citizens in the euro area were to hold this maximum individual level of CBDC in 2021, the amount of CBDC in circulation would be roughly 34% of quarterly GDP.[40] Based on euro area data for 2021

[36] Without loss of generality, welfare effects displayed in figure D.3 correspond to quantity rule (ii) and welfare criterion "B".

[37] By steady state effects, we refer to the impacts that are relevant when it comes to the shift from one steady state (i.e., baseline - no CBDC - scenario) to the other (i.e., CBDC policy scenario). That is, this section disregards effects that only occur over the cycle or during the transition from one steady state to the other.

[38] Recall that, strictly speaking, interest rate rule (i) cannot be refered to as an optimal CBDC policy rule (since there is no policy parameter with respect to which it can be optimized), but rather as a CBDC policy scenario.

[39] Note that the choice of these two variables has been inspired by the two variables for which we present our empirical findings in section 2 (i.e., banks' market valuations and bank lending to firms). In our analysis, the present value of banks is proxied by the objective function of the representative bank.

[40] This number has been obtained after having rounded up the size of the population in the euro area to 340 million citizens and average quarterly GDP in 2021 to EUR 3,000 billions.

again, such an amount precisely lies between 15% and 45% of quarterly real GDP.[41]

In addition, the analysis further confirms that optimal CBDC interest rate rules are associated to a larger quantity of CBDC in equilibrium and, thus, to a more sizable bank disintermediation effect. It follows, then, that welfare gains under optimal interest rate rules are comparatively larger due to a more beneficial stabilization/bank disintermediation trade-off faced by borrowers.

Steady State Effects of Related Policies Importantly, regulators and the central bank have the capacity to alter the magnitude (and even the sign) of certain steady state effects not only by calibrating the amount of CBDC in circulation through a policy rule but also by changing the design of other related policies. Panel A of figure 9 displays the steady state impact that ceteris paribus changes in the reserve requirement parameter, θ_R, have on the size of the central bank balance sheet, F.[42] As the reserve requirement increases, a larger proportion of the adjustment in the face of a CBDC issuance is made via a reduction in the stock of bank reserves and, hence, a lower fraction of the adjustment is made by means of a central bank's balance sheet expansion. That is, as the proportion of the adjustment made via drawing down reserves increases, the magnitude of the bank disintermediation effect diminishes. This is valid for any circumstance that motivates banks to increase the fraction of the adjustment to be made via reducing their stock of reserves.[43]

Finally, note that the net steady state impact of CBDC on bank lending crucially depends on the design and calibration of the central bank collateral framework (captured by expression 15). In practice, a central bank collateral framework often allows for different eligible asset classes, usually differing from one another in their associated haircut and in their weight in the collateral pool. Consider the following general version of equation (15):

$$f_t \leq \sum_{i=0}^{N} \theta_{i,t} E_t \left(\frac{Q_{i,t}}{R_{f,t}} \pi_{t+1} \right), \tag{40}$$

where $Q_{i,t}$ denotes holdings of eligible asset "i" by the representative bank in period t, N is the number of eligible assets, and $\theta_{i,t}$ refers to the possibly time-varying fraction of asset "i" holdings that can be financed with central bank funding. Interestingly, under specification (40) of collateral requirements, $\theta_{i,t}$ can be interpreted not only as the complementary of the haircut on asset "i" holdings, but also as the weight of such asset in the collateral pool.

[41] In practice, the CBDC-to-GDP ratio under a EUR 3,000 limit on individual holdings would likely be lower than 34% and probably closer to the levels implied by optimal quantity rules (i.e., 15% - 30%) for at least two reasons. First, not all citizens in the euro area hold money and have bank accounts. Second, due to their preferences and/or to their availability of funds, not all citizens are likely to exhaust the regulatory limit. See Adalid et al. (2022).

[42] Without loss of generality, the steady state effects illustrated in figure 9 are those that apply under welfare criterion B for optimal CBDC policy rule within the class of quantity rules (ii).

[43] For instance, as the stock of reserves has been increasing in advanced economies over the last years (mainly due to the implementation of certain unconventional monetary policy measures), when compared to the pre-Global Financial Crisis era, it could be that in the current situation banks decided to more prominently adjust via reserves.

Depending on which assets are eligible as collateral in monetary policy operations with the central bank and on how they weigh in the collateral pool, the steady state rebalancing effects on banks' balance sheets may vary and the impact on bank lending may differ. Consider the following particular case within the general class of collateral requirements referred by expression (40):

$$f_t \leq \theta_{b,t} E_t \left(\frac{b_{b,t}}{R_{f,t}} \pi_{t+1} \right) + \theta_{l,t} E_t \left(\frac{L_{e,t}}{R_{f,t}} \pi_{t+1} \right), \tag{41}$$

where $\theta_{l,t} = \theta_l \varepsilon_{\theta l,t}$ provides information on the haircut on loans to firms as well as on the weight of this asset class in the collateral pool. Panel B of figure 9 plots the steady state effect of ceteris paribus changes in θ_l, on bank lending to firms, $L_{e,t}$. As θ_l (and, thus, the weight of $L_{e,t}$ in the collateral pool) increases, the structural impact of issuing CBDC on bank lending to non-financial corporations diminishes.

4.2.2 Impulse Responses

As shown in section 3.3, while the magnitude of CBDC cyclical effects also depends on the amount of central bank digital currency in circulation, even the sign of these effects may differ from that of steady state impacts. In this section we give a brief overview of how key selected aggregates respond to exogenous shocks under optimal CBDC policy rules. We differentiate between two broad groups of shocks. First, shocks which - due to the absence of data on CBDC holdings and flows - have been omitted from the baseline calibration and the welfare analysis, but which are a key determinant of CBDC supply and demand dynamics (i.e., CBDC supply shocks, $\varepsilon_{cbdc,t}$, and CBDC preference shocks, ϑ_t).[44]Second, all other shocks; which are considered in the baseline calibration and the welfare analysis and whose impacts (especially on real aggregates) through the issuance of CBDC are of a different order of magnitude.

On the similarities between the implications of these two main groups of shocks under optimal CBDC policy rules and regardless of the particular type of shock under consideration, it is worth noting that the same transmission channels described in section 3.3 apply. On the differences, there are two aspects that stand out. First, CBDC policy rules clearly matter for economic stabilization only under CBDC supply and preference shocks. Figures 4 and 10 suggest that, the magnitude of the response of real GDP and other aggregates of the real economy to exogenous CBDC supply and preference shocks increases with the amount of CBDC in equilibrium (which depends on the specification and calibration of the policy rule). Second, a positive CBDC (supply or preference) shock tends to have a positive net cyclical impact on bank lending and real GDP as the fiscal expansion effect dominates the bank disintermediation effect. This is the case due to the fact that these types of shocks have a relatively more sizable impact on the central bank balance sheet and

[44]As for the case of CBDC supply shocks, the size of CBDC preference shocks, σ_ϑ, is set equal to 0.1.

profits (and, thus, on tax revenues).

With regards to the rest of the shocks (i.e., those considered in the baseline calibration), the difference in the impact exogenous shocks have on real economic activity across different CBDC policy scenarios tends to be moderate to negligible. These shocks only indirectly affect CBDC dynamics and, hence, the transmission through the central bank balance sheet and profits is more moderate despite the fact that the stabilization effect applies to all types of shocks (recall figure D.3). Figure D.4 illustrates these results by plotting the responses of selected aggregates to a negative technology shock. First, the increase in CBDC holdings does not lead to an economic expansion and the choice of the CBDC policy rule has no significant consequences from a stabilization perspective. Second, the bank disintermediation effect is present, and the increase in CBDC holdings leads to the previously discussed readjustment in the composition of banks' assets and liabilities. Third, the stabilization effect applies and the deviation of bank deposits from their steady state levels is more moderate under CBDC policy scenarios. In line with the previously discussed findings on the welfare effects of CBDC policy rules, such stabilization effect is more sizable under interest rate rules (starred and dotted lines) than under optimal quantity rules (diamond lines).

4.2.3 Robustness Checks

In this section, we first investigate the robustness of the welfare effects of a CBDC quantity rule of type (i) to changes in key parameters.[45] This paper focuses the attention on assessing the effects and trade-offs of issuing CBDC under different policy rules. Nevertheless, there are other CBDC design features which we have not explicitly modelled but which can fundamentally affect parameters that greatly matter to CBDC demand. These parameters include the elasticity of substitution across monetary instruments, η_z, and the cash storage cost parameter, ψ_m (which captures the technological superiority of CBDC relative to cash).

As the degree of substitutability across different forms of money declines (lower η_z), the lower weight of bank deposits, ω_D, in the liquidity services CES aggregator, z_t, implies that the premium deposits need to offer for the marginal utility of holding the different monetary instruments to remain unchanged is higher. Deposit holdings decrease even if the rate at which they are remunerated is higher under an equilibrium with a lower η_z. The interest rate on CBDC adjusts downwards to also allow for the required premium on bank money holdings, given the fixed CBDC supply. Cash holdings increase up to the point that a higher marginal cash storage cost keeps ensuring that, in the margin, savers are indifferent between holding any type of monetary instrument. Savers benefit from an overall increase in liquidity services. Borrowers are negatively affected since the reduction in bank deposit funding adversely impacts lending supply (see figure 11).

[45] In order to do so, and without loss of generality, we consider quantity rule of type (i).

As the value of parameter ψ_m decreases, welfare levels unambiguously increase (see figure 12). Savers benefit from, overall, higher levels of liquidity services as holding cash (and adjusting its level) becomes less costly. Consequently, the fraction of the adjustments that is borne by cash holdings increases, thereby exerting a smoothing effect on deposit holdings and bank lending. Borrowers benefit from such credit smoothing effect.

Similarly, the value of certain parameters associated to the design and calibration of related policies plays an important role in determining the optimal quantity of CBDC in equilibrium. Motivated by the previous discussion on the key transmission mechanisms, this section focuses on the sensitivity of the main welfare and macroeconomic effects to changes in the value of the central bank collateral requirement parameter, θ_b. Figure 13 shows that, as the central bank collateral policy tightens (i.e., decrease in θ_b or higher haircut on government bonds), the welfare level of CBDC holders increases whereas that of borrowers declines. For a given size of the central bank balance sheet required to supply a certain quantity of CBDC, banks now need to hold more public debt. Even if the balance sheet of the representative bank expands there is a crowding out of bank loans, which negatively affects borrowers' welfare. Under a more stringent collateral policy, there is a fraction of government bond holdings which was previously financed by central bank funding that is now financed via deposits. Savers benefit from larger liquidity services.

Lastly, figure 14 displays - for interest rate rule (i) - the same impulse responses as in figure 10. As the collateral policy of the central bank becomes more restrictive (i.e., lower θ_b), the deviation of central bank funding and banks' government holdings from their steady state levels become more moderate and the effects of the fiscal expansion on real output and bank lending are mitigated.

In a nutshell, although quantitative differences may arise, the main conclusions of this section are robust across key parameter values that depend on CBDC design features and related policies. Regardless of the policy rule under consideration, the issuance of CBDC is subject to certain welfare trade-offs and, from this perspective, there is an optimal quantity of central bank digital currency which is sensitive to the design of the CBDC policy rule as well as to that of other related policies.

5 Conclusion

The recent and growing literature on central bank digital currencies identifies a trade-off between the benefits of having access to a digital currency issued by a central bank for retail payment purposes and the potential risk of bank disintermediation through deposit substitution. We present novel evidence on bank stock price reactions to CBDC news in the euro area suggesting that market participants expect the impact of introducing a CBDC on the banks' valuations and lending conditions to crucially depend on the design features aimed at controlling the amount of central bank digital currency in circulation.

Against this background, we develop a quantitative macro-banking DSGE model that incorporates these trade-offs and a selection of mechanisms through which the issuance of a CBDC is expected to affect bank intermediation and the real economy. Liquidity (reserves) requirements, the central bank's balance sheet and the collateral framework of the monetary authority are mechanisms that interact with one another and play a key role in the transmission of CBDC-induced effects to the banking sector and the macroeconomy.

Welfare-maximizing CBDC policy rules are effective in mitigating the risk of bank disintermediation and induce significant welfare gains for both, patient households (i.e., CBDC holders) and impatient households (i.e., borrowers who do not hold CBDC). Based on a social welfare maximization approach, the model suggests that the optimal amount of CBDC in circulation for the case of the euro area would lie between 15% and 45% of quarterly real GDP in equilibrium. In line with what our empirical analysis suggests, if CBDC were to be issued under no quantity limits and no remuneration, the amount of CBDC in circulation would be larger (i.e., of roughly 65% of quarterly real GDP) and the steady state effects on banks' valuations and lending would be comparatively more sizable. While changes in the value of key parameters could quantitatively affect these results to some extent, the main findings of our quantitative analysis are shown to be particularly robust across different CBDC policy scenarios, welfare criteria and parameterizations of the model.

The simplicity of the model is instrumental to clearly identify the effects of issuing CBDC and the mechanisms through which they are transmitted. Yet, it comes at the cost of omitting ingredients which are present in reality and that could possibly change some of the results. The model could be extended along different dimensions so as to allow for a more accurate quantification of the impact issuing a certain amount of CBDC could have on bank intermediation.

On the one hand, there are assumptions of the model which could possibly be leading to an overstatement of the potential risk of bank disintermediation. Among others, the design of the central bank's collateral requirement (which only considers public debt as eligible asset) and the implicit assumption that it is always binding in a neighborhood of the steady state; the simplifying assumption according to which banks do not obtain revenues from offering CBDC-related services; the absence of other digital currencies and payment methods that would in practice compete with a CBDC in the segment of retail payments; and the omission of a more explicit modelling of some of the unconventional monetary policy measures which had contributed to the build-up of a large stock of excess reserves in the system of many advanced economies, a channel through which a larger proportion of the adjustment could take place in practice.

On the other hand, there are other assumptions due to which the model could be underestimating the impact of introducing a CBDC on the banking sector and the macroeconomy. First, the simple specification of the liquidity (reserves) requirement implies that, in practice, banks are likely to be more limited when deciding how to rebalance the asset and liabilities sides of their balance sheets in the face of a CBDC issuance. Second, the fiscal expansion effect could in practice be of a different nature and order of magnitude, not having the impact on private consumption and real GDP that the model predicts.

Finally, the tractability of the model allows for a more detailed and extended inspection of the interactions between CBDC policy and other related policies and regulations (e.g., monetary policy and the associated collateral framework, fiscal policy, capital and liquidity regulation).

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Table 1: Determinants of abnormal stock market returns during digital euro events

	(1)	(2)	(3)	(4)	(5)	(6)
Deposit ratio	-0.057**	-0.050*	-0.054*	-0.058*	-0.077**	-0.081*
	(0.027)	(0.028)	(0.028)	(0.032)	(0.038)	(0.047)
Assets		1.463	1.316	1.305	-0.298	-1.475
		(0.942)	(0.957)	(0.954)	(0.948)	(1.164)
Reliance on TLTROs			-0.016	-0.020	0.003	-0.001
			(0.038)	(0.041)	(0.043)	(0.058)
Securities holdings			-0.013	-0.016	0.019	0.080
			(0.046)	(0.048)	(0.067)	(0.088)
Excess liquidity holdings			0.014	0.014	0.020	0.048
			(0.025)	(0.026)	(0.029)	(0.038)
ROA			-0.060	-0.043	-0.070	-0.122
			(0.100)	(0.099)	(0.106)	(0.141)
NPL ratio				-0.009		
				(0.025)		
CDS spread					0.003	0.001
					(0.002)	(0.002)
Event FE	YES	YES	YES	YES	YES	YES
Bank FE	YES	YES	YES	YES	YES	YES
Observations	1,601	1,601	1,601	1,601	1,146	1,146
R-squared	0.055	0.057	0.057	0.057	0.074	0.160

Notes: The specification is as in model (2). Dependent variable is bank-specific abnormal returns identified with the estimation of model (1). Observations are an unbalanced sample of 53 banks and 28 events. All controls are lagged by one month with respect to the month in which each event took place. Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 2: Impact on lending from digital euro events

Dependent variable: Growth rate of loans	(1)	(2)	(3)	(4)	(5)
Reaction of stock prices	0.136**	0.355***	0.365***	0.290***	0.199*
	(0.063)	(0.054)	(0.058)	(0.052)	(0.106)
Assets	-3.155***	-3.106***	-3.003***	-2.365***	-2.311*
	(0.762)	(0.470)	(0.353)	(0.781)	(1.264)
Reliance on TLTROs		1.025***	1.041***	0.730*	1.322**
		(0.123)	(0.133)	(0.421)	(0.620)
Securities holdings		-0.400***	-0.422***	-0.537**	0.606**
		(0.139)	(0.141)	(0.224)	(0.285)
Excess liquidity holdings		0.288*	0.286**	0.134	-0.308
		(0.144)	(0.133)	(0.169)	(0.360)
ROA		3.583**	3.890***	4.417**	-4.490*
		(1.358)	(1.411)	(1.625)	(2.378)
NPL ratio			0.173		
			(0.175)		
CDS spread				0.038*	-0.048
				(0.022)	(0.036)
Industry - Location - Sector FE	YES	YES	YES	YES	-
Firm FE	-	-	-	-	YES
Observations	1,523,078	1,523,078	1,523,078	1,358,450	375,877
R-squared	0.110	0.112	0.112	0.120	0.454

Notes: The specification is as in model (3). Dependent variable is the percentage change in corporate loan volumes. Reaction of stock prices is the (cumulated) abnormal returns in October 2020. All controls are measured in September 2020. Standard errors clustered at the bank level in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 3: Baseline pre-set parameter values

Parameter	Description	Value
φ	Inverse of the Frisch elasticity	1.0000
σ_h	HH Risk aversion param	2.0000
ω_T	Fraction of taxes paid by HH_p	0.5000
δ_0^k	Depreciation rate of physical capital	0.0250
$\delta_1^k; \delta_2^k$	Endogenous depr. rate params	$r_{k_e}^{ss}; 0.1\text{x}r_{k_e}^{ss}$
m_H	LTV ratio on HH housing	0.7000
$\gamma_{\widetilde{R}}$	Debt-to-assets, reserves risk-adjusted	1.0000
γ_b	Debt-to-assets, gov. bonds risk-adjusted	1.0000
ε	Elast. of subst. intermediate goods	6.0000
θ	Calvo probability	0.8200
χ_π	Inflation indexation parameter	0.2300
ρ_r	Taylor rule: smoothing parameter	0.9000
α_π	Taylor rule: inflation response param	2.5000
α_y	Tayor rule: GDP growth response param	0.1000

Note: Parameters are set to standard values in the literature. Abreviations HH, HHp and LTV refer to households, patient households and loan-to-value, respectively.

Table 4: Baseline calibrated parameter values: Part I

Parameter	Description	Value	Target ratio
β_p	Savers' discount factor	0.9930	$R_d^{ss} = (1.023)^{1/4}$
β_i	Borrowers' discount factor	0.9800	$(r_{le}^{ss} - r_d^{ss})x\ 400 = 3.0474$
j_p	Savers' housing services weight	0.0100	$C^{ss}/Y^{ss} = 0.5479$
j_i	Borrowers' housing services weight	8.7902	$l_i^{ss}/(Y^{ss}) = 2.0918$
χ_z	Savers' liquidity services weight	0.0541	$M^{ss}/Y^{ss} = 0.3443$
ω_d	Deposits weight in liquidity services	0.7100	$D^{ss}/A^{ss} = 0.8081$
η_z	Elast. of subst. liquidity services	3.5800	$(r_R^{ss} - r_d^{ss})x\ 400 = 0.2650$
ψ_m	Cash storage cost parameter	0.0020	$M^{ss}/F^{ss} = 0.5118$
m_K	LTV ratio on NFC physical capital	0.2140	$l_e^{ss}/A^{ss} = 0.3675$
α	Capital share in production	0.3300	$I^{ss}/Y^{ss} = 0.2124$
ν	Real estate share in production	0.0100	$l_e^{ss}/Y^{ss} = 1.7820$
γ_e	Debt-to-assets, NFC risk-adjusted	0.8950	$e^{ss}/l^{ss} = 0.1050$
γ_i	Debt-to-assets, HH risk-adjusted	0.9200	$l_i^{ss}/A^{ss} = 0.4313$
δ^e	Depreciation rate of bank capital	0.0710	$\widetilde{R}_b^{ss}/A^{ss} = 0.0677$
θ_R	Banks' liquidity (reserves) requirement	0.0874	$\widetilde{R}_b^{ss}/Y^{ss} = 0.3284$
θ_b	Central bank funding collateral requirement	0.9950	$f^{ss}/A^{ss} = 0.0861$
ϕ_{Bp}	Fiscal rule: HH gov. bonds response param	0.4010	$b_b^{ss}//Y^{ss} = 0.6473$
ϕ_{Bb}	Fiscal rule: Banks' gov. bonds response param	0.2300	$b_b^{ss}//A^{ss} = 0.1335$
ϱ	Public consumption-to-GDP ratio	0.2070	$G^{ss}/Y^{ss} = 0.2070$
$\overline{\pi}$	Gross inflation target	1.0050	$(\pi - 1)x\ 400 = 2.0000$
μ	Lending-deposit facility corridor param	0.0059	$(r_f^{ss} - r_R^{ss})x\ 400 = 1.3860$
ϕ_Y	CBDC quantity rule: CBDC supply parameter	0.0000	$\widetilde{R}^{ss}/F^{ss} = 0.4882$

Note: Parameters are calibrated to match steady state data targets. Abreviations HH, NFC and LTV refer to households, non-financial corporations (entrepreneurs) and loan-to-value, respectively.

Table 5: Steady state ratios

Variable	Description	Model	Data
Bank statistics			
l_i^{ss}/Y^{ss}	HH loans-to-GDP ratio	2.0431	2.0918
l_e^{ss}/Y^{ss}	NFC loans-to-GDP ratio	1.7585	1.7820
b_b^{ss}/Y^{ss}	Bank government bonds-to-GDP ratio	0.6825	0.6473
l_i^{ss}/A^{ss}	HH loans-to-bank assets ratio	0.4243	0.4313
l_e^{ss}/A^{ss}	NFC loans-to-bank assets ratio	0.3652	0.3675
$\widetilde{R}_b^{ss}/A^{ss}$	Reserves-to-bank assets ratio	0.0671	0.0677
$b_b^{ss}//A^{ss}$	Bank government bonds-to-bank assets ratio	0.1417	0.1335
D^{ss}/A^{ss}	Deposits-to-bank assets ratio	0.7877	0.8081
f^{ss}/A^{ss}	Central bank funding-to-bank assets ratio	0.1400	0.0861
e^{ss}/l^{ss}	Equity-to-risk weighted assets ratio	0.0916	0.1050
Central bank statistics			
$\widetilde{R}^{ss}/Y^{ss}$	Reserves-to-GDP ratio	0.3315	0.3284
M^{ss}/Y^{ss}	Cash-to-GDP ratio	0.3428	0.3443
$\widetilde{R}^{ss}/F^{ss}$	Reserves-to-CB assets ratio	0.4917	0.4882
M^{ss}/F^{ss}	Cash-to-CB assets ratio	0.5083	0.5118
Macroeconomic statistics			
C^{ss}/Y^{ss}	Private consumption-to-GDP ratio	0.5549	0.5479
I^{ss}/Y^{ss}	Gross fixed capital formation-to-GDP ratio	0.2125	0.2124
G^{ss}/Y^{ss}	Public consumption-to-GDP ratio	0.2070	0.2070

Note: All series in Euros are seasonally adjusted and deflated. Data targets have been constructed from euro area quarterly data for the period 2000:I-2021:II. The exception is the target for the bank capital-to-risk weighted assets, which has been based on the Basel III regime. Abreviations HH, NFC refer to households, and non-financial corporations (entrepreneurs), respectively.Data sources are Eurostat and ECB.

Table 6: Steady state rates and spreads

Variable	Description	Model	Data
$(r_{le}^{ss} - r_{d}^{ss})x\ 400$	Annualized Bank lending (to NFCs) spread	3.2316	3.0474
$(r_{f}^{ss} - r_{R}^{ss})x\ 400$	Annualized lending-deposit facility corridor	2.3600	1.3860
$(r_{R}^{ss} - r_{d}^{ss})x\ 400$	Annualized Reserves-deposits spread	0.2682	0.2650
$r_{d}^{ss}\ x\ 400$	Annualized interest rate on bank deposits	2.2376	2.3000
$(\bar{\pi} - 1)\ x\ 400$	Inflation target	2.0000	2.0000

Note: Data targets for spreads and interest rates have been constructed from euro area quarterly data. While the period for which data targets for spreads have been constructed is 2000:I-2021:II, as standard in this strand of the macro-banking literature, the data target for the nominal interest rate on bank deposits is based on the pre-crisis period. The data target for the inflation target corresponds to the quantitative definition of the ECB's price stability objective. Abreviation NFC refers to non-financial corporations (entrepreneurs). Data sources are Eurostat and ECB.

Table 7: Baseline calibrated parameter values: Part II

Parameter	Description	Value	Source/Target ratio
ψ_I	Investment adj. cost param	0.0920	$\sigma_I/\sigma_Y = 2.0193$
σ	Banker EIS	6.4000	$\sigma_{\Omega_b}/\sigma_Y = 9.6434$
σ_A	Std. productivity shock	0.0016	$\sigma_Y\ x\ 100 = 3.3368$
σ_h	Std. housing pref. shock	0.0090	$\sigma_C\ /\ \sigma_Y = 1.1626$
σ_η	Std. elast. of subst. liquidity services shock	0.0012	$\sigma_D\ /\ \sigma_Y = 2.4620$
σ_z	Std. liquidity pref. shock	0.0043	$\sigma_M\ /\ \sigma_Y = 2.6871$
σ_{mh}	Std. HH collateral shock	0.0072	$\sigma_L\ /\ \sigma_Y = 2.4741$
σ_{mk}	Std. NFC collateral shock	0.0201	$\sigma_e/\sigma_Y = 2.8820$
σ_{θ_R}	Std. reserves requirement shock	0.1540	$\sigma_R\ /\ \sigma_Y = 11.8348$
σ_{θ_b}	Std. Central bank funding collateral shock	0.0015	$\sigma_F\ /\ \sigma_Y = 5.0259$
σ_r	Std. interest rate shock	0.0008	$\sigma_{r_d}\ /\ \sigma_Y = 7.1691$

Note: Parameters are calibrated to match second moment data targets. Abreviations HH, NFC, EIS and Std refer to households, non-financial corporations (entrepreneurs), elasticity of intertemporal substitution and standard deviation, respectively.

Table 8: Second moments (relative volatilities)

Variable	Description	Model	Data
Bank statistics			
σ_{Ω_b} / σ_Y	Std. bank dividends/Std(GDP)	9.7168	9.6434
σ_L / σ_Y	Std.bank loans/Std(GDP)	2.3979	2.4741
σ_e / σ_Y	Std. bank capital/Std(GDP)	2.1877	2.8820
σ_D / σ_Y	Std. bank deposits/Std(GDP)	2.7164	2.4620
σ_{r_d} / σ_Y	Std. bank deposit interest rate/Std(GDP)	5.1142	7.1691
Central bank statistics			
σ_M / σ_Y	Std. banknotes/Std(GDP)	3.2769	2.6871
σ_R / σ_Y	Std. reserves/Std(GDP)	11.9641	11.8348
σ_F / σ_Y	Std. central bank assets/Std(GDP)	5.2022	5.0259
Macroeconomic statistics			
σ_I / σ_Y	Std. investment/Std(GDP)	2.5411	2.0193
σ_C / σ_Y	Std consumption/Std(GDP)	0.8208	1.1626
σ_Y x 100	Std(GDP) x 100	3.3593	3.3368

Note: Series expressed in Euro amounts are seasonally adjusted and deflated, and their log value has been linearly detrended before computing standard deviation targets.These data targets have been constructed from euro area quarterly data for the period 2000:I-2021:II. For each variable, its relative volatility has been computed by dividing its standard deviation (Std) by the standard deviation of quarterly real GDP. The standard deviation of GDP is in quarterly percentage points. The standard deviation of bank dividends has been taken from the dataset used in Muñoz (2021).

Table 9: Welfare gains of optimal CBDC quantity rules

	Savers	Borrowers	Social
(A) Welf criterion "A" (i.e., $\zeta_{\varkappa} = 0.5$)			
(i) $\phi_Y^* = 0.241$	1.2519	0.0603	0.6561
(ii) $\phi_Y^* = 0.276$	1.3808	0.0658	0.7233
(iii) $\phi_Y^* = 0.279$; $\phi_X^* = -5$	1.3917	0.0771	0.7344
(B) Welf criterion "B" (i.e., $\zeta_{\varkappa} = 1 - \beta_{\varkappa}$)			
(i) $\phi_Y^* = 0.178$	1.0046	0.0675	0.0084
(ii) $\phi_Y^* = 0.204$	1.1087	0.0738	0.0092
(iii) $\phi_Y^* = 0.206$; $\phi_X^* = -5$	1.1164	0.0852	0.0095

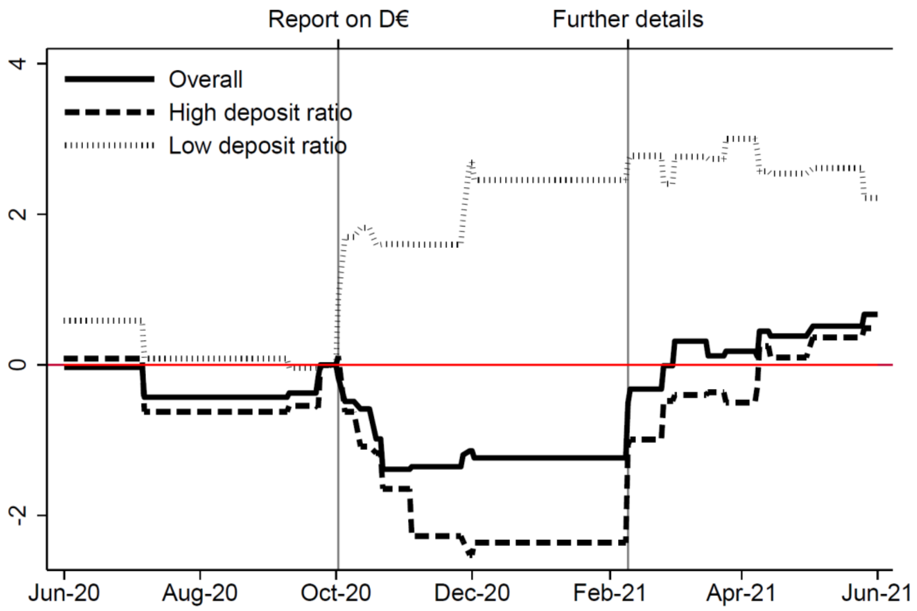
Note: Second-order approximation to the welfare gains associated to the optimal CBDC quantity rules and the corresponding optimized policy parameter for each of the two proposed welfare criteria. Welfare gains are expressed in percentage permanent consumption. Policy parameter values marked with an asterisk correspond to those for which social welfare is maximized under the corresponding welfare weighting criterion.

Table 10: Welfare gains of CBDC interest rate rules

	Savers	Borrowers	Social
(A) Welf criterion "A" (i.e., $\zeta_{\varkappa} = 0.5$)			
(i) $\phi_r = 0.000$	2.5483	0.0773	1.3128
(ii) $\phi_r^* = -0.313$	1.9488	0.1182	1.0335
(iii) $\phi_r^* = -0.384$	1.8433	0.1302	0.9868
(B) Welf criterion "B" (i.e., $\zeta_{\varkappa} = 1 - \beta_{\varkappa}$)			
(i) $\phi_r = 0.000$	2.5483	0.0773	0.0194
(ii) $\phi_r^* = -0.520$	1.6497	0.1268	0.0141
(iii) $\phi_r^* = -0.581$	1.5784	0.1379	0.0138

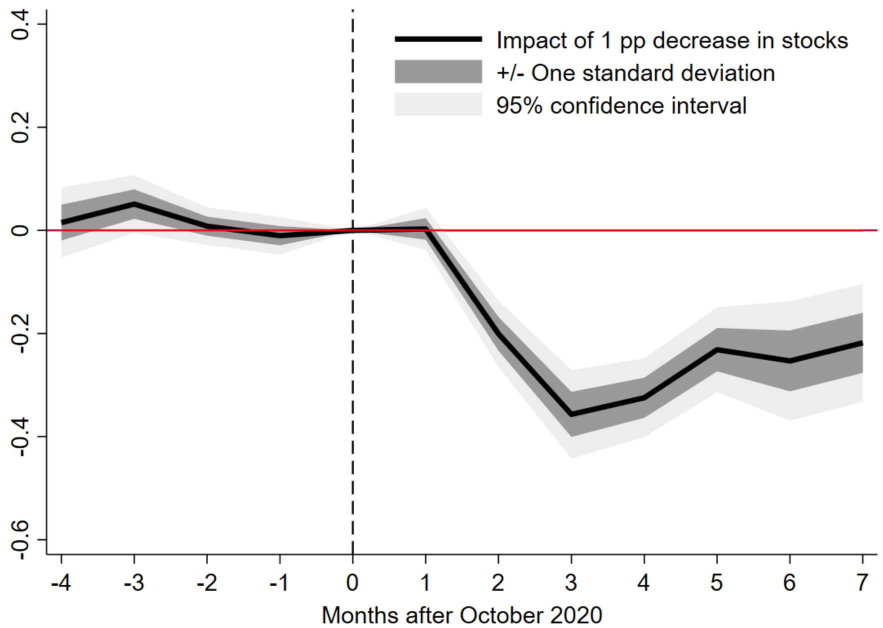
Note: Second-order approximation to the welfare gains associated to the CBDC interest rate rules and the corresponding optimized policy parameter for each of the two proposed welfare criteria. Welfare gains are expressed in percentage permanent consumption. Policy parameter values marked with an asterisk correspond to those for which social welfare is maximized under the corresponding welfare weighting criterion.

Figure 1: Stock market reactions to CBDC news by euro area banks (percentage points)



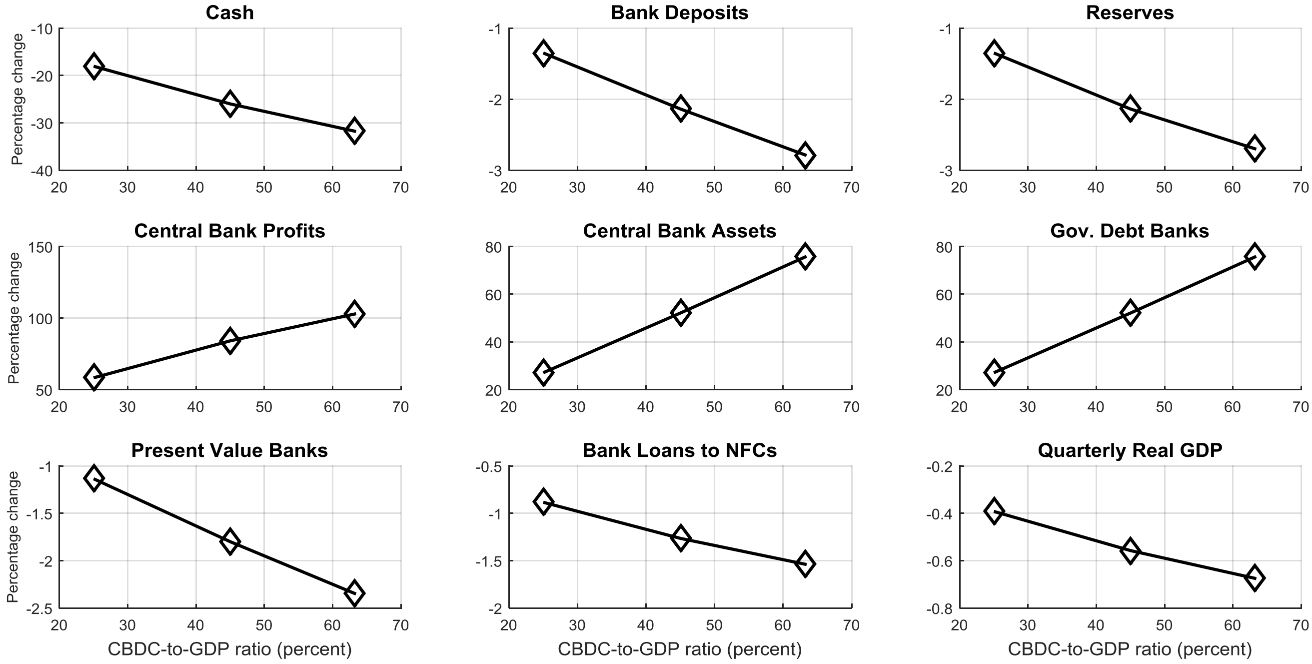
Notes: The figure reports the results of the estimation of model (1). Each horizontal segment reports the cumulated abnormal returns up to the latest key event, relative to the level on 1 October 2020. The solid line reports the average across all banks in the sample. The dashed and dotted lines report the average within two groups of banks, those with deposit ratio above or below the median, respectively. The two grey vertical lines indicate the publication of the ECB report on a digital euro on 2 October 2020 and the interview on 9 February 2021.

Figure 2: Change in loan volumes to firms associated with reactions of bank stock prices (percentages of volumes in October 2020)



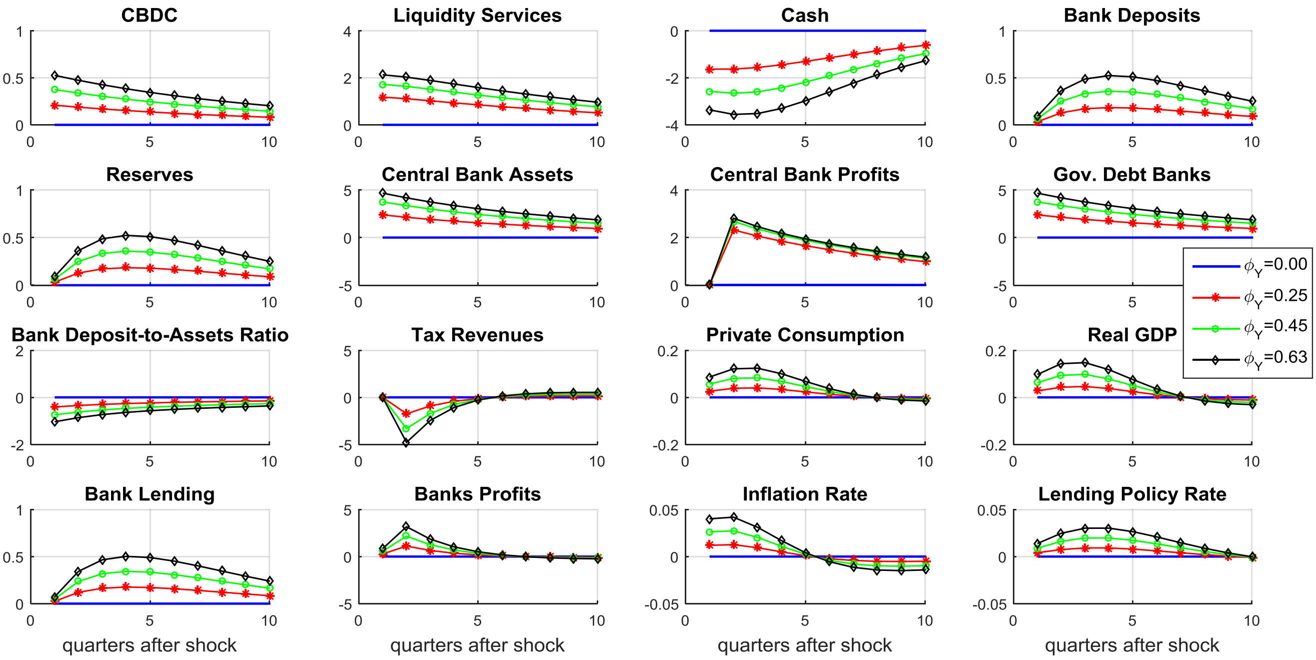
Notes: The figure reports the results of the estimation of model (3) with the specification of Table 3 (column 3). The solid line reports, for each monthly horizon from October 2020 indicated on the horizontal axis, the impact of 1 pp decrease in (cumulated) abnormal returns in October 2020. Shaded areas represent confidence intervals based on standard errors clustered at the bank level.

Figure 3: Transmission and steady state effects of CBDC issuance



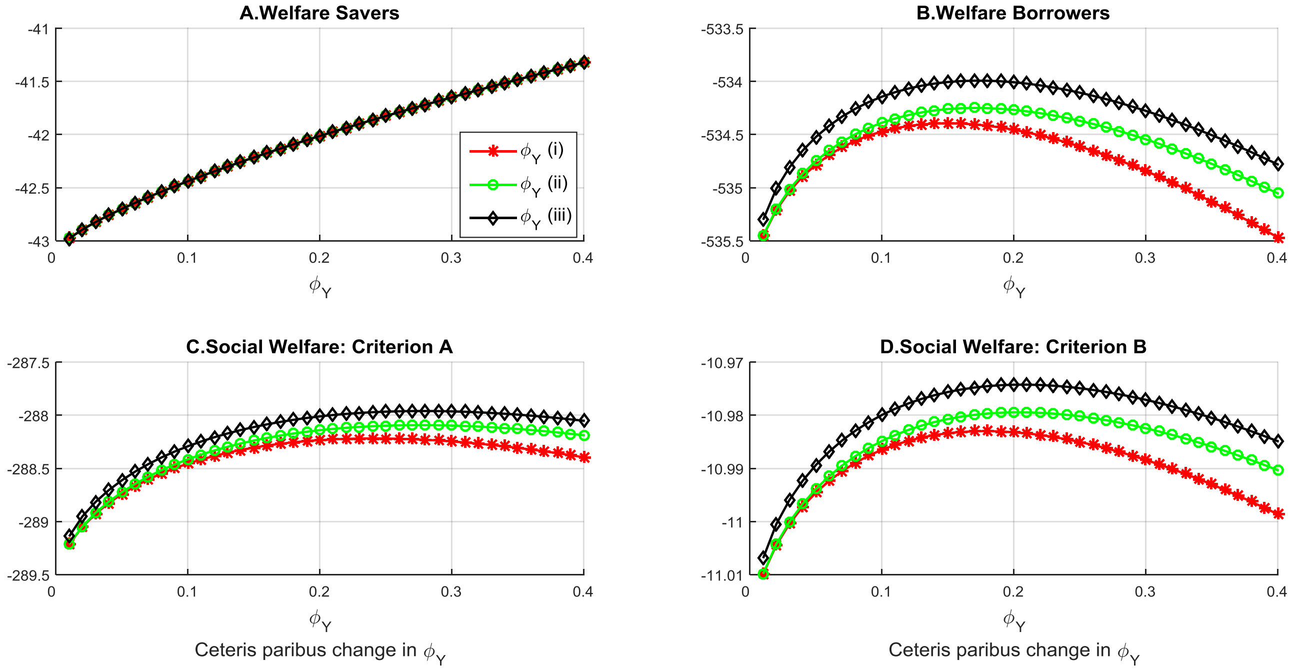
Notes: The figure reports the percentage changes in the steady state level of key selected aggregates arising when the economy moves from the no CBDC scenario to alternative CBDC scenarios under which CBDC supply in equilibrium is assumed to be equal to 25%, 45% and 63.2% of quarterly real GDP, respectively.

Figure 4: Transmission and cyclical effects. Impulse-responses to a positive CBDC supply shock



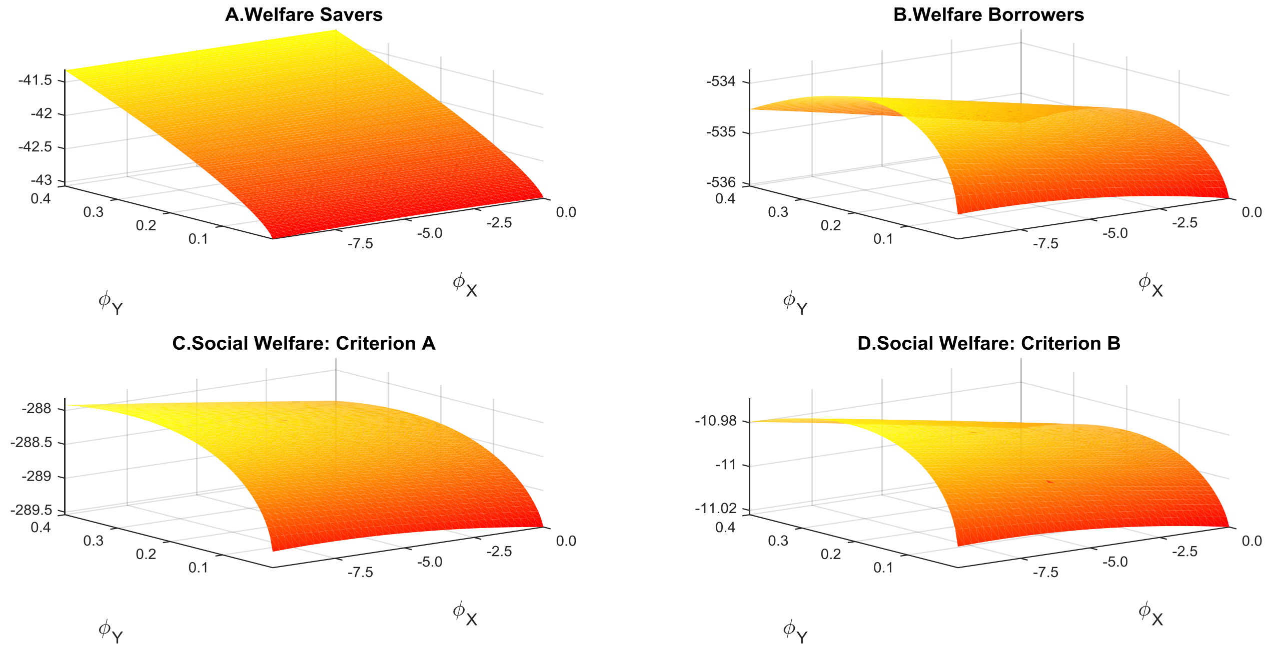
Notes: Variables are expressed in percentage deviations from the steady state with the exceptions of CBDC, the inflation rate and the lending policy rate, which are shown as absolute deviations from the steady state. These two rates have been annualized and are expressed in percentage points. The solid line refers to the baseline (no CBDC) scenario. The starred, dotted, and diamond lines make reference to alternative scenarios under which CBDC supply in equilibrium is equal to 25%, 45% and 63.2% of quarterly real GDP, respectively.

Figure 5: Welfare effects of CBDC quantity rules (welfare effects of ceteris paribus changes in ϕ_Y)



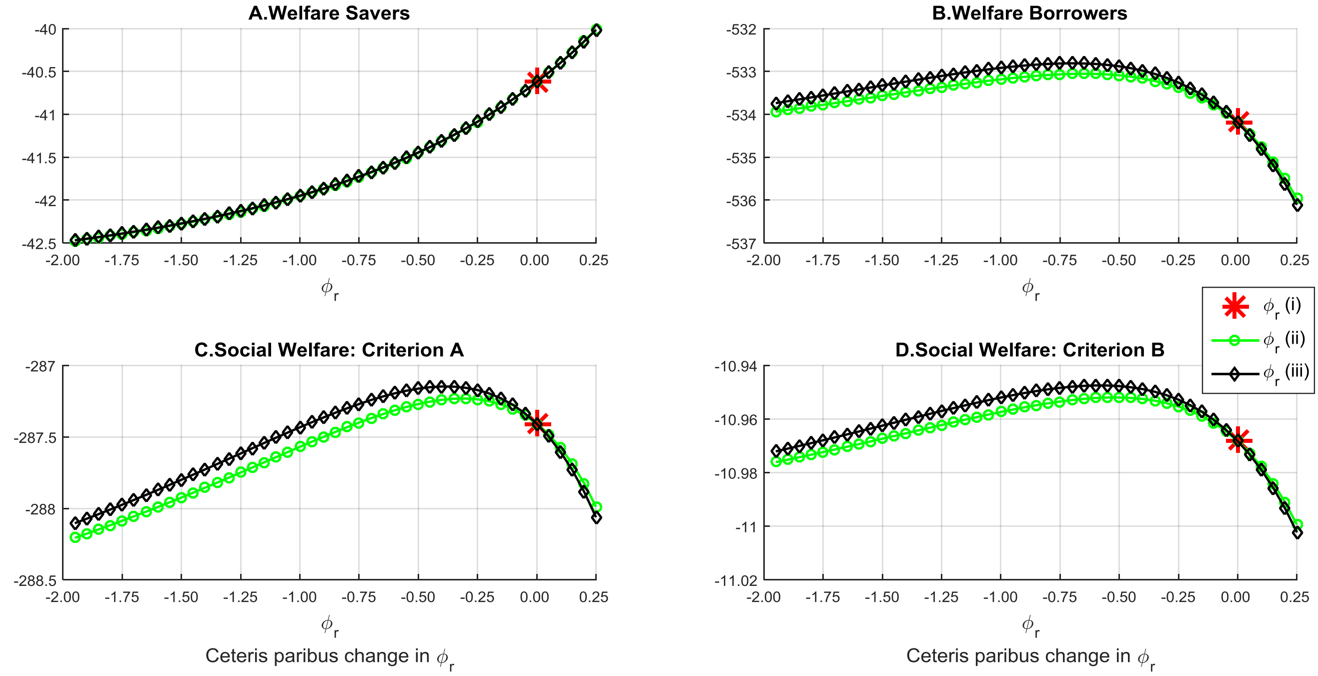
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria “A” and “B” as a function of CBDC policy parameter ϕ_Y. The starred line, the dotted line, and the diamond line relate to CBDC quantity rules (i), (ii) and (iii), respectively.

Figure 6: Welfare effects of CBDC quantity rule (iii) (welfare effects of ceteris paribus changes in ϕ_Y - ϕ_X)



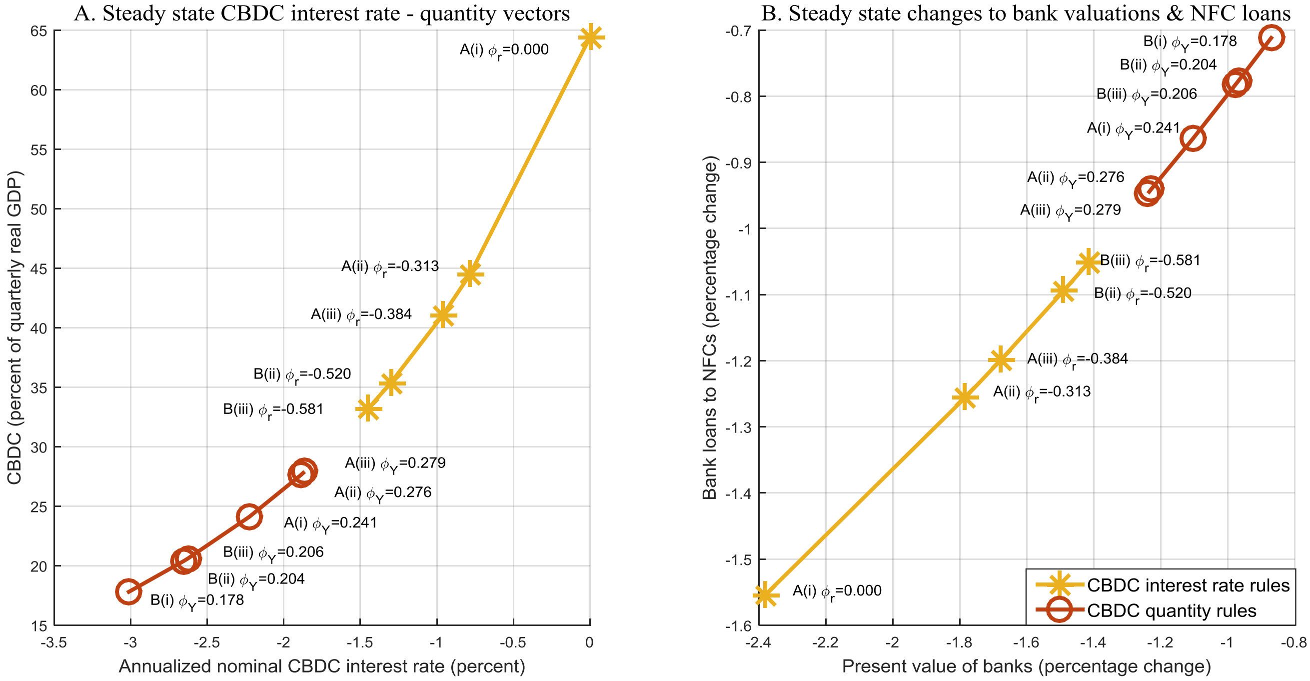
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria “A” and “B” as a function of CBDC policy parameters ϕ_Y and ϕ_X under CBDC quantity rule (iii).

Figure 7: Welfare effects of CBDC interest rate rules (welfare effects of ceteris paribus changes in ϕ_r)



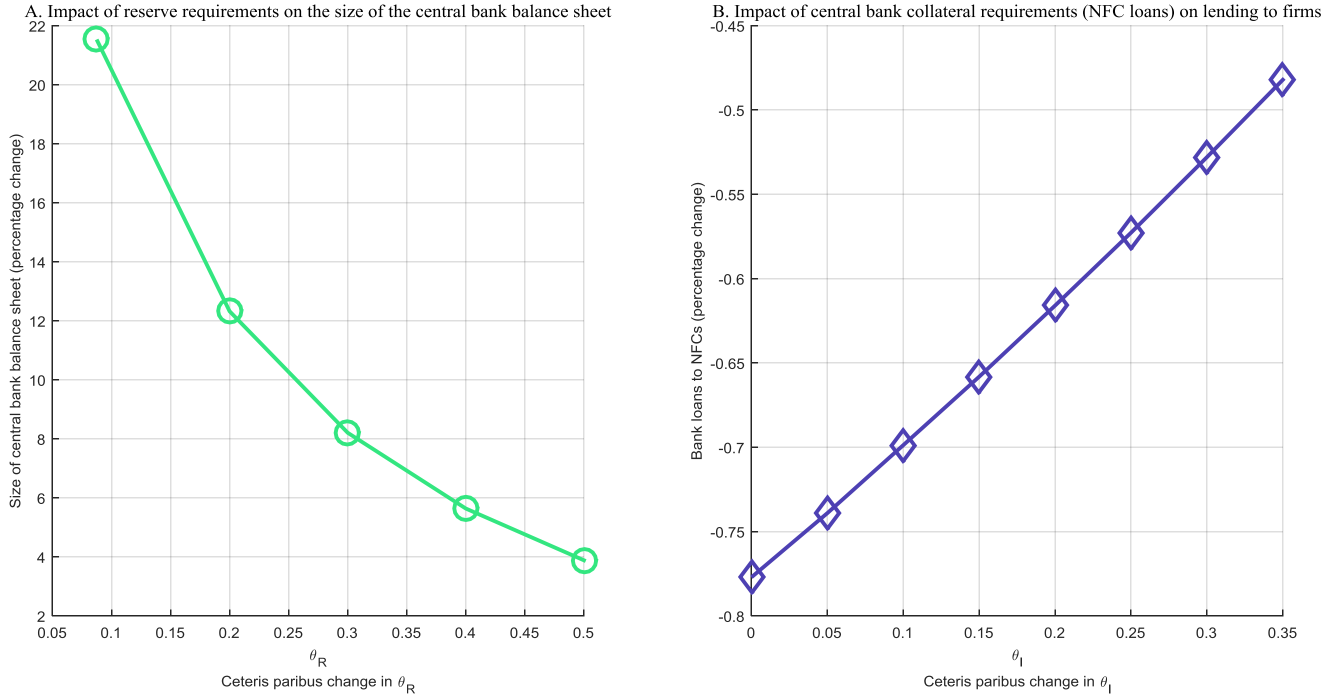
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria "A" and "B" as a function of CBDC policy parameter ϕ_r. The star, the dotted line, and the diamond line refer to CBDC interest rate rules (i), (ii) and (iii), respectively.

Figure 8: Steady state effects of CBDC policy rules



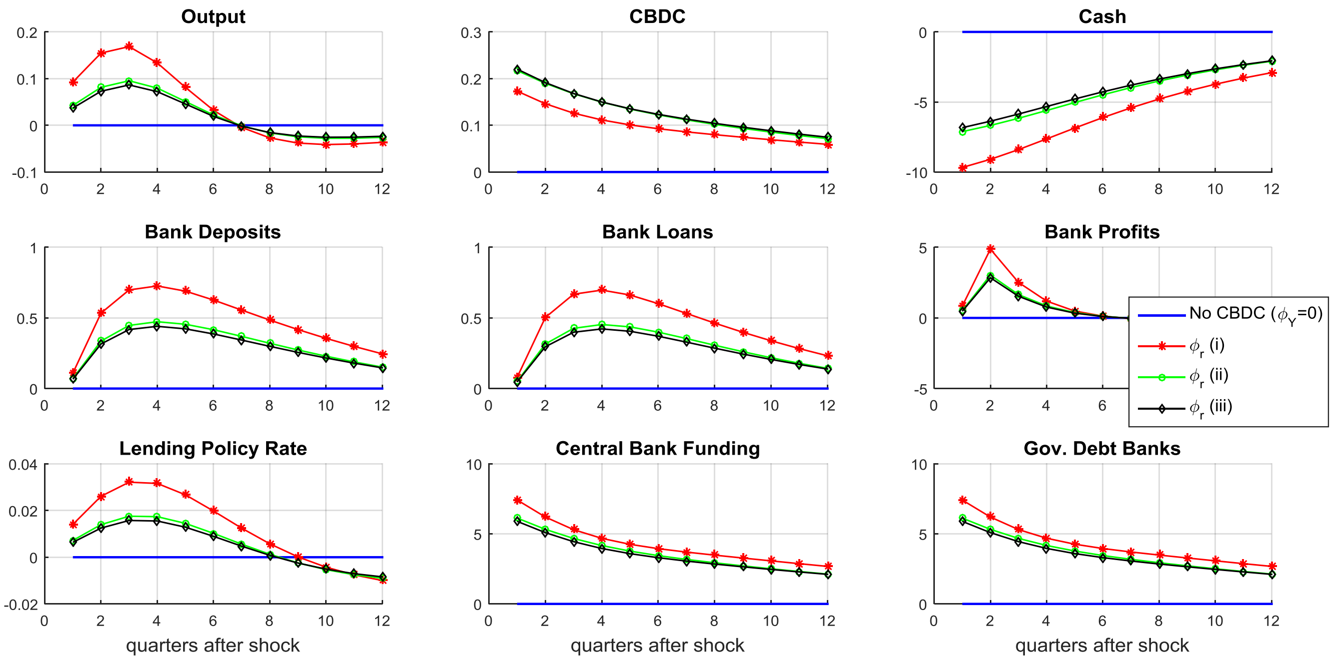
Notes: For each of the six considered specifications of the CBDC policy rule and for welfare weighting criteria "A" and "B", panel A reports the annualized nominal CBDC interest rate and the CBDC-to-real GDP ratio associated to each welfare-maximizing CBDC policy rule. For the same optimal policy rules, panel B displays the steady state impact on bank valuations and aggregate bank loans to firms. Bank valuations in the model are proxied by the recursive value of the representative bank (i.e., the objective function of banks' optimization problem).

Figure 9: Steady state effects of related policies under optimal CBDC quantity rule (ii)



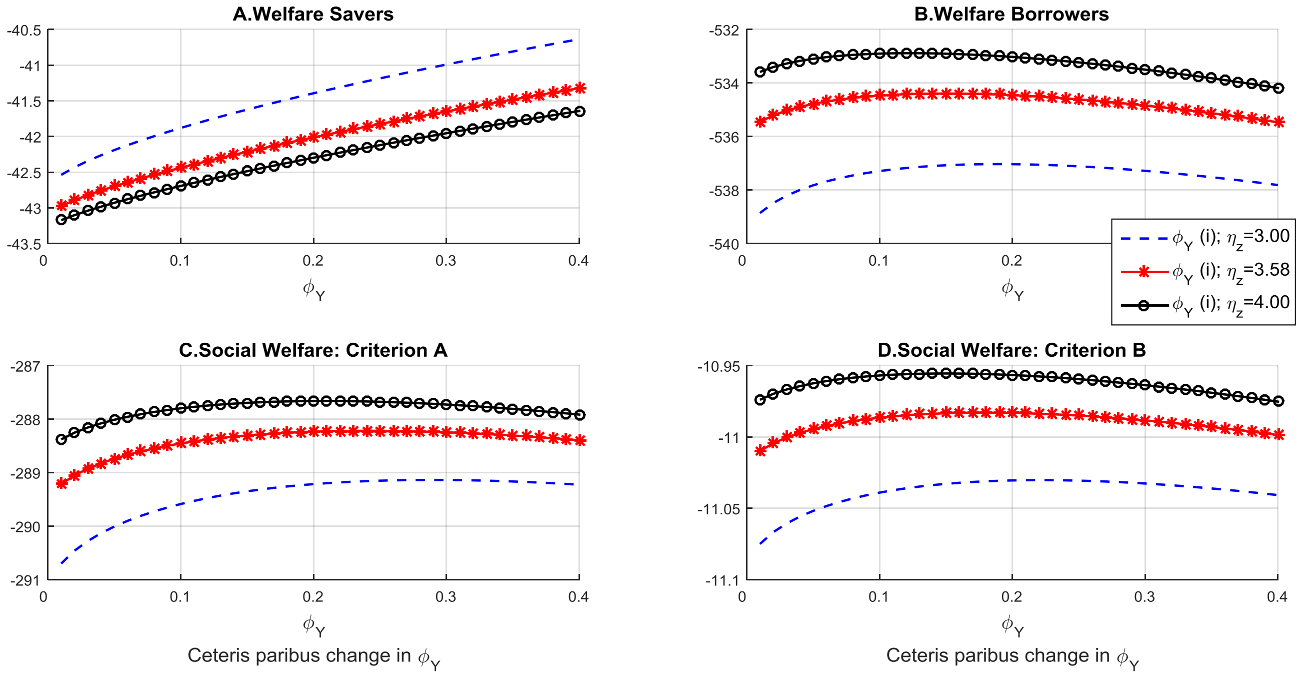
Notes: Given the CBDC policy rule within the class of quantity rules (ii) that maximizes social welfare under welfare criterion "B", panel A reports the steady state impact of issuing CBDC on the size of the central bank's balance sheet for different values of the reserves requirement parameter, θ_R, whereas panel B displays the steady state impact of introducing a CBDC on aggregate bank loans to firms for different values of central banks' collateral requirement parameter, θ_I.

Figure 10: Impulse-responses to a positive CBDC preference shock under optimal CBDC policy rules



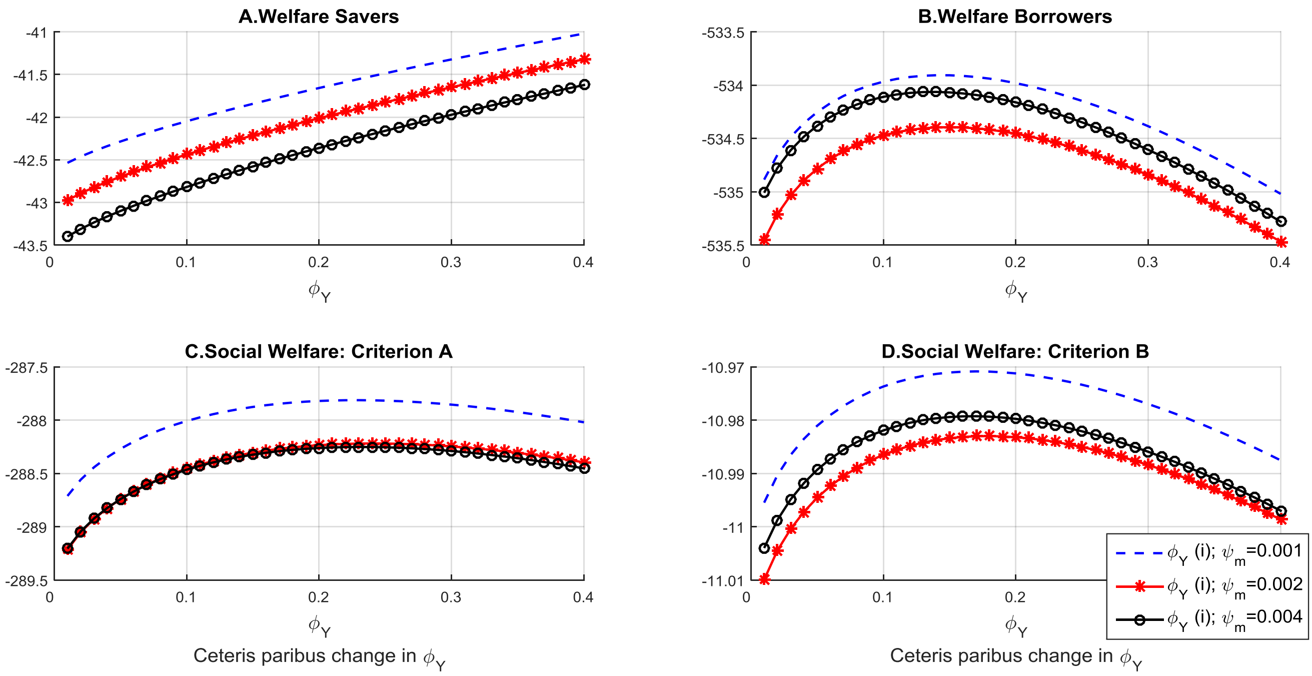
Notes: Variables are expressed in percentage deviations from the steady state with the exception of the lending policy rate, which is shown as absolute deviations from the steady state and expressed in percentage points. Social welfare has been maximized under welfare criterion "B". The solid line refers to the baseline (no CBDC) scenario. The starred line corresponds to interest rate rule (i). The dotted line relates to the CBDC optimal policy rule within the class of interest rate rules (ii). The diamond line makes reference to the CBDC interest rate rule of type (iii) that maximizes social welfare.

Figure 11: Robustness Checks: η_z (welfare effects of ceteris paribus changes in ϕ_Y)



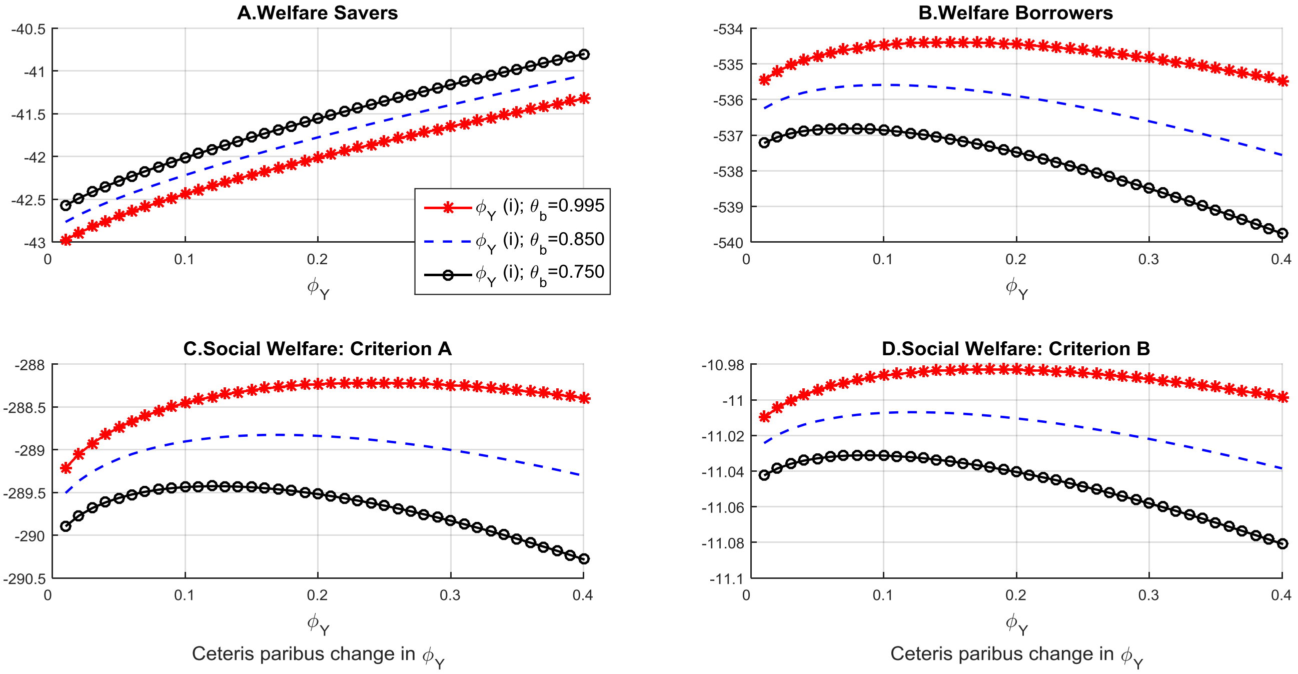
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria "A" and "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the elasticity of substitution across forms of money, η_z. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure 12: Robustness Checks: ψ_m (welfare effects of ceteris paribus changes in ϕ_Y)



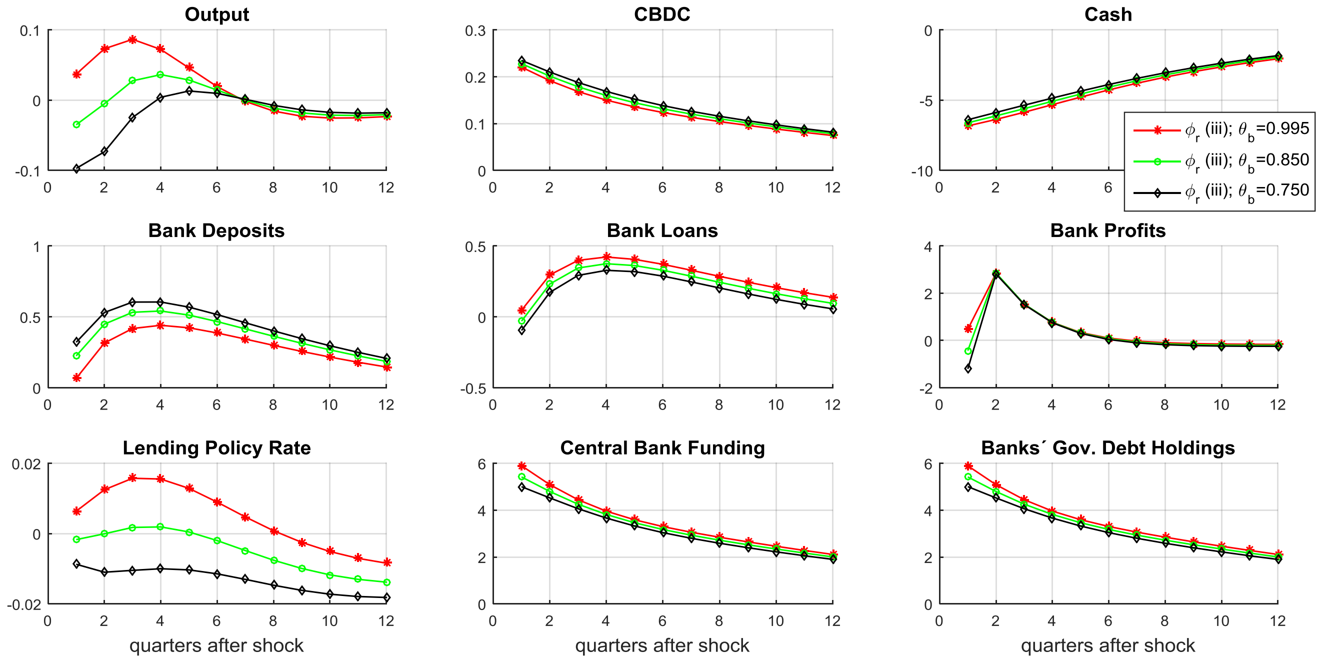
Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria "A" and "B" for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the cash storage cost parameter, ψ_m. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure 13: Robustness Checks: θ_b (welfare effects of ceteris paribus changes in ϕ_Y)



Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria “A” and “B” for CBDC quantity rule (i) as a function of policy parameter ϕ_Y, for alternative values of the central bank’s collateral requirement parameter for government bonds, θ_b. The starred line refers to the baseline calibration whereas the dotted and dashed lines relate to alternative parameterization scenarios.

Figure 14: Impulse-responses to a positive CBDC preference shock under different haircuts on gov. bonds



Notes: Variables are expressed in percentage deviations from the steady state with the exception of the lending policy rate, which is shown as absolute deviations from the steady state and expressed in percentage points. Social welfare has been maximized under welfare criterion “B” and for interest rate rule (iii). The starred line refers to the baseline calibration whereas the dotted and diamond lines relate to alternative parameterization scenarios.

A List of Digital Euro Events

Table A.1: List of digital euro events

Date	Type of event	Subject
29-Jul-21	INTERVIEW	Luis de Guindos: Interview with Handelsblatt
29-Jul-21	INTERVIEW	Fabio Panetta: Interview with Corriere della Sera
14-Jul-21	PRESS RELEASE	Eurosystem launches digital euro project
14-Jul-21	THE ECB BLOG	Fabio Panetta: Preparing for the euro's digital future
20-Jun-21	INTERVIEW	Fabio Panetta: Interview with Financial Times
26-May-21	INTERVIEW	Fabio Panetta: Interview with Nikkei
03-May-21	INTERVIEW	Luis de Guindos: Interview with La Repubblica
14-Apr-21	SPEECH	Fabio Panetta: A digital euro to meet the expectations of Europeans
14-Apr-21	PRESS RELEASE	ESB publishes the results of the public consultation on a digital euro
11-Apr-21	INTERVIEW	Fabio Panetta: Interview with El País
09-Apr-21	INTERVIEW	Isabel Schnabel: Interview with Der Spiegel
08-Apr-21	SPEECH	Christine Lagarde: IMFC Statement
25-Mar-21	THE ECB BLOG	Fabio Panetta: Digital central bank money for Europeans - getting ready for the future
17-Mar-21	INTERVIEW	Frank Elderson: Q&A on Twitter
02-Mar-21	INTERVIEW	Luis de Guindos: Interview with Público
25-Feb-21	INTERVIEW	Isabel Schnabel: Interview with LETA
10-Feb-21	SPEECH	Fabio Panetta: Evolution or revolution? The impact of a digital euro on the financial system
09-Feb-21	INTERVIEW	Fabio Panetta: Interview with Der Spiegel
31-Jan-21	INTERVIEW	Isabel Schnabel: Interview with Deutschlandfunk
02-Dec-20	THE ECB BLOG	Fabio Panetta: Money in the digital era
30-Nov-20	INTERVIEW	Christine Lagarde: The future of money - innovating while retaining trust
27-Nov-20	SPEECH	Fabio Panetta: From the payments revolution to the reinvention of money
04-Nov-20	SPEECH	Fabio Panetta: The two sides of the (stable)coins
22-Oct-20	SPEECH	Fabio Panetta: On the edge of a new frontier: European payments in the digital age
19-Oct-20	INTERVIEW	Christine Lagarde: Interview with Le Monde
12-Oct-20	SPEECH	Fabio Panetta: A digital euro for the digital era
05-Oct-20	VOXEU COLUMN	Ulrich Bindseil & Fabio Panetta: CBDC remuneration in a world with low or negative nominal interest rates
02-Oct-20	PRESS RELEASE	ECB intensifies its work on a digital euro
02-Oct-20	THE ECB BLOG	Fabio Panetta: We must be prepared to issue a digital euro
24-Sep-20	INTERVIEW	Philip R. Lane: Q&A on Twitter
23-Sep-20	INTERVIEW	Yves Mersch: Interview with Bloomberg
10-Sep-20	SPEECH	Christine Lagarde: Payments in a digital world
07-Jul-20	SPEECH	Fabio Panetta: Unleashing the euro's untapped potential at global level
11-May-20	SPEECH	Yves Mersch: An ECB digital currency - a flight of fancy?
08-Jan-20	INTERVIEW	Christine Lagarde: Interview with "Challenges" magazine

B Data and Sources

This appendix presents the full data set employed to calibrate the model in section 3.2.

Gross Domestic Product: Gross domestic product at market prices, Euro area 19 (fixed composition), Domestic (home or reference area), Total economy, Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

GDP Deflator: Gross domestic product at market prices, Euro area 19 (fixed composition), Domestic (home or reference area), Total economy, Index, Deflator (index), Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

Private Consumption: Private final consumption, Individual consumption expenditure, Euro area 19 (fixed composition), World (all entities, including reference area, including IO), Households and non profit institutions serving households (NPISH), Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

Public Consumption: Government final consumption, Final consumption expenditure, Euro area 19 (fixed composition), World (all entities, including reference area, including IO), General government, Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data. Source: ESA2010 National accounts, Main aggregates, Eurostat.

Gross fixed capital formation: Gross fixed capital formation, Euro area 19 (fixed composition), World (all entities, including reference area, including IO), Total economy, Fixed assets by type of asset (gross), Euro, Current prices, Non transformed data, Calendar and seasonally adjusted data.

Bank Deposits (Counterpart: MFIs): Deposit liabilities vis-a-vis euro area MFI reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector, Deposit liabilities, Total maturity, Euro, Euro area (changing composition) counterpart, Monetary financial institutions (MFIs) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Deposits (Counterpart: Non-MFIs): Deposit liabilities vis-a-vis euro area non-MFI reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Deposit liabilities, Total maturity, Euro - Euro area (changing composition) counterpart, Non-MFIs sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Capital and Reserves: Capital and reserves reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Capital and reserves, All currencies combined, World not allocated (geographically) counterpart, Unspecified counterpart sector sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance

Sheet Items (BSI Statistics), European Central Bank.

Bank Loans to Households: Loans vis-a-vis euro area households reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector, Loans, Total maturity, All currencies combined, Euro area (changing composition) counterpart, Households and non-profit institutions serving households (S.14 and S.15) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Loans to NFCs: Loans vis-a-vis euro area NFC reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Non-Financial corporations (S.11) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Bank Holdings of Government Debt: Holdings of debt securities issued by euro area General Government reported by MFI excluding ESCB in the euro area (stock), Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector, Debt securities held, Total maturity, All currencies combined, Euro area (changing composition) counterpart, General Government sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Reserves: Liabilities to euro area credit institutions related to MPOs denominated in euro - Eurosystem, Euro area (changing composition), Eurosystem reporting sector, Liabilities to euro area credit institutions related to MPOs denominated in euro, Euro, Euro area (changing composition) counterpart. Source: Internal Liquidity Management (ILM Statistics), European Central Bank.

Banknotes (Cash): Banknotes in circulation - Eurosystem, Euro area (changing composition), Eurosystem reporting sector, Banknotes in circulation, Euro, World not allocated (geographically) counterpart. Source: MFI Balance Sheet Items (BSI Statistics), European Central Bank.

Deposit Interest Rate: Bank interest rates, overnight deposits from households - euro area, Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector, Overnight deposits, Total original maturity, New business coverage, Households and non-profit institutions serving households (S.14 and S.15) sector, denominated in Euro. Source: MFI Interest Rate Statistics (MIR Statistics), European Central Bank.

NFC Loans Interest Rate: Bank interest rates, loans to corporations with an original maturity of up to one year (outstanding amounts) - euro area, Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector, Loans, Up to 1 year original maturity, Outstanding amount business coverage, Non-Financial corporations (S.11) sector, denominated in Euro. Source: MFI Interest Rate Statistics (MIR Statistics), European Central Bank.

Deposit Facility Rate: ECB Deposit facility, date of changes (raw data), Level. Euro area (changing composition), Key interest rate, ECB Deposit facility, date of changes (raw data), Level, Euro, provided by ECB. Source: Financial market data (MF Statistics), European Central Bank.

Lending Facility Rate: ECB Marginal lending facility - date of changes (raw data) - Level. Euro area (changing composition), Key interest rate, ECB Marginal lending facility, date of changes (raw data), Level, Euro, provided by ECB. Source: Financial market data (MF Statistics), European Central Bank.

C Equations of the Model

This section presents the full set of equilibrium equations of the DSGE model.

C.1 Patient Households

Patient households seek to maximize their objective function subject to the following budget constraint:

$$\begin{aligned} c_{p,t} + q_t(h_{p,t} - h_{p,t-1}) + m_t + f(m_t) + cbdc_t + d_t + b_{p,t} + R_{g,t}b_{p,t-1} + \omega_T T_t \\ = \frac{m_{t-1}}{\pi_t} + R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} + R_{d,t-1}\frac{d_{t-1}}{\pi_t} + R_{g,t-1}\frac{b_{p,t-1}}{\pi_t} + w_t n_{p,t} + \Omega_t, \end{aligned} \tag{C.1}$$

Their choice variables are $c_{p,t}$, $h_{p,t}$, d_t, m_t, $cbdc_t$, $b_{p,t}$ and $n_{p,t}$. The optimality conditions of the problem read

$$\lambda_t^p = \left[c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)}\right]^{-\sigma_h}, \tag{C.2}$$

$$q_t\lambda_t^p = \frac{j_{p,t}}{h_{p,t}} + \beta_p E_t\left(q_{t+1}\lambda_{t+1}^p\right), \tag{C.3}$$

$$\lambda_t^p = \beta_p E_t \left(\lambda_{t+1}^p R_{d,t}/\pi_{t+1}\right) + \frac{\chi_{z,t}}{z_t}\omega_d \left(\frac{z_t}{d_t}\right)^{1/\eta_{z,t}}, \tag{C.4}$$

$$\lambda_t^p = \beta_p E_t \left(\lambda_{t+1}^p R_{cbdc,t}/\pi_{t+1}\right) + \frac{\chi_{z,t}}{z_t}\vartheta_t \left(\frac{z_t}{cbdc_t}\right)^{1/\eta_{z,t}}, \tag{C.5}$$

$$\lambda_t^p (1 + f_m) = \beta_p E_t \left(\lambda_{t+1}^p/\pi_{t+1}\right) + \frac{\chi_{z,t}}{z_t} \left(\frac{z_t}{m_t}\right)^{1/\eta_{z,t}}, \tag{C.6}$$

$$\lambda_t^p = \beta_p E_t \left(\lambda_{t+1}^p R_{g,t}/\pi_{t+1}\right), \tag{C.7}$$

$$w_t = n_{p,t}^{\phi}, \tag{C.8}$$

where λ_t^p is the Lagrange multiplier on the budget constraint of the representative patient household.

C.2 Impatient Households

The representative impatient household chooses the trajectories of consumption $c_{i,t}$, property housing $h_{i,t}$, hours worked $n_{i,t}$, and demand for loans $l_{i,t}$ that maximizes its objective function subject to a budget constraint and a borrowing limit:

$$c_{i,t} + q_t \left(h_{i,t} - h_{i,t-1}\right) + R_{i,t-1}\frac{l_{i,t-1}}{\pi_t} + (1 - \omega_T) T_t = l_{i,t} + w_t n_{i,t}, \tag{C.9}$$

$$l_{i,t} \leq m_{H,t} E_t \left(\frac{q_{t+1}}{R_{i,t}} h_{i,t}\pi_{t+1}\right). \tag{C.10}$$

The resulting optimality conditions are

$$\lambda_t^i = \left[c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)}\right]^{-\sigma_h}, \tag{C.11}$$

$$\lambda_t^i \left[q_t - E_t \left(m_{H,t}\frac{q_{t+1}}{R_{i,t}}\pi_{t+1}\right)\right] = \frac{j_{i,t}}{h_{i,t}} + \beta_i E_t \left[q_{t+1}\lambda_{t+1}^i \left(1 - m_{H,t}\right)\right], \tag{C.12}$$

$$w_t = n_{i,t}^{\phi}. \tag{C.13}$$

where λ_t^i is the Lagrange multiplier on the budget constraint of the representative impatient household.

C.3 Banks

Banks maximize their objective function subject to a balance sheet identity, a cash flow restriction, a capital adequacy constraint, a liquidity (reserves) requirement and a central banks' collateral requirement

$$L_{i,t} + L_{e,t} + b_{b,t} + \widetilde{R}_{b,t} = e_t + D_t + f_t, \tag{C.14}$$

$$\Omega_{b,t} + e_t - (1-\delta^e)\frac{e_{t-1}}{\pi_t}$$
$$= \frac{\left(r_{i,t-1}L_{i,t-1} + r_{e,t}L_{e,t-1} + r_{g,t-1}b_{b,t-1} + r_{\widetilde{R},t-1}\widetilde{R}_{b,t-1} - r_{d,t-1}D_{t-1} - r_{f,t-1}f_{t-1}\right)}{\pi_t}, \tag{C.15}$$

$$D_t + f_t \leq \gamma_i L_{i,t} + \gamma_e L_{e,t} + \gamma_b b_{b,t} + \gamma_{\widetilde{R}}\widetilde{R}_{b,t}, \tag{C.16}$$

$$\theta_{R,t} D_t \leq \widetilde{R}_{b,t}, \tag{C.17}$$

$$f_t \leq \theta_{b,t} E_t\left(\frac{b_{b,t}}{R_{f,t}}\pi_{t+1}\right). \tag{C.18}$$

The law of motion for bank equity reads

$$e_t = J_{b,t} - \ \Omega_{b,t} + \ (1-\delta^e)e_{t-1}/\pi_t. \tag{C.19}$$

Their choice variables are $\Omega_{b,t}$, $L_{i,t}$, $L_{e,t}$, $b_{b,t}$, $\widetilde{R}_{b,t}$, D_t and f_t.The resulting optimality conditions read

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}} + \mu_{e,t}\gamma_e = E_t\left[\Lambda_{t,t+1}\frac{\left(r_{e,t+1} + 1 - \delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.20}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}} + \mu_{e,t}\gamma_i = E_t\left[\Lambda_{t,t+1}\frac{\left(r_{i,t} + 1 - \delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.21}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}} + \mu_{\widetilde{R},t} + \mu_{e,t} = E_t\left[\Lambda_{t,t+1}\frac{\left(r_{\widetilde{R},t} + 1 - \delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.22}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}}+\mu_{f,t}\theta_{f,t}E_t\left(\frac{\pi_{t+1}}{R_{f,t}}\right)+\mu_e=E_t\left[\Lambda_{t,t+1}\frac{\left(r_{g,t}+1-\delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.23}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}}+\mu_{e,t}+\mu_{\widetilde{R},t}\theta_{R,t}=E_t\left[\Lambda_{t,t+1}\frac{\left(r_{d,t}+1-\delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right], \tag{C.24}$$

$$\frac{1}{\Omega_{b,t}^{\frac{1}{\sigma}}}+\mu_{e,t}+\mu_{f,t}=E_t\left[\Lambda_{t,t+1}\frac{\left(r_{f,t}+1-\delta^e\right)/\pi_{t+1}}{\Omega_{b,t+1}^{\frac{1}{\sigma}}}\right]. \tag{C.25}$$

where $\mu_{e,t}$, $\mu_{\widetilde{R},t}$, and $\mu_{f,t}$ are the multipliers on the capital adequacy constraint, the reserve requirement, and the central bank's collateral constraint, respectively.

C.4 Entrepreneurial Managers

Entrepreneurs seek to maximize their objective function subject to subject to a budget constraint and the corresponding borrowing limit:

$$\Omega_{e,t}+R_{e,t}\frac{l_{e,t-1}}{\pi_t}+q_{k,t}\left[k_{e,t}-(1-\delta_t^k)k_{e,t-1}\right]+q_t(h_{e,t}-h_{e,t-1})=r_{h,t}h_{e,t-1}+r_{k,t}u_tk_{e,t-1}+l_{e,t}+J_{er,t}, \tag{C.26}$$

$$l_{e,t}\leq m_{K,t}E_t\left[\frac{q_{k,t+1}}{R_{e,t+1}}(1-\delta_{t+1}^k)k_{e,t}\pi_{t+1}\right], \tag{C.27}$$

where

$$\delta_t^k\left(u_t\right)=\delta_0^k+\delta_1^k(u_t-1)+\frac{\delta_2^k}{2}\left(u_t-1\right)^2. \tag{C.28}$$

Their choice variables are $\Omega_{e,t}$, $l_{e,t}$, $k_{e,t}$, $h_{e,t}$ and u_t. The following optimality condition can be derived from the first order conditions of the problem

$$\Omega_{e,t}^{-\frac{1}{\sigma}}q_t=\Lambda_{t,t+1}E_t\left[{\Omega_{e,t+1}}^{-\frac{1}{\sigma}}\left(q_{t+1}+r_{h,t+1}\right)\right], \tag{C.29}$$

$$\Omega_{e,t}^{-\frac{1}{\sigma}}\left\{q_{k,t}-m_{K,.t}E_t\left[\frac{q_{k,t+1}}{R_{e,t+1}}(1-\delta_{t+1}^k)\pi_{t+1}\right]\right\}$$
$$=\Lambda_{t,t+1}E_t\left\{{\Omega_{e,t+1}}^{-\frac{1}{\sigma}}\left[q_{k,t+1}(1-\delta_t^k-m_{K,t})+u_{t+1}r_{k,t+1}\right]\right\}, \tag{C.30}$$

$$\delta_1^k + \delta_2^k \left(u_t - 1\right) = r_{k,t}. \tag{C.31}$$

C.5 Emtrepreneurial Retailers

There is a continuum of entrepreneurial retailers (also referred to as intermediate non-housing good producers). Each intermediate good producer j operates the following Cobb-Douglas production function:

$$Y_t(j) = A_t \left[u_t(j)k_{e,t-1}(j)\right]^{\alpha} h_{e,t-1}(j)^{\nu} N_t(j)^{(1-\alpha-\nu)}, \tag{C.32}$$

Intermediate good producers solve a two-stage problem. In the first stage, they choose the trajectories of $k_{e,t-1}(j)$, $h_{e,t-1}(j)$ and $N_t(j)$ that minimize total real costs, $r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_t N_t(j)$:

$$\frac{w_t}{r_{k,t}} = \frac{(1-\alpha-\nu)}{\alpha}\frac{u_t k_{e,t-1}}{N_t}, \tag{C.33}$$

$$\frac{r_{h,t}}{r_{k,t}} = \frac{\nu}{\alpha}\frac{u_t k_{e,t-1}}{h_{e,t-1}}, \tag{C.34}$$

$$mc_t = \frac{\left(w_t\right)^{(1-\alpha-\nu)}\left(r_{k,t}\right)^{\alpha}\left(r_{h,t}\right)^{\nu}}{A_t\left(1-\alpha-\nu\right)^{(1-\alpha-\nu)}\alpha^{\alpha}\nu^{\nu}}. \tag{C.35}$$

The firms that can change prices in period t set them to satisfy:

$$g_t^1 = \lambda_t^p mc_t Y_t + \beta_p \theta E_t \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\varepsilon} g_{t+1}^1, \tag{C.36}$$

$$g_t^2 = \lambda_t^p \pi_t^* Y_t + \beta_p \theta E_t \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\varepsilon} \left(\frac{\pi_t^*}{\pi_{t+1}^*}\right) g_{t+1}^2, \tag{C.37}$$

$$\varepsilon g_t^1 = \left(\varepsilon - 1\right) g_t^2. \tag{C.38}$$

The price level and price dispersion υ_t , respectively, evolve according to:

$$1 = \theta \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{1-\varepsilon} + \left(1-\theta\right)\pi_t^{*1-\varepsilon}, \tag{C.39}$$

and

$$\upsilon_t = \theta \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{-\varepsilon} \upsilon_{t-1} + \left(1-\theta\right)\pi_t^{*-\varepsilon}. \tag{C.40}$$

Profits from each intermediate good producer j are transferred to entrepreneurial managers:

$$J_{er,t}(j) = Y_t(j) - \left[r_{k,t} k_{e,t-1}(j) + r_{h,t} h_{e,t-1}(j) + w_t N_t(j) \right]. \tag{C.41}$$

C.6 Capital Goods Producers

Capital-good-producing firms seek to maximize their objective function with respect to net investment in physical capital, I_t. The resulting optimal condition is

$$1 = q_{k,t} \left[1 - \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \psi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + E_t \left[\Lambda_{t,t+1} q_{k,t+1} \psi_I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]. \tag{C.42}$$

The law of motion for physical capital reads

$$K_t = (1 - \delta_t^k) K_{t-1} + I_t \left[1 - \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right]. \tag{C.43}$$

C.7 Government

Tax revenues are collected from households in a lump-sum fashion and determined according to a fiscal rule

$$T_t = \phi_p b_{p,t-1} + \phi_b b_{b,t-1}. \tag{C.44}$$

Government spending is assumed to be equal to a constant fraction of steady state real output

$$G_t = \varrho Y^{ss}. \tag{C.45}$$

Supply of short-term government bonds is endogenously determined by the following intertemporal budget constraint

$$R_{g,t-1} \frac{B_{g,t-1}}{\pi_t} + G_t = T_t + B_{g,t} + \Omega_{cb,t}. \tag{C.46}$$

C.8 Central Bank

The central bank sets the lending facility rate $r_{f,t}$ according to a Taylor-type policy rule:

$$r_{f,t} = \rho_r r_{f,t-1} + (1-\rho_r)\left(r_f^{ss} + \alpha_\pi \tilde{\pi}_t + \alpha_Y \tilde{y}_t\right) + e_{rf,t}. \tag{C.47}$$

A constant corridor of width $\alpha > 0$ is assumed to be maintained between the lending facility rate and the deposit facility rate,

$$r_{\widetilde{R},t} = r_{f,t} - \alpha. \tag{C.48}$$

According to the balance sheet of the central bank:

$$F_t = \widetilde{R}_t + M_t + CBDC_t. \tag{C.49}$$

Central bank´s profits evolve as

$$\Omega_{cb,t} = \widetilde{R}_t + M_t + CBDC_t + R_{f,t-1}\frac{F_{t-1}}{\pi_t} - R_{\widetilde{R},t-1}\frac{\widetilde{R}_{t-1}}{\pi_t} - \frac{M_{t-1}}{\pi_t} - R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} - F_t. \tag{C.50}$$

In the baseline scenario, CBDC supply is set according to the following policy rule:

$$CBDC_t = \phi_Y Y^{ss}. \tag{C.51}$$

C.9 Aggregation and Market Clearing

Market clearing is implied by the Walras' law, by aggregating all the budget constraints. The aggregate resource constraint of the economy represents the equilibrium condition for the final goods market:

$$Y_t = C_t + q_{k,t} I_t + G_t + \delta^e e_{t-1} + f(m_t). \tag{C.52}$$

Similarly, in equilibrium labor demand equals total labor supply,

$$N_t = n_{p,t} + n_{i,t}. \tag{C.53}$$

The stock of physical capital produced by capital goods producers must equal the demand for this good coming from households

$$K_t = k_{e,t}. \tag{C.54}$$

The stock of real estate must equal the demand coming from households and entrepreneurs

$$\overline{H} = h_{p,t} + h_{i,t} + h_{e,t}. \tag{C.55}$$

Similarly, in equilibrium demand for loans of households and entrepreneurs equals bank credit supply

$$l_{i,t} + l_{e,t} = L_t. \tag{C.56}$$

In equilibrium, the supply of government bonds equals the demand for this asset coming from patient households and banks

$$b_{p,t} + b_{b,t} = B_{g,t}. \tag{C.57}$$

Bank´s reserves are a liability of the central bank

$$\widetilde{R}_{b,t} = \widetilde{R}_t. \tag{C.58}$$

CBDC issued by the central bank equals demand for that means of payment

$$CBDC_t = cbdc_t. \tag{C.59}$$

Cash issued by the central bank equals demand for that monetary instrument

$$M_t = m_t. \tag{C.60}$$

The stock of bank deposits held by households must be equal to banks' deposit funding

$$d_t = D_t. \tag{C.61}$$

In equilibrium, banks' demand for central bank funding equals central bank's supply of funding to banks

$$f_t = F_t. \tag{C.62}$$

C.10 Shocks

The following zero-mean, AR(1) shocks are present in the baseline calibration model: $\varepsilon_{h,t}$, $\varepsilon_{z,t}$, $\varepsilon_{\eta,t}$, $\varepsilon_{mh,t}$, $\varepsilon_{mk,t}$, A_t, $\varepsilon_{\theta_R,t}$, $_{\theta_b,t}$. These shocks follow the processes given by:

$$\log \varepsilon_{h,t} = \rho_h \log \varepsilon_{h,t-1} + e_{h,t}, \;\; e_{h,t} \sim N(0, \sigma_h), \tag{C.63}$$

$$\log \varepsilon_{z,t} = \rho_z \log \varepsilon_{z,t-1} + e_{z,t},\ \ e_{z,t} \sim N(0, \sigma_z), \tag{C.64}$$

$$\log \varepsilon_{\eta,t} = \rho_z \log \varepsilon_{\eta,t-1} + e_{\eta,t},\ \ e_{\eta,t} \sim N(0, \sigma_\eta), \tag{C.65}$$

$$\log \varepsilon_{mh,t} = \rho_{mh} \log \varepsilon_{mh,t-1} + e_{mh,t},\ \ e_{mh,t} \sim N(0, \sigma_{mh}), \tag{C.66}$$

$$\log \varepsilon_{mk,t} = \rho_{mk} \log \varepsilon_{mk,t-1} + e_{mk,t},\ \ e_{mk,t} \sim N(0, \sigma_{mk}), \tag{C.67}$$

$$\log A_t = \rho_A \log A_{t-1} + e_{A,t},\ \ e_{A,t} \sim N(0, \sigma_A). \tag{C.68}$$

$$\log \varepsilon_{\theta_R,t} = \rho_{\theta_R} \log \varepsilon_{\theta_R,t-1} + e_{\theta_R,t},\ \ e_{\theta_R,t} \sim N(0, \sigma_{\theta_R}). \tag{C.69}$$

$$\log \varepsilon_{\theta_b,t} = \rho_{\theta_b} \log \varepsilon_{\theta_b,t-1} + e_{\theta_b,t},\ \ e_{\theta_b,t} \sim N(0, \sigma_{\theta_b}). \tag{C.70}$$

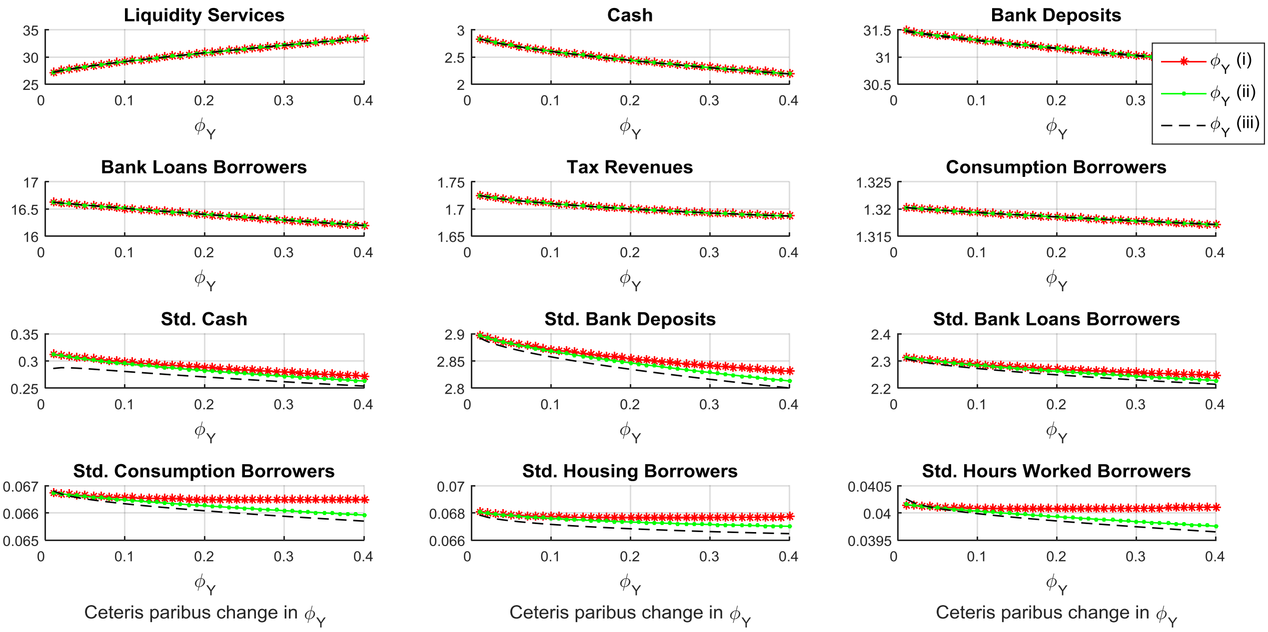
In addition to the above defined shocks, the model also allows for zero-mean, AR(1), CBDC preference and supply shocks, ϑ_t and $\varepsilon_{cbdc,t}$, under CBDC policy scenarios:

$$\log \vartheta_t = \rho_\vartheta \log \vartheta_{t-1} + e_{\vartheta,t},\ \ e_{\vartheta,t} \sim N(0, \sigma_\vartheta), \tag{C.71}$$

$$\log \varepsilon_{cbdc,t} = \rho_{cbdc} \log \varepsilon_{cbdc,t-1} + e_{cbdc,t},\ \ e_{cbdc,t} \sim N(0, \sigma_{cbdc}). \tag{C.72}$$

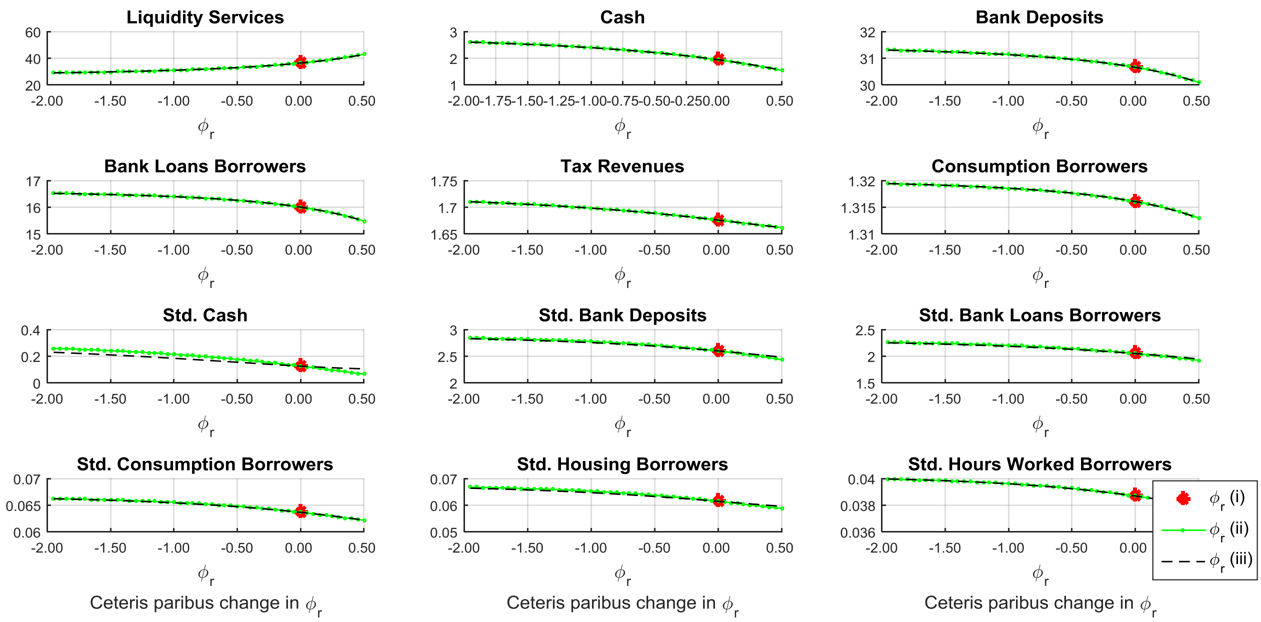
D Complementary Figures to Section 4.2

Figure D.1: Mean and volatility effects of CBDC quantity rules (welfare effects of ceteris paribus changes in ϕ_Y)



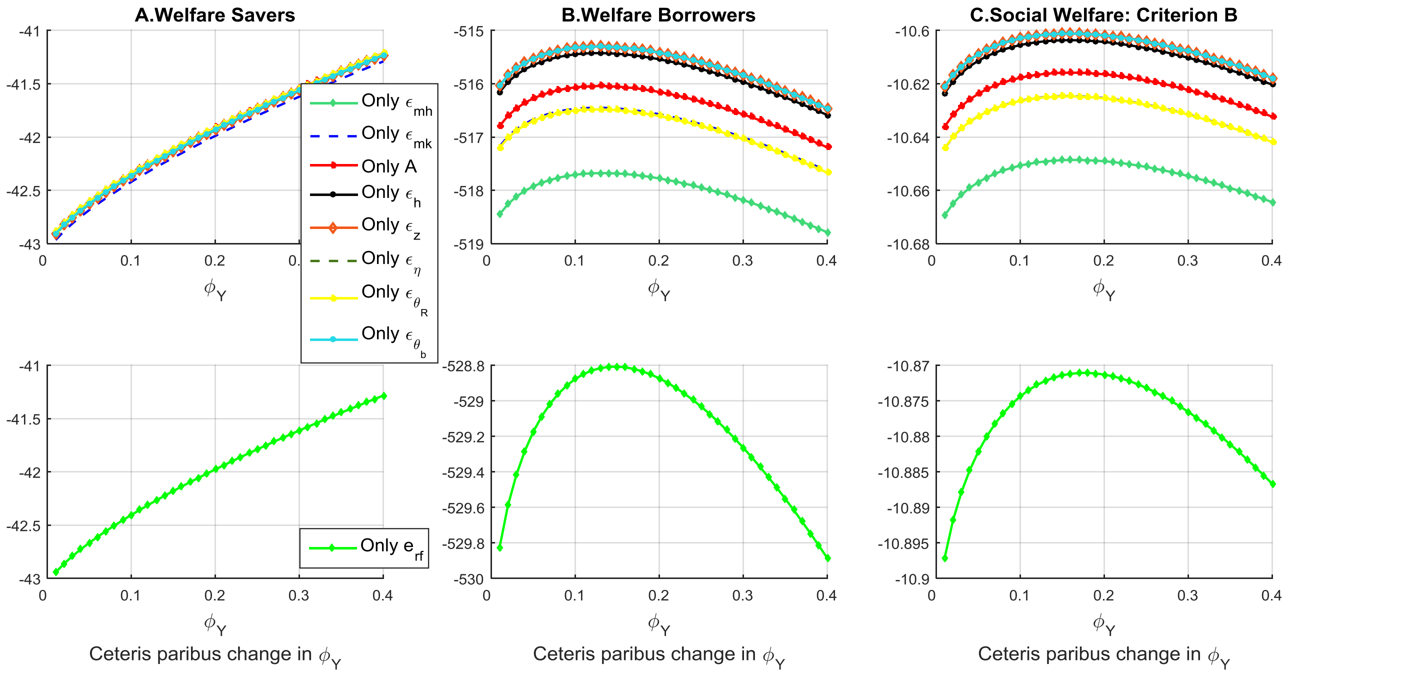
Notes: Second-order approximation to the unconditional mean and standard deviation of key selected aggregates as a function of CBDC policy parameter ϕ_Y. The starred line, the dotted line, and the dashed line relate to CBDC quantity rules (i), (ii) and (iii), respectively.

Figure D.2: Mean and volatility effects of CBDC interest rate rules (welfare effects of ceteris paribus changes in ϕ_r)



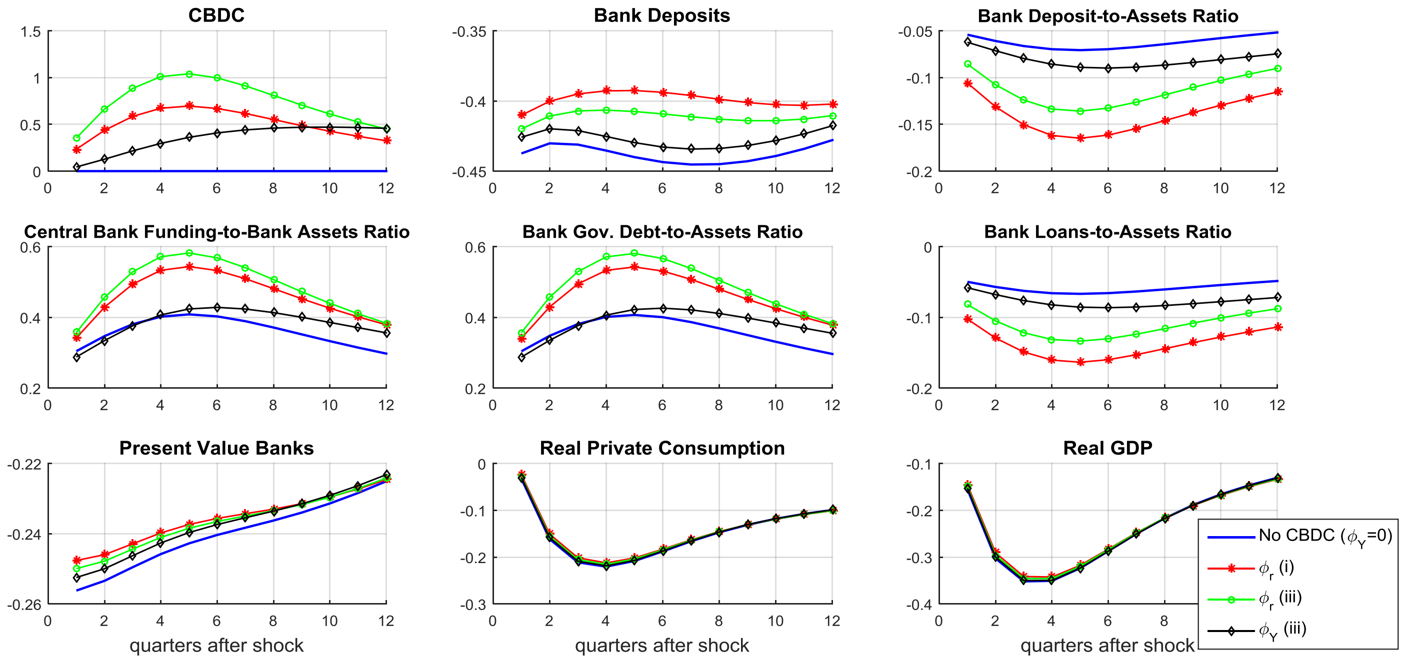
Notes: Second-order approximation to the unconditional mean and standard deviation of key selected aggregates as a function of CBDC policy parameter ϕ_r. The star line, the dotted line, and the dashed line relate to CBDC interest rate rules (i), (ii) and (iii), respectively.

Figure D.3: Welfare effects of CBDC quantity rules by types of shocks (shutting down shocks)



Notes: Second-order approximation to the unconditional welfare of savers and borrowers as well as to the unconditional social welfare under welfare criteria “A” and “B” as a function of CBDC policy parameter ϕ_Y under quantity rule of type (ii). Each of the 9 lines informs about the welfare effects of ceteris paribus changes in ϕ_Y when only one of the 9 types of shocks that are considered under the baseline calibration hits this model economy.

Fig D.4: Impulse-responses to a negative technology shock



Notes: Variables are expressed in percentage deviations from the steady state. Social welfare has been maximized under welfare criterion “B”. The solid line refers to the baseline (no CBDC) scenario. The starred line corresponds to interest rate rule (i). The dotted line relates to the CBDC optimal policy rule within the class of interest rate rules (iii). The diamond line makes reference to the CBDC quantity rule of type (iii) that maximizes social welfare.