Private Money and Self-Fulfilling Prophecies*

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Abstract

This paper develops a model to study how the use of private assets as payment instruments can lead to self-fulfilling booms and busts. In the model, privately issued payment instruments are claims on the profits of firms that operate in frictional markets. In these markets, agents devote effort to get matched to trading partners and need money to settle transactions. When agents search more intensely, economic activity expands and private money has higher value. As a result, agents' liquidity constraints loosen and it becomes attractive for them to search more intensely. A strategic complementarity in search effort arises and it is sufficiently strong to generate endogenous booms and busts. To stabilize the economy, monetary policy should stabilize inflation and ensure that agents' liquid wealth remains unaffected by changes in the value of private money. Helicopter money, unsecured lending, or a troubled-asset relief program are shown to be effective in this respect.

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1 Introduction

How does private money creation enable self-fulfilling booms and busts, and what can policy do to stabilize the economy? To answer this question, this paper develops a money-search model in which privately issued assets are used as payment instruments. In the model, a strategic complementarity operating through the value of private assets generates a coordination problem in real economic activity, which is reminiscent of that in Diamond (1982). Due to the coordination problem, I find that the usage of private assets as payment instruments can give rise to self-fulfilling prophecies. These can be eliminated when a government issues fiat money and conducts an active monetary policy. That means, inflation needs to be stable and in case of a financial panic, which can turn into a bust, policies reminiscent of TARP and emergency lending need to be conducted.

What motivates the current paper is the fact that in developed economies, many privately issued assets have money-like properties. For example, the rapid advance of exchange-traded funds (ETFs) is making it increasingly easy to trade well-diversified portfolios of stocks and commercial bonds at short notice and low cost (Lettau & Madhavan, 2018). Such properties allow ETFs to become, just like the liabilities of commercial banks (mostly deposits), a near-substitute for fiat money. The usage of private assets as monetary substitutes is however perceived to make economies vulnerable to financial panics.[1] This was demonstrated by the experience of the 2007/2008 financial crisis, which in turn has led to a hot debate on restricting and regulating private money creation.[2]

Furthermore, as a response to the financial crisis and the role of private, money-like assets therein, central banks have resorted to unconventional policies such as asset purchasing programs, which nowadays even include the purchase of ETFs.[3] Such policies are controversial and furthermore, they are oftentimes financed by the issuance of fiat money. Because fiat money is, by definition, intrinsically useless, it can be a source of other self-fulfilling phenomena, such as cyclical inflation dynamics. Moreover, the sources underlying the destabilizing nature of private money may interact with those giving rise to the self-fulfilling phenomena inherent to fiat money.

The aim of this paper is to gain a better theoretical understanding of these sources and how they interact with each other, as well as to study what monetary policy can do to stabilize the economy. For this purpose, I develop a tractable model of a monetary economy akin to that of Lagos and Wright (2005). In the model, buyers and firms participate in alternating frictional markets, in which there is a transactions-based demand for assets, and frictionless markets, in which the agents re-balance their asset positions.

[1] This idea goes back to Fisher (1936) and other proponents of the so-called Chicago Plan.

[2] In 2018 Switzerland held a referendum on a popular initiative to provide the SNB (the Swiss Central Bank) with the sole authority to create money. The initiative was rejected by 76% of the voters.

[3] The Bank of Japan began purchasing equity ETFs in October 2010 and the U.S. Federal Reserve started purchasing commercial bond ETFs in May 2020.

In the frictional market, bilateral meetings between buyers and firms take place. In the spirit of Pissarides (1984), buyers and firms are matched according to a constant returns-to-scale matching function and the matching probabilities depend on buyers' search effort. In this sense, the frictional market can be thought of as a market in which buyers want to purchase complicated, tailor-made goods. This requires the buyers to search for firms which have the expertise to produce such goods. Furthermore, due to information frictions in the spirit of Kocherlakota (1998), there is a transactions-based role for assets, which gives rise to a liquidity constraint for the buyers. That means, the buyers need to settle transactions on the spot by paying with assets.

A key feature of the framework is that private assets are modeled as tradable claims on the profits of the firms. Due to this feature, which also allows the assets to be interpreted as ETFs, there is a strategic complementarity in search effort: If buyers increase their search effort, more matches get realized and the firms' profits increase. This raises the value of private assets, which loosens the liquidity constraint of a buyer so that the surplus of a realized match with a firm increases. In turn, this incentivizes more search effort.

The strategic complementarity in search effort can give rise to a coordination problem in economy activity. To understand it, I first analyze matters when only private assets are used as payment instruments. Due to the coordination problem, I find a multiplicity of steady states – one in which buyers devote high search effort (a boom) and one in which buyers devote low search effort (a bust). Furthermore, there is a continuum of cyclical and stochastic equilibria in which search effort fluctuates over time. Contrasting existing models from the money-search literature, these results arise even though private assets are one-period lived – a feature which implies that private assets are priced at fundamental value, so that there are no asset price bubbles.

To introduce a role for policy, I proceed by adding fiat money to the set of payment instruments. The supply of fiat money is managed by a government that targets inflation. In an environment in which the inflation target is attained but the target itself is away from the Friedman rule, so that fiat money acts as a risk-free but costly substitute for private assets, I find a unique deterministic equilibrium – the multiplicity of steady states and the existence of deterministic cycles disappears. However, because holding fiat money is costly, private assets remain accepted in payment, and because asset positions can only be adjusted in the frictionless market, search effort in the frictional market can still fluctuate stochastically. So, even though fiat money is available as a risk-free alternative for private assets, there still exist stochastic equilibria exhibiting boom-bust dynamics.

To eliminate the remaining stochastic equilibria, I show that policies similar to those proposed by Berentsen and Waller (2011, 2015) need to be conducted in the frictional market. Basically, when agents believe that private assets have low value – a situation resembling a financial panic – the government should temporarily inject additional fiat money into the economy. This injection serves to prevent a tightening of liquidity constraints, which would otherwise incentivize low search effort and rationalize the financial panic. The injection is thus set according to a feedback rule, which responds to the value of private assets. Because the creation of liquidity in real terms is crucial to prevent a tightening of liquidity constraints, the interaction between a monetary authority and a fiscal authority matters for the effectiveness of policy. I shed light on this issue by separating the government into a central bank and treasury department, and then proceed by studying three empirically relevant ways in which the central bank can inject liquidity.

First, I analyze helicopter money, meaning that the central bank lump-sum distributes freshly printed money to the buyers. Only when undone by future lump-sum taxation, these injections have real effects. Fiscal support for the central bank is therefore crucial.

Second, I analyze a troubled-asset relief program (TARP), in which the central bank injects liquidity by buying troubled private assets at a premium. That means, when there is a financial panic (a fear of coordination on low search effort), the central bank offers to purchase private assets at the high search equilibrium price. For TARP to work, again fiscal support for the central bank is crucial. Otherwise, when the buyers devote low search effort, the central bank incurs losses which imply a permanent injection of fiat money – money is injected by purchasing assets at price exceeding fundamental value, generating inflation which renders the purchases ineffective.

Third, if the central bank can enforce repayment, it can provide emergency loans to the buyers in case of a panic. These loans have no inflationary effects as long as they are repaid – the injected money is withdrawn upon the repayment of the loans. Should default nevertheless occur, the central bank again requires support by the treasury.

Finally, I study how the strategic complementarity in search effort can interact with self-fulfilling inflation dynamics. For this purpose, I drop the assumption of inflation targeting and study matters when monetary policy is passive. That means, the supply of fiat money grows at a fixed rate. It turns out that the strategic complementarity, which arises due to the nature of private assets, increases the scope for self-fulfilling inflation dynamics. In this sense, my paper complements the subfield of the money-search literature which focuses on such dynamics.[4]

This paper also provides two other contributions to the money-search literature. First, it identifies a feature making private assets special in causing macroeconomic instability: their fundamental value depends on economic activity and this gives rise to a strategic complementary in search effort. Second and related to the strategic complementarity, the paper demonstrates that in a monetary economy, the fundamental value of an asset need not be determined uniquely. This contrasts existing papers which have focused on self-fulfilling dynamics in liquidity premia and inflation.

[4]See for example Lagos and Wright (2003), who study self-fulfilling inflation dynamics a the money-search model of fiat money, or Azariadis (1981), who studies such dynamics a OLG model of fiat money.

Besides contributing to the money-search literature, in its analysis of stabilization policies, this paper complements a growing literature on the importance of the coordination between fiscal and monetary policy. Finally, by focusing on the interaction between liquidity creation, liquidity constraints, and economic activity, this paper also adds to a diverse literature on the inherent instability of financial intermediation.

Section 2 of this paper reviews the literature mentioned above in more detail. In Section 3 I set up the model and in Section 4 I analyze matters with only private assets. Section 5 analyzes how matters change with risk-free fiat money and Section 6 considers how monetary policy should respond to financial panics. Section 7 considers self-fulfilling inflation dynamics when monetary policy is passive. Section 8 concludes the paper.

2 Related literature

This paper relates closely to money-search papers with a role for assets other than fiat money.[5] Some, following Lucas (1978), treat dividends paid by private assets as exogenous. Examples are Geromichalos, Licari, and Suárez-Lledó (2007), Lagos (2010), Rocheteau and Wright (2013), and Geromichalos and Herrenbrueck (2016, 2017). Others let dividends be determined by outcomes in frictionless markets. Examples are Lagos and Rocheteau (2008), Andolfatto, Berentsen, and Waller (2016), Altermatt (2017), and Van Buggenum (2021). In these setups, the fundamental value of assets affects outcomes in frictional markets, but does not depend on outcomes in frictional markets. Self-fulfilling asset price dynamics can arise in these models, but only when assets are infinitely lived and oftentimes also only for specific utility functions and bargaining protocols.

Altermatt, Iwasaki, and Wright (2021) provide a very rich model to study self-fulfilling asset price and inflation dynamics in a money-search environment, as they study a setup with both fiat money and private assets. Private assets in their model bear an exogenous dividend, and they find cycles only if (i) money supply grows at a fixed rate or (ii) private assets are infinitely lived. Also, to obtain cycles, they have to rely on specific parametrizations for utility functions. In my framework, with utility satisfying assumptions that are standard in the literature, self-fulfilling prophecies arise even with one-period lived assets and an active monetary policy that stabilizes inflation.

Closest to my paper in terms of modeling private assets is an extension of the baseline model in Rocheteau and Wright (2013) and the model of Branch and Silva (2019). Rocheteau and Wright (2013) study a setup somewhat similar to the environment in the current paper, in which the fundamental value of liquid assets is determined in markets in which these assets are used in payment. However, Rocheteau and Wright (2013) do not include fiat money in their model, so they cannot study monetary policy, and their mecha-

[5]See Lagos, Rocheteau, and Wright (2017) for a review of the money-search literature.

nism does not work through search intensity but trough firm entry. For such a mechanism, or others that endogenize sellers' participation decisions, regardless of the nature of liquid assets there can exist multiple equilibria due to coordination issues, whereas my approach identifies features making private assets special as drivers of coordination failures.[6]

Branch and Silva (2019) study an economy akin to that of Mortensen and Pissarides (1994), with households self-insuring against Aiyagari (1994) style liquidity shocks by holding bonds as well as shares in the Mortensen-Pissarides firms. This setup produces a strong aggregate demand effect of employment, potentially producing multiple equilibria. The authors consider an environment with a passive supply of bonds, whereas I introduce a government that actively manages money supply. Also, they find multiplicity only for specific parameterizations, whereas multiplicity in my environment is more general.

Conceptually, the current paper also relates to papers from the labor-search literature that study self-fulfilling prophecies in unemployment. Howitt and McAfee (1987) show that when the labor market matching function exhibits increasing returns-to-scale, there are multiple equilibrium unemployment rates. Howitt and McAfee (1992) and Kaplan and Menzio (2016) show a similar result but consider constant returns-to-scale in the labor market matching function and incorporate a demand effect of low unemployment. The current paper incorporates a demand effect of search effort – when search effort increases, liquidity constraints loosen as the value of private assets increases.

In its analysis of stabilization policies, this paper contributes to the literature pioneered by Sargent and Wallace (1981), which studies the interaction between the budget constraint of the central bank and that of the treasury department. This topic received renewed attention due to the deployment of unconventional monetary policy, as substantial losses from these policies may leave a central bank with negative equity – a potential threat to price stability which may call for fiscal intervention (Reis, 2015; Tanaka, 2021). Similar to me, albeit in a New Keynesian model, Del Negro and Sims (2015) show that with large central bank balance sheets, controlling inflation requires fiscal backing. Benigno (2020) shows that a central bank can control inflation without fiscal backing, but this requires sufficient ex-ante capitalization of the central bank by the treasury.

Finally, this paper fits into a broad literature which shows how various aspects of financial intermediation, for example the provision of liquidity insurance (Peck & Shell, 2003), market-making (Rubinstein & Wolinsky, 1987), the role of intermediaries' reputation (Gu, Mattesini, Monnet, & Wright, 2013), the creation of information insensitive liabilities (Gorton & Ordoñez, 2014), *etcetera*, generate instability. Gu, Monnet, Nosal, and Wright (2020) review many of these aspects analytically. My contribution is to focus on the creation of liquid claims backed by economic activity, and in a setup in which the economy is affected by liquidity constraints and search frictions.

[6] See Rocheteau and Wright (2005), Berentsen, Menzio, and Wright (2011), and Nosal and Rocheteau (2011) for models with coordination failures driven by endogenous participation decisions by sellers.

3 Model

Time is discrete and denoted with $t \in \mathbb{N}$. The time horizon is infinite. At time t, two markets convene sequentially: first a decentralized market (DM) and then a centralized market (CM). The DM is a frictional market in which payment instruments and search effort are essential. The CM is a frictionless market in which agents re-balance their asset portfolios. There are two fully perishable and perfectly divisible goods: special goods and general goods, which are traded in the DM and the CM, respectively. General goods are used as the numeraire, so all prices and real values are expressed in general goods.

The economy is populated by a unit measure of infinitely lived buyers, overlapping generations of finitely lived firms, and a government. There is a single-coincidence of wants in the DM because buyers want to consume the special good while only firms can produce the special good.

Buyers' preferences are described by the flow utility function

$$\mathcal{U}(q, e, y) = u(q) - s(e) + y \tag{1}$$

and they discount utility between periods at a rate $\beta \in (0, 1)$. In Equation (1), q is the consumption of special goods, e is search effort, and y is the net consumption of general goods. In the DM, buyers choose search effort $e \in E \subseteq [0, 1]$ at utility cost $s : E \to \mathbb{R}_+$. Search effort is normalized to equal the probability of being able to acquire special goods in the DM, which will be described in further detail below, and search costs are increasing and strictly convex in search effort. Additionally, u is twice continuously differentiable and satisfies $u(0) = 0$, $u' > 0$, $u'' < 0$, $\lim_{q \to 0} u'(q) = \infty$, and $\lim_{q \to \infty} u'(q) = 0$.

In each CM a measure one of firms, owned by the buyers and living until the next CM, arises.[7] In the DM firms receive an endowment of y general goods from which they can produce q specialized goods by using $c(q)$ general goods as an input, where $c(0) = 0$, $c' > 0$, and $c'' \geq 0$. General goods unused in the DM are stored until the CM.

Two assets are available in the economy. The first are perfectly divisible ownership shares of firms, labeled as private assets, and I normalize the amount of shares issued by each firm to one. The second asset is a perfectly divisible and intrinsically useless object called fiat money, which is issued by the government. Once I turn towards analyzing the coordination problem in search effort, I will first study matters when fiat money is in zero supply, and then I study how supplying fiat money can stabilize the economy.

Formally, all the aggregate uncertainty in the economy comes from a sunspot – a random variable irrelevant for preferences and technologies. Before markets convene at time t, the sunspot generates a realization which is observed by all agents, who in turn

[7] The results hold true when the firms are owned by, for instance, entrepreneurs who have no transactions-based motive to hold assets. This is because these entrepreneurs will then find it attractive to sell claims on their profits to the buyers.

coordinate their behavior based on this realization. As a tool to simplify the analysis, I assume the existence of tradable claims on general goods, which are in zero net supply and which generate a one-shot payoff in the time t CM, contingent on the full history of realizations of the sunspot up to and including time t. These assets resemble Arrow securities and would arise naturally if the firms can freely securitize their profits.[8]

In what follows, to simplify the notation, I will index all prices, quantities, and values with t rather than with the full history of the sunspot. Variables and functions indexed with t are therefore to be treated as (potentially) stochastic objects.

3.1 Centralized markets

The CM is a perfectly competitive market in which the buyers re-balance their asset positions and the incumbent firms pay dividends. Due to the existence of state-contingent Arrow securities, the law of one price (LOOP) implies that an asset which has value x_{t+1} in CM_{t+1}, trades at a price

$$\mathbb{E}_t \left\{ \beta(1 + \iota_{t+1}) x_{t+1} \right\} \tag{2}$$

in CM_t, where $\beta(1+\iota_{t+1})$ is the stochastic discount factor. I write the stochastic discount factor as $\beta(1+\iota_{t+1})$ because, as follows from to the quasi-linear utility structure, the asset would be priced at $\mathbb{E}_t \left\{ \beta x_{t+1} \right\}$ when it would not be tradable in DM_{t+1}. This notation therefore allows ι_{t+1} to be interpreted as a stochastic liquidity premium.

The CM_t prices of fiat money and private assets are, respectively, ϕ_t and Υ_t. By construction, the price of a private asset is the market value of a newborn firm. For the price of fiat money, the LOOP (2) implies

$$\phi_t = \mathbb{E}_t \left\{ \beta(1 + \iota_{t+1}) \phi_{t+1} \right\}. \tag{3}$$

Buyers: Let a_{t+1} be the value of the asset portfolio that a buyer carries into DM_{t+1}. According to the LOOP, the CM_t value of it equals $\mathbb{E}_t\{\beta(1 + \iota_{t+1})a_{t+1}\}$. Here, a_{t+1} can be chosen contingent on $\mathcal{H}_{t+1}$, that is the full history of the sunspot's realizations up to and including time $t + 1$. Buyers are however characterized by limited commitment, so $a_{t+1} \geq 0$ – buyers can neither short-sell assets nor issue assets. Write $V_{t+1}(a_{t+1})$ for the utility value of entering DM_{t+1} with assets worth a_{t+1} and let τ_t denote a lump-sum tax imposed on the buyers in CM_t. Since a measure one of newborn firms, owned by the buyers, arise in CM_t, the buyers also receive private assets worth Υ_t. The utility value

[8] When an asset market without state-contingent claims would produce different outcomes, it is in the interest of the firms to securetize their profits into state-contingent liabilities. Doing so increases the set of choices for the buyers and these state-contingent assets can therefore be issued at a premium. Furthermore, when buyers have access to a real risk-free asset, it turns out that the state-contingent assets are not affecting the results presented in this paper. Details are available on request.

of entering CM_t with assets worth a_t is therefore:

$$W_t(a_t) = \max_{y_t, \{a_{t+1}\} \forall \mathcal{H}_{t+1}} \{y_t + \beta \mathbb{E}_t\{V_{t+1}(a_{t+1})\}\} \tag{4}$$
$$\text{s.t.} \quad y_t + \mathbb{E}_t\{\beta(1+\iota_{t+1})a_{t+1}\} + \tau_t \leq a_t + \Upsilon_t \quad \text{and} \quad a_{t+1} \geq 0 \; \forall \mathcal{H}_{t+1}.$$

The budget constraint in Equation (4) binds for optimal choices and since utility is linear in the consumption of general goods, the value function is affine in a_t. It can therefore be written without the need to account explicitly for history:

$$W_t(a_t) = a_t + \Upsilon_t - \tau_t + \beta \mathbb{E}_t \left\{ \max_{a_{t+1} \geq 0} \{V_{t+1}(a_{t+1}) - a_{t+1}(1+\iota_{t+1})\} \right\}. \tag{5}$$

Firms that are about to die: Consider a firm that is about to die, which holds assets worth a_t and an inventory o_t of general goods. The dividend paid by this firm is

$$F_t(a_t, o_t) = a_t + o_t.$$

Newborn firms: For a newborn firm in CM_t, dividends paid in CM_{t+1} depend on the sunspot's history at time $t+1$, as well as on idiosyncratic factors related to outcomes in DM_{t+1}. Let F^e_{t+1} denote the history-contingent expected dividends paid by the firm in CM_{t+1}. Because idiodyncratic risk is not priced, the CM_t value of a newborn firm can be written as:

$$\Upsilon_t = \mathbb{E}_t\{\beta(1+\iota_{t+1})F^e_{t+1}\}. \tag{6}$$

Government: The government can print money and levy lump-sum taxes on (or provide subsidies to) the buyers. The nominal supply of fiat money, measured at the end of CM_t, is denoted with M_t. Lump-sum taxation is such that the government's budget constraint holds:

$$\tau_t = \phi_t(M_{t-1} - M_t). \tag{7}$$

3.2 Decentralized markets

In the DM, the buyers are randomly matched to the firms. The probability that a buyer ends up in a match with a firm equals the level of search effort devoted by the buyer. A firm finds a match with a buyer with a probability equal to the average search effort across buyers, the latter being denoted with $\tilde{e}$.[9] In Appendix A I show that a setup with two sided search effort produces the same result as the model presented here.

[9] The setup can be microfounded with a constant returns-to-scale matching function $\min\{b, f\}$, where f is the measure of firms and b is the effective measures of buyers – the measure of buyers multiplied by their average level of search effort. The measure of realized matches is then $\min\{\tilde{e}, 1\}$, the probability a buyers finds a match is $e \min\{\tilde{e}, 1\}/(\tilde{e}) = e$, and the probability a firm finds a match is $\min\{\tilde{e}, 1\} = \tilde{e}$.

In what follows, I make two assumptions. First, for the set of feasible levels for search effort, I use $E = \{l, h\}$, with $0 \leq l < h \leq 1$ and $s(h) - s(l) = k$. This makes the mechanism more transparent and is not critical for the results.[10] Second, I assume that $l > 0$, which rules out recurrent market freezes. This is without loss of generality.

In the DM, monitoring and record-keeping are sufficiently bad to rule out credit arrangements. Assets are therefore needed as a payment instruments. Furthermore, communication is limited in the sense that a matched buyer and firm are unable to observe what happens in other matches.

Bargaining: Let (q, p) denote the terms of trade in a pairwise DM meeting, with q the amount of special goods received by the buyer and p the value of the assets received by the firm. Using the linearity of W_t, the utility surplus for the buyer is then $u(q) - p$. Furthermore, since the firm uses $c(q)$ units of general goods as an input in production, profit from the transaction for the firm is $p - c(q)$.

In bargaining, firms are interested in maximizing the utility of their shareholders. Due to limited communication, during bargaining the firm and the buyer rationally disregard the effects of changes in the firm's profits on other DM matches. The reason is that in other DM matches, the expected profits of the firm in question are taken as given. Furthermore, since their is a continuum of firms and matching is random, changes in the profit of the firm leave the value of the buyer (with which the firm bargains) his/her assets unaffected. Hence, since expressed in general goods, the firm's profits from the transaction directly represent its shareholders' utility gain from the transaction.

Given the above, total surplus from the transaction equals $u(q)-c(q)$. The transaction is subject to a liquidity constraint $p \leq a$, where a denotes the value of the buyer's assets, and a capacity constraint $c(q) \leq y$. Rather than imposing a specific bargaining solution, I follow the more general approach developed by Gu and Wright (2016). This means the bargaining outcome is summarized by a payment protocol $v : q \mapsto p$, mapping the traded quantity of special goods into a required payment by the buyer. When q special goods are traded, utility surplus of the buyer is then $L(q) = u(q) - v(q)$ and profits for the firm are $\Pi(q) = v(q) - c(q)$. A buyer chooses q to maximize $L(q)$ subject to $v(q) \leq a$ and $c(q) \leq y$. In what follows, I assume that the capacity constraint is always slack.

Let q^* solve $u'(q) = c'(q)$ – the first-best level for q. Assume that the payment protocol

[10] When facing a liquidity premium associated with carrying assets, increased search effort makes it more attractive for buyers to also increase their asset holdings. This is because assets can then be spend on special goods with a higher probability. Taking this complementarity between search effort and asset holdings into account, marginal benefits of exerting search effort are increasing in the level of search effort. Therefore, though optimal search effort will be generically unique if E is a convex set, the set of search effort levels implementable in equilibrium exhibits gaps when costs of search are close to linear – search effort may jump from a high level to a low level for an infinitesimally small change in the liquidity premium. If search cost would be linear or concave, then for convex $E = [\underline{e}, \overline{e}]$ we get that, depending on asset holdings, buyers either choose $\underline{e}$ or $\overline{e}$.

is twice continuously differentiable and such that $v(0) = 0$, $v' > 0$, $L(q)$ attains a unique global maximum at $\hat{q} \in (0, q^*]$ and is strictly increasing in q for $q \in (0, \hat{q})$, $\Pi(q) > 0$ for $q \in (0, \hat{q}]$, and $\Pi'(q) > 0$ for $q \in (0, \hat{q}]$. These conditions are satisfied for a broad set of bargaining solutions, including Nash (1950) bargaining, proportional bargaining à la Kalai (1977), and gradual bargaining as in Rocheteau, Hu, Lebeau, and In (2021), as well as a payment protocol representing constant markup pricing.

Given v, the liquidity constraint binds when $a < v(\hat{q})$. The terms of trade therefore become

$$q = \begin{cases} v^{-1}(a) & \text{if } a < v(\hat{q}) \\ \hat{q} & \text{if } a \geq v(\hat{q}) \end{cases} \quad \text{and} \quad p = \begin{cases} a & \text{if } a < v(\hat{q}) \\ v(\hat{q}) & \text{if } a \geq v(\hat{q}) \end{cases}.$$

To ensure the capacity constraint is indeed always slack, I impose $y \geq c(\hat{q})$.

Buyers: When exerting search effort $e \in \{l, h\}$, a buyer is matched to a firm with probability e. Accounting for the linearity of buyers' CM_t value function, for a buyer the value of entering DM_t with assets worth a_t is:

$$V_t(a_t) = \max_{e \in \{l,h\}} \left\{ eL\left(\min\{v^{-1}(a_t), \hat{q}\}\right) - s(e) \right\} + a_t + W_t(0). \tag{8}$$

A buyer is willing devote search effort level $e = h$ during the DM if and only if $(h - l)L\left(\min\{v^{-1}(a_t), \hat{q}\}\right) \geq k$. Similarly, it is willing devote search effort level $e = l$ during the DM if and only if $(h - l)L\left(\min\{v^{-1}(a_t), \hat{q}\}\right) \leq k$. This implies a positive relationship between asset holdings and search effort.[11]

Firms: Let $G_t(a', e')$ denote the history-contingent probability that a randomly drawn buyer in DM_t holds assets worth $a'' \leq a'$ and devotes search effort $e'' \leq e'$. Using the properties of F_t, a firm is therefore expected to pay as history-contingent CM_t dividend

$$F_t^e = \iint e'\Pi\left(\min\{v^{-1}(a'), \hat{q}\}\right) \mathrm{d}G_t(a', e') + y. \tag{9}$$

Intuitively, upon entering DM_t, firms receive an endowment of y general goods. Then, each firm draws a buyer from the CDF G_t. This buyer carries assets worth a' and devotes search effort e'. A match with the buyer occurs with probability e' and results in additional profits $\Pi\left(\min\{v^{-1}(a'), \hat{q}\}\right)$. The firm can thus be thought of as a one-period lived asset in the spirit of Lucas (1978), but with a partially endogenous dividend.

[11]In the model, there are no income effects due to the quasi-linear utility structure. Even with income effects, a positive relationship between asset holdings and search effort arises. The reason is that optimal search effort depends only on the surplus of a match for the buyer, which in turn should depend positively on the value of the buyer's asset holdings even in the presence of income effects.

3.3 Equilibrium and welfare

Using Equation (8), we can write $W_{t-1}(a) = a + \Upsilon_{t-1} - \tau_{t-1} + \beta \mathbb{E}_t \left\{ \widetilde{W}_t + W_t(0) \right\}$, where $\widetilde{W}_t$ captures the buyer's maximization problems corresponding to time t:

$$\widetilde{W}_t = \max_{\{a_t, e_t\} \in \mathbb{R}_+ \times \{l,h\}} \left\{ e_t L \left(\min\{v^{-1}(a_t), \hat{q}\} \right) - s(e_t) - \iota_t a_t \right\}. \tag{10}$$

In equilibrium, asset holdings and search effort should be in line with Equation (10). Furthermore, the aggregate value of assets at the beginning of time t is $\phi_t M_{t-1} + F_t^e$. That is, the value of fiat money (if supplied by government) plus the history-contingent dividends paid by the firms. An equilibrium can therefore be defined as follows:

Definition 1. *Given a (potentially stochastic) process $\{M_{t-1}\}_{t=0}^{\infty}$ for fiat money supply, equilibrium is a stochastic process $\{G_t : \mathbb{R}^2 \rightarrow [0,1], F_t^e, \iota_t, \phi_t, \Upsilon_t\}_{t=0}^{\infty}$ such that:*

1. *The LOOP holds: ϕ_t satisfies Equation (3) and Υ_t satisfies Equation (6) .*
2. *Markets clear: $\iint a' G_t(a', e') = \phi_t M_{t-1} + F_t^e$ with F_t^e given by Equation (9).*

For welfare, I consider a utilitarian measure. Welfare as of time t is then given by integrating over the DM_t value functions of the buyers, taking into accounting the asset distribution at time t: $U_t = \iint V_t(a') \mathrm{d}G_t(a', e')$.

Lemma 1. *Equilibrium welfare satisfies $U_t = \mathcal{W}_t + \beta \mathbb{E}_t \{U_{t+1}\}$, where*

$$\mathcal{W}_t = \iint \left[e'(u - c) \circ \min\{v^{-1}(a'), \hat{q}\} - s(e') \right] \mathrm{d}G_t(a', e') + y. \tag{11}$$

Relevant for flow welfare – denoted with $\mathcal{W}_t$ – is the surplus from DM activity plus the firms' endowment of general goods. The former equals aggregated surplus across DM matches minus the search costs incurred by the buyers.

In what follows I will focus on symmetric equilibria, in which all buyers behave identically. Goods markets, asset markets, and welfare then behave as discussed below.

Goods markets: Taking as given the realization of the stochastic liquidity premium ι_t, consider activity in DM_t. From Equation (10), it follows that asset demand becomes infinitely large when the stochastic liquidity premium is negative. Hence, focus on cases in which $\iota_t \geq 0$. The optimality condition for buyers' asset holdings then implies

$$\iota_t = e_t L'(q_t) / v'(q_t), \tag{12}$$

meaning that the liquidity benefits of the marginal asset equal the realization of the stochastic liquidity premium. To keep things simple, I ensure that q_t is determined uniquely as a function of e_t and ι_t by means of the following assumption:

Assumption 1. *The payment protocol is such that* $L'(q)/v'(q)$ *is strictly decreasing in* q *on the domain* $(0, \hat{q})$.

The liquidity value of the marginal asset is, given Assumption 1, decreasing in the value of the buyer's assets.[12] The work of Gu and Wright (2016) then implies q_t is continuous in ι_t/e_t, decreasing in ι_t/e_t, strictly decreasing in ι_t/e_t for $\iota_t/e_t \in (0, \bar{\iota})$, and equals $\hat{q}$ for $\iota_t/e_t = 0$ and zero for $\iota_t/e_t \geq \bar{\iota} \equiv \lim_{q \to 0} L'(q)/v'(q)$.

Regarding search effort, recall that $k = s(h) - s(l)$. As a result, buyers are willing to search at $e = h$ if and only if

$$\max_{q \geq 0} \{hL(q) - \iota_t v(q)\} - \max_{q \geq 0} \{lL(q) - \iota_t v(q)\} \geq k.$$

Similarly, buyers are willing to search at $e = l$ if and only if

$$\max_{q \geq 0} \{hL(q) - \iota_t v(q)\} - \max_{q \geq 0} \{lL(q) - \iota_t v(q)\} \leq k.$$

Here, $\max_{q \geq 0} \{hL(q) - \iota_t v(q)\} - \max_{q \geq 0} \{lL(q) - \iota_t v(q)\}$ is decreasing in ι_t, and strictly decreasing in ι_t for $\iota_t \in (0, h\bar{\iota})$. In words, buyers are more likely to search intensely when the stochastic liquidity premium is low. When $k > (h - l)L(\hat{q})$, buyers are unwilling to search intensely even when the liquidity premium equals zero. To generate some action in terms of search effort, I therefore assume:

Assumption 2. $k \leq (h - l)L(\hat{q})$.

Given Assumption 2, buyers are willing to search intensely when the stochastic liquidity premium equals zero. However, when the stochastic liquidity premium becomes sufficiently large, buyers will, for a uniquely determined threshold $\tilde{\iota}$ which depends on k, switch to devoting low search effort:

$$e_t = \begin{cases} h & \text{if } \iota_t < \tilde{\iota} \\ h \quad \text{or} \quad l & \text{if } \iota_t = \tilde{\iota} \\ l & \text{if } \iota_t > \tilde{\iota} \end{cases}. \tag{13}$$

Concluding, e_t, and therefore also q_t are, except for a knife-edge case with $\iota_t = \tilde{\iota}$, determined uniquely by ι_t. Observe that when $\iota_t = \tilde{\iota} > 0$, consumption drops from $\underline{q}_H$ to $\overline{q}_L$ when buyers switch from searching at $e = h$ to $e = l$. Here, $\underline{q}_H$ and $\overline{q}_L$ solve $\tilde{\iota} = hL'(\underline{q}_H)/v'(\underline{q}_H)$ and $\tilde{\iota} = lL'(\overline{q}_L)/v'(\overline{q}_L)$, and we have $\overline{q}_L < \tilde{q} < \underline{q}_H$ where $(h - l)L(\tilde{q}) = k$.

[12]When terms of trade within DM matches are determined by proportional bargaining and gradual bargaining, this property is always satisfied. When terms of trade are determined by Nash bargaining, this property is satisfied when the bargaining power of the buyer is sufficiently large.

Asset markets: Optimality conditions associated with Equation (10) imply the DM demand for assets satisfies

$$a_t \geq v(q_t), \quad \text{with equality if } \iota_t > 0. \tag{14}$$

That means, when facing a strictly positive liquidity premium, buyers hold exactly the amount of assets needed to purchase the desired quantity of special goods q_t. When the liquidity premium is zero, asset demand is indeterminate but subject to a lower bound, so that buyers hold at least the assets required to purchase the desired quantity q_t.

Asset supply consists of the value of private assets, which equals the expected CM_t dividend payment by firms, and (if supplied by the government) the value of fiat money:

$$a_t = \phi_t M_{t-1} + e_t \Pi(q_t) + y. \tag{15}$$

Welfare: In a symmetric equilibrium, taking into account the relationship between a_t and q_t as described by Equation (14), the expression for flow welfare (11) becomes:

$$\mathcal{W}_t = e_t \left[u(q_t) - c(q_t)\right] - s(e_t) + y.$$

Using (12) and (13), flow welfare can be expressed as a function $\mathscr{W}(\iota_t)$ of the stochastic liquidity premium.[13] Welfare is maximized when the stochastic liquidity premium is at the zero lower bound. When the realization of the stochastic liquidity premium increases, buyers start economizing on asset holdings and trade of special goods within DM matches falls below $\hat{q}$, resulting in reduced welfare.

Also, when ι_t increases beyond $\tilde{\iota}$, buyers reduce their search effort. Though buyers are indifferent between searching at a high or low level when $\iota_t = \tilde{\iota}$, welfare jumps down when buyers reduce their search effort. This is because buyers fail to internalize the effect of search effort on firms. Moreover, when $\iota_t > 0$, a change in search effort also implies a jump in the amount of assets held by the buyers.

4 Matters in an economy with only private assets

Having established some useful properties of goods markets, asset markets, and welfare in symmetric equilibria, I now turn towards a more detailed characterization of these equilibria. To understand how the use of private assets as payment instruments can lead to a coordination problem in search effort, in this section I first consider matters when fiat money is in zero supply.

[13] Strictly speaking $\mathscr{W}$, is a correspondence as allocations are not uniquely determined by ι_t when $\iota_t = \tilde{\iota}$. Because this is a knife-edge case, we can treat $\mathscr{W}$ as a function without loss of generality.

Setting $M_{t-1} = 0$ and then combining the equations for asset demand (14) and asset supply (15) with $q_t \leq \hat{q}$ (with equality if and only if $\iota_t = 0$), we obtain:

$$e_t \Pi(q_t) + y \geq v(q_t), \quad \text{with} = \text{if } q_t < \hat{q}. \tag{16}$$

Both asset demand (the RHS of (16)) and asset supply (the LHS of (16)) are increasing in q_t. However, the demand effect always dominates since $\Pi'(q) \leq v'(q)$. Intuitively, more trade in special goods requires the firms to devote more inputs to production, so the matched firms' revenues increase faster than profits. Equation (16) therefore uniquely maps levels of search effort e_t into values for $q_t \in [0, \hat{q}]$.

Additionally, according to Equation (12), the tuple (e_t, q_t) can be mapped into the realization for the stochastic liquidity premium. In turn, according to Equation (13), this premium has to rationalize the level of devoted search effort. Therefore, we need

$$\frac{e_t L'(q_t)}{v'(q_t)} \begin{cases} \leq \tilde{\iota} & \text{if } e_t = h \\ \geq \tilde{\iota} & \text{if } e_t = l \end{cases}. \tag{17}$$

It follows that a private asset equilibrium (henceforth PAE) is suffiently described by a stochastic process $\{e_t, q_t\}_{t=0}^{\infty}$ that satisfies the system (16)-(17).

Recall that we have two feasible levels for search effort and that given search effort, we have a unique value for the traded amount of special goods. Let q_h (q_l) denote the amount of special goods traded in DM matches when buyers devote search effort h (resp. l). Clearly, q_l can be observed on the equilibrium path if and only if $lL'(q_l)/v'(q_l) \geq \tilde{\iota}$, and q_h can be observed on the equilibrium path if and only if $hL'(q_h)/v'(q_h) \leq \tilde{\iota}$. Dependent on the value for $\tilde{\iota}$, we therefore have either a unique PAE or a continuum of PAEs. In the latter case, due to a coordination problem in search effort, there are two steady states – one with high search effort and one with low search effort – as well as a continuum of cyclical equilibria because any deterministic or stochastic process governing the selection of $(e_t, q_t) \in \{(l, q^l), (h, q^h)\}$ is an equilibrium.

Proposition 1. *A PAE always exists. Furthermore, there is set of search costs k and endowments y for which there is a continuum of PAEs. This set has positive measure.*

A parameterized example: To gain some understanding for the multiplicity of PAEs and the underlying coordination problem, I consider a parameterized example with closed form solutions. Let $u(q) = q^{1-\varrho}/(1-\varrho)$ with $\varrho \to 1$ so that $u'(q) \to 1/q$. Let $c(q) = q$ and consider a payment protocol representing constant mark-up pricing. Specifically, $v(q) = (1+\sigma)q$ with $\sigma > 0$ so that $L(q) = q^{1-\varrho}/(1-\varrho) - (1+\sigma)q$, $\Pi(q) = \sigma q$, and

$\hat{q} = 1/(1+\sigma)$. The asset market clearance condition (16) then becomes

$$\sigma e_t q_t + y \geq (1+\sigma) q_t, \quad \text{with} = \text{if } q_t < 1/(1+\sigma).$$

In turn, this implies

$$q_t = \min\left\{ \frac{y}{1+\sigma(1-e_t)}, \frac{1}{1+\sigma} \right\}.$$

Due to the positive effect of search effort on firms' profits we have that $q_l \leq q_h$ – more search effort, through the firms' profits, increases the value of private assets which loosens the buyers' liquidity constraint and allows for more trade within matches. The necessary and sufficient condition for $q_l < q_h$ is $q_l \leq \hat{q} = 1/(1+\sigma)$. In turn, this requires the firms' endowment y to be sufficiently small. Intuitively, the endowment acts as a lower bound on the value of a firm and therefore, through the liquidity constraint, as a lower bound on the traded amount of special goods. At the same time, to ensure the firms' capacity constraint is always slack we need the endowment to be sufficiently large: $y \geq \hat{q} = 1/(1+\sigma)$. Summarizing, we need

$$y \in \left[\frac{1}{1+\sigma}, \frac{1+\sigma(1-l)}{1+\sigma} \right), \tag{18}$$

which is a non-empy set for $\sigma > 0$.

Revisiting from the buyers' DM value function, househols determine search effort in the DM to maximize $eL(q) - s(e)$. Since the surplus from a DM match is increasing in q on the domain $[0, \hat{q}]$, with two feasible levels of search effort there exists a threshold $\tilde{q}$ below (above) which buyers devote a low (resp. high) level of search effort. This threshold depends on the search costs k according to $(h-l)L(\tilde{q}) = k$. Choosing $k \in [(h-l)L(q_l), (h-l)L(q_h)]$, which is a non-empty set with positive measure whenever $q_l < q_h$, we can therefore rationalize multiple potential outcomes in the DM.

At the root of multiple DM outcomes is a coordination problem in search effort. Because claims on the profits of the firms are accepted in payment in the DM and because the profits of these firms depend on DM activity, there is a strategic complementary in devoting search effort. Specifically, if all other buyers in the economy devote more search effort, expected profits of the firms increase as these firms get matched to buyers with a greater probability. In turn, because the claims on the profits of the firms are accepted in payment, these higher profits loosen a buyer's liquidity constraint. As a result, the surplus from being matched in the DM increases and it becomes attractive for a buyer to increase search effort.

To ensure a true multiplicity of equilibria, meaning the coordination problem manifests itself as an event that occurs with positive probability, we also need to take into account buyers' choice for asset holdings. Using Equation (12), for the current parameterization we find

$$\iota_t = e_t \max\left\{\frac{1+\sigma(1-e_t)}{y(1+\sigma)} - 1, 0\right\}.$$

Revisiting from the analysis of the goods markets, there exists a threshold $\tilde{\iota}$ for the stochastic liquidity premium above (below) which buyers' choice of asset portfolios is such that they exert a low (resp. high) level of search effort in the DM. To rationalize the occurrence of both high and low search effort in equilibrium, we therefore need

$$h \max\left\{\frac{1+\sigma(1-h)}{y(1+\sigma)} - 1, 0\right\} \leq l\left[\frac{1+(1-l)}{y(1+\sigma)} - 1\right]. \tag{19}$$

When Condition (19) is satisfied, there exist k rationalizing the buyers' choice of asset portfolios in the presence of equilibrium multiplicity. This set for k is a strict subset of the one which rationalizes multiple potential outcomes in the DM. Furthermore, when (19) holds with strict inequality, this set has positive measure. Note that (19) is always satisfied when $h[1+\sigma(1-h)] \leq l[1+\sigma(1-l)]$ and becomes less likely to hold when y decreases in case $h[1+\sigma(1-h)] > l[1+\sigma(1-l)]$. Combining with the previously derived set for y in (18), we find that for all

$$y \in \left[\max\left\{\frac{h[1+\sigma(1-h)] - l[1+\sigma(1-l)]}{(h-l)(1+\sigma)}, \frac{1}{1+\sigma}\right\}, \frac{1+\sigma(1-l)}{1+\sigma}\right),$$

which is a non-empty set, there exist k for which we have a continuum of PAEs. Specifically, any process governing the selection of $(e_t, q_t) \in \{(l, q_l), (h, q_h)\}$, where

$$q_l = \frac{y}{1+\sigma(1-l)} \quad \text{and} \quad q_h = \min\left\{\frac{y}{1+\sigma(1-h)}, \frac{1}{1+\sigma}\right\},$$

is a an equilibrium when search costs k are chosen appropriately.

Discussion: Well-known in monetary theory is that with finitely lived assets, there cannot be self-fulfilling dynamics in liquidity premia. This insight is based on models in which, following Lucas (1978), assets earn an exogenously specified dividend. Since a finitely lived asset is priced fundamentally when it matures, through backwards induction ruling out self-fulfilling price dynamics, a unique equilibrium arises.

In the current environment, private assets are the sole medium of exchange and also finitely lived. They are also priced fundamentally when traded in the DM – Equation (15) demonstrates that the value of private assets equals the aggregate dividend payment by the firms. Nevertheless, the dividend payment by the firms depends on activity in the DM. In turn, through the buyers' liquidity constraint, activity in the DM depends on the value of the private assets. This intricate relationship between economic activity and the fundamental value of assets gives rise to a coordination problem in search effort, entailing the existence of multiple equilibria which resemble booms and busts.

In a boom (bust), buyers devote high (resp. low) search effort so that many (resp. few) matches are realized in the DM and the fundamental value of private assets is high (resp. low). In turn, this high (low) value for private assets incentivizes high (resp. low) search effort because the liquidity constraints are relatively loose (resp. tight). The economy can switch between a boom and bust in both a deterministic and stochastic fashion. The use of private assets as in payment thus gives rise to rich boom-bust dynamics.

5 Fiat money as an alternative means of payment

To introduce a role for government in stabilizing the economy, I now turn towards studying matters when government issues fiat money to compete with private assets as a means of payment. In doing so, I am going to suppose that the government wants to achieve a time-invariant inflation target to ensure that fiat money earns a risk-free return. Specifically, the nominal price of general goods should increase at a gross rate π in between periods.

To achieve its inflation target, the government lets the supply of fiat money be driven by demand. That means, M_t is such that given ϕ_t, supply of money balances equals demand for money balances given the achievement of the target. Profits (losses) from seignorage (resp. redemption) are then automatically turned into lump-sum subsidies for (resp. taxes on) buyers so that the government's budget constraint (7) is respected. I assume that the government indeed achieves its objective, so that the CM price of fiat money develops deterministically according to $\pi\phi_{t+1} = \phi_t$.[14] In Section 7, I will relax this assumption and study how the coordination problem in search effort can interact with self-fulfilling inflation dynamics.

Because the price of fiat money develops deterministically in the current setup, fiat money is a risk-free asset. Using Equation (3) and the law of motion $\pi\phi_{t+1} = \phi_t$, we then obtain the following relationship between inflation and the stochastic liquidity premium:

$$i \equiv (\pi - \beta)/\beta = \mathbb{E}_{t-1}\{\iota_t\} \qquad \text{if } \phi_t > 0. \tag{20}$$

The LHS of (20) is often referred to as the Fisher nominal interest rate – the nominal rate which compensates buyers exactly for inflation and their rate of time preference. When fiat money is in positive real supply, the Fisher rate pins down the risk-free rate, given by the RHS of (20), through a no-arbitrage condition.

[14]This feature implies a deviation from papers which show that when M_t grows at a constant rate, there can be equilibria in which ϕ_t develops in a stochastic or cyclical fashion. From an empirical perspective, focusing on stable inflation is realistic since inflation dynamics tend to be smooth and inflation expectations are well-anchored. Levin, Natalucci, and Piger (2004), Demertzis, Marcellino, and Viegi (2009), and Gürkaynak, Swanson, and Levin (2010) show that this is especially true for inflation targeting regimes.

Furthermore, in equilibria with the coexistence of fiat money and private assets, private assets should not drive fiat money out of existence. Combining (14) and (15), this requires

$$e_t \Pi(q_t) + y \leq v(q_t) \quad \text{if} \quad \iota_t > 0. \tag{21}$$

Combining (21) with Equations (12) and (13) produces a set $\mathcal{I}$ so that for all $\iota_t \in \mathcal{I}$, Condition (21) is satisfied. Just like in standard money-search models, with constant search effort this set takes the form $\mathcal{I}_e = [0, \hat{\iota}_e]$, where $\hat{\iota}_e = eL'(q_e)/v'(q_e)$ and q_e is the unique solution of $v(q) \leq e\Pi(q) + y$ (with $=$ if $q < \hat{q}$) which we have considered in the analysis of PAEs. Combining with the critical threshold $\tilde{\iota}$, we therefore have $\mathcal{I} = ([0, \hat{\iota}_h] \cap [0, \tilde{\iota}]) \cup ([0, \hat{\iota}_l] \cap [\tilde{\iota}, \infty))$, which can be a non-convex set.

In what follows, I first look at deterministic coexistence equilibria (henceforth DCEs) and then at stochastic coexistence equilibria (henceforth SCEs). In a DCE, all uncertainty regarding matters at time t is resolved already at time $t-1$. Equation (20) then implies $\iota_t = i$. Combining the latter with Equations (12) and (13) implies that a DCE is sufficiently described by a pair (e, q) satisfying

$$i = eL'(q)/v'(q) \quad \text{and} \quad e = \begin{cases} h & \text{if } i < \tilde{\iota} \\ l \text{ or } h & \text{if } i = \tilde{\iota} \\ l & \text{if } i > \tilde{\iota} \end{cases}.$$

DCEs exist if and only if $i \in \mathcal{I}$ and, except for a knife-edge case with $i = \tilde{\iota}$, there is a unique DCE. In a DCE, fiat money and private assets are perfect substitutes and therefore earn the same return when held in between CMs. This return is determined by inflation through the Fisher rate, pinning down the liquidity premium commanded by all assets in the economy. In turn, the liquidity premium uniquely determines buyers' search effort and asset demand except for a knife-edge case with $i = \tilde{\iota}$.Because inflation pins down the liquidity premium through the Fisher rate, welfare in the DCE is decreasing in inflation and maximized when $\pi = \beta$. Then, $i = 0$ and all liquidity constraints are slack. Setting $\pi = \beta$ is commonly known as the Friedman rule.

Next, consider stochastic coexistence equilibria (SCEs). In these equilibria, at time $t-1$ it is still uncertain what the economic outcomes will be at time t. That means, agents coordinate their DM_t behavior on the realization of the sunspot. With the price of fiat money developing according to $\pi\phi_{t+1} = \phi_t$, combining (14) and (15) implies

$$e_t \Pi(q_t) + y + \phi_{t-1} M_{t-1}/\pi \geq v(q_t), \quad \text{with } = \text{if } q_t < \hat{q}. \tag{22}$$

In DM_t, the value of fiat money balances acts as a predetermined variable, so that according to a similar argument as in Section 4, q_t is pinned down by e_t through Equation (22).

Because search effort can only take two values, with stable inflation we can therefore focus on a sunspot with two realizations, H and L. Agents coordinate on high (low) search effort when the realization of the sunspot is H (resp. L). Because of the quasi-linear utility structure, history in period $t-1$ affects allocations in period t only through the expectations operator $\mathbb{E}_{t-1}$. It therefore suffices to characterize SCEs for sunspot processes that are independently and identically distributed over time, so let ρ_H (ρ_L) denote the probability that the sunspot's realization is H (resp. L). Obviously we have $\rho_L + \rho_H = 1$, and we can index quantities and values with the sunspot's realization instead of with t. Using Equations (6), (9), and (13)-(20), an SCE is sufficiently described by a tuple $(\rho_L, \rho_H, q_L, q_H) \in \Delta^1 \times [q_l, \hat{q}] \times [q_h, \hat{q}]$ satisfying the following system of equations:

$$i = \rho_L l L'(q_L)/v'(q_L) + \rho_H h L'(q_H)/v'(q_H), \tag{23}$$

$$hL'(q_H)/v'(q_H) \leq \tilde{\imath} \leq lL'(q_L)/v'(q_L), \tag{24}$$

$$(1-h)\Pi(q_H) + c(q_H) \leq (1-l)\Pi(q_L) + c(q_L), \quad \text{with} = \text{if } q_H < \hat{q}. \tag{25}$$

Equation (23), which combines (13) and (20), ensures that the stochastic liquidity premium satisfies the LOOP (3) and rationalizes the quantity of trade within DM matches. Equation (24), following from (13), ensures it is optimal for buyers to exert search effort h (l) when the realization of the sunspot is H (resp. L). Equation (25) ensures the existence of a value for fiat money to clear the asset market. Because we restrict attention to $(q_L, q_H) \in [q_l, \hat{q}] \times [q_h, \hat{q}]$, this value for fiat money is not only independent of the sunspot's realization but also positive – fiat money is not driven out of existence.

Proposition 2. *There exists a set of Fisher rates, search costs, and endowments for which SCEs exist. This set has positive measure. If for given parameters an SCE exists, then SCEs exist for Fisher rates $i \in (\underline{i}, \bar{\imath})$. For given parameters, if $i < \tilde{\imath}$ then for given probabilities (ρ_L, ρ_H) an SCE exists if and only if $\rho_L \in [\underline{\widetilde{\rho_L}}, \overline{\rho_L}]$. Similarly, if $i > \tilde{\imath}$ then for given probabilities (ρ_L, ρ_H) an SCE exists if and only if $\rho_H \in (0, \min\{\overline{\rho_H}, 1 - \underline{\widetilde{\rho_L}}\}]$, and in a knife-edge case with $i = \tilde{\imath}$ an SCE exists for all probabilities such that $\rho_L \geq \underline{\widetilde{\rho_L}}$. Finally, for fixed parameters, a fixed Fisher rate i, and fixed probabilities (ρ_L, ρ_H), there can only exist a unique SCE.*

The characterizations of $\underline{i}$, $\bar{\imath}$, $\overline{\rho_L}$, $\underline{\widetilde{\rho_L}}$ and $\overline{\rho_H}$ are in the proof of Proposition 2. Intuitively, because private assets are still accepted in payment, the underlying coordination problem in search effort is still present in the DM. Therefore, after portfolio decisions have been made in CM_{t-1}, search effort can still behave stochastically in DM_t.

The bounds $\overline{\rho_L}$ and $\overline{\rho_H}$ follow from the system (23)-(25). If in the DCE buyers exert high (low) search effort then an SCE exists when the probability of coordinating on low (resp. high) search effort is sufficiently small. To understand why, consider a case in which the DCE is characterized by high search effort. When buyers coordinate on low search with a probability approaching one ($\rho_L \to 1$), the stochastic liquidity premium when the sunspot turns out L approaches the Fisher rate ($\iota_L \to i$). As a result, holding assets is relatively cheap and therefore buyers want to devote high search effort even when the realization of the sunspot is L.

The bound $\widetilde{\underline{\rho_L}}$, which is independent from the Fisher rate, follows from the requirement $(q_L, q_H) \in [q_l, \hat{q}] \times [q_h, \hat{q}]$ – it ensures that fiat money is not driven out of existence. Intuitively, and proven when turning towards qualitative properties of sunspot equilibria, buyers' demand for fiat money balances decreases when the probability ρ_L of coordinating on low search effort falls. To ensure fiat money is not driven out of existence, an additional lower bound on ρ_L therefore arises.[15]

Corollary 1. *The closer is the Fisher rate, i, to the critical threshold $\tilde{\imath}$ triggering a change in buyers' search decisions, the larger is the set of sunspot probabilities for which SCEs exist – $\partial\overline{\rho_L}/\partial i > 0$, $\partial\overline{\rho_H}/\partial i < 0$, $\lim_{i\uparrow\tilde{\imath}} \overline{\rho_L} = \lim_{i\downarrow\tilde{\imath}} \overline{\rho_H} = 1$, and $\lim_{i\downarrow\underline{i}} \overline{\rho_L} = \lim_{i\uparrow\bar{\imath}} \overline{\rho_H} = 0$.*

According to Corollary 1, the set of sunspot probabilities for which we have an SCE grows as the Fisher rate i approaches the critical threshold $\tilde{\imath}$. This is because buyers are indifferent between exerting search effort h or l when $i = \tilde{\imath}$.

Corollary 2. *For a given parametrization of the model we have that as policy approaches the Friedman rule ($i \to 0$), except for a knife-edge case with $k = (h - l)L(\hat{q})$, the set of sunspot probabilities for which an SCE exists vanishes.*

As we approach the Friedman rule, $i \to 0$. However, in an SCE the realization of the stochastic liquidity premium ι when the sunspot turns out L is bounded from below by the threshold $\tilde{\imath}$. Otherwise, buyers devote high search effort when the realization of the sunspot is L. Except for the knife-edge case $k = (h - l)L(\hat{q})$, the threshold $\tilde{\imath}$ is strictly positive. Hence, when approaching the Friedman rule the probability ρ_L of buyers coordinating on low search effort becomes arbitrarily small, as otherwise $\mathbb{E}_{t-1}\{\iota_t\} > i$.

This result points towards the fact that opportunity costs associated with holding fiat money are undesirable because of two reasons. First, as holds true in many microfounded models of money, these costs lead to binding liquidity constraints, which in turn leads to less economic activity and lower welfare. Second, as Corollary 2 demonstrates, these opportunity costs increase the reliance of the economy on privately created means of payment, which opens the door for the coordination problem in search effort. The welfare consequences thereof are however ambiguous, as I discuss further below.

A parameterized example: To gain further appreciation for SCEs and their existence, I consider the same parametrization as before. Let $m = \phi_t M_{t-1}$ denote the value of fiat

[15] This bound is zero when a high-search DCE exists.

money balances carried into DM_t, which is independent of the realization of the sunspot since $\pi\phi_t = \phi_{t-1}$. The asset market clearance condition becomes

$$\sigma e_t q_t + y + m \geq (1+\sigma) q_t, \quad \text{with} = \text{if } q_t < \hat{q},$$

where the value of fiat money is now incorporated into asset supply. Intuitively, firms' aggregate endowment and the value of fiat money constitute the exogenous part of asset supply in the DM. Like before, the endogenous part of asset supply equals firms' profit from operating in the DM. Trade in special goods therefore satisfies

$$q_t = \min\left\{\frac{y+m}{1+\sigma(1-e_t)}, \frac{1}{1+\sigma}\right\}$$

and as in PAEs, we have $q_L \leq q_H$. Focusing on the case in which the capacity constraint is always slack and $q_L < q_H$, we now need

$$y \in \left[\frac{1}{1+\sigma}, \frac{1+\sigma(1-l)}{1+\sigma} - m\right), \tag{26}$$

which is a non-empty set when the value of fiat money is sufficienty small. For all y in (26) and according to a similar reason as in the PAE, a coordination problem arises due to the strategic complementarity in search effort.

While being taken as given in DM_t, m is determined endogenously in CM_{t-1} and depends on the behavior of the stochastic liquidity premium

$$\iota_t = e_t \max\left\{\frac{1}{1+\sigma}\frac{1+\sigma(1-e_t)}{y+m} - 1, 0\right\}.$$

For the LOOP (3) to hold, we therefore need

$$i = \rho_L l \left(\frac{1+\sigma(1-l)}{(1+\sigma)(y+m)} - 1\right) + \rho_H h \max\left\{\frac{1+\sigma(1-h)}{(1+\sigma)(y+m)} - 1, 0\right\}.$$

Summarizing, the sunspot probabilities and, through the Fisher rate, the government's inflation target determine the equilibrium value for m.

To demonstrate the possibility of sunspot equilibria, I let $\rho_L \to 0$. This represents a setup in which a low search effort outcome in the DM is deemed possible but also highly unlikely. The equilibrium value for m and the allocations in case of a high search outcome in the DM then approach their DCE values:

$$q_h = \frac{1}{1+\sigma}\frac{h}{i+h} \quad \text{and} \quad m \geq \frac{h[1+\sigma(1-h)]}{(i+h)(1+\sigma)} - y, \quad \text{with} = \text{if } i > 0. \tag{27}$$

Ceteris paribus, a lower Fiser rate i increases trade within DM matches as well as the demand for assets. Increased trade within matches increases the supply of assets through the firms' dividends but, as established earlier, this effect is dominated by the increase in the demand for assets. To clear the asset market, m therefore adjusts endogenousy.

Without loss of generality, I focus on matters when policy is away from the Friedman rule. Otherwise, according to the LOOP (3), the stochastic liquidity premium always needs to be at the ZLB, which in turn implies $q_L = q_H = \hat{q}$ and is an equilibrium only for the knife-edge case with $\tilde{\imath} = 0$ or, equivalently, $k = (h - l)L(\hat{q})$.

Away from the Friedman rule, we have $q_H < \hat{q}$ and therefore $q_L < q_H$ is satisfied automatically – we can ignore the upper bound in (26). To ensure $m \geq 0$ and the capacity constraint is always slack, the relevant set for the firms' endowment becomes

$$y \in \left[\frac{1}{1+\sigma}, \frac{1+\sigma(1-h)}{(1+\sigma)(1+i/h)}\right), \tag{28}$$

which is a strict subset of (26) and a non-empty set whenever $i \leq \sigma h(1-h)$.

With m positive and pinned down uniquely by Equation (27) q_l satisfies

$$q_l = \frac{1}{1+\sigma}\frac{h}{i+h}\frac{1+\sigma(1-h)}{1+\sigma(1-l)},$$

which is decreasing in i since higher opportunity cost of holding assets reduce asset demand. In turn, this leads to less trade on the intensive margin as the liquidity constraint becomes tighter.

Finally, we need buyers' portfolio decisions to be rationalized in an environment where both high and low search effort occur with strictly positive probability. In a state in which buyers coordinate on low search effort, this requires the stochastic liquidity premium to exceed the Fisher rate:

$$i \leq l\left[\frac{1+\sigma(1-l)}{(1+\sigma)(y+m)} - 1\right]. \tag{29}$$

With (29) satisfied, there exist values for the critical threshold $\tilde{\imath}$ such that buyers indeed devote low (high) search effort when the realization of the sunspot is L (resp. H). Using (27) in (29), we find an additional upper bound for the Fisher rate

$$i \leq \frac{hl(h-l)}{h[1+\sigma(1-h)] - l[1+\sigma(1-l)]} \quad \text{if } h[1+\sigma(1-h)] > l[1+\sigma(1-l)]. \tag{30}$$

When (30) holds with strict inequality or when $h[1+\sigma(1-h)] \leq l[1+\sigma(1-l)]$, the set of values for the critical threshold $\tilde{\imath}$ (or equivalently set of values for k) such that buyers indeed devote low (high) search effort when the realization of the sunspot is L (resp. H), has positive measure. Thus, for i and y such that (28) and (30) hold, we indeed obtain a sunspot equilibrium in which buyers devote low search effort with a small but strictly positive probability. As Proposition 2 demonstrates, this result does not hinge on the choice of parameters or letting $\rho_L \to 0$.

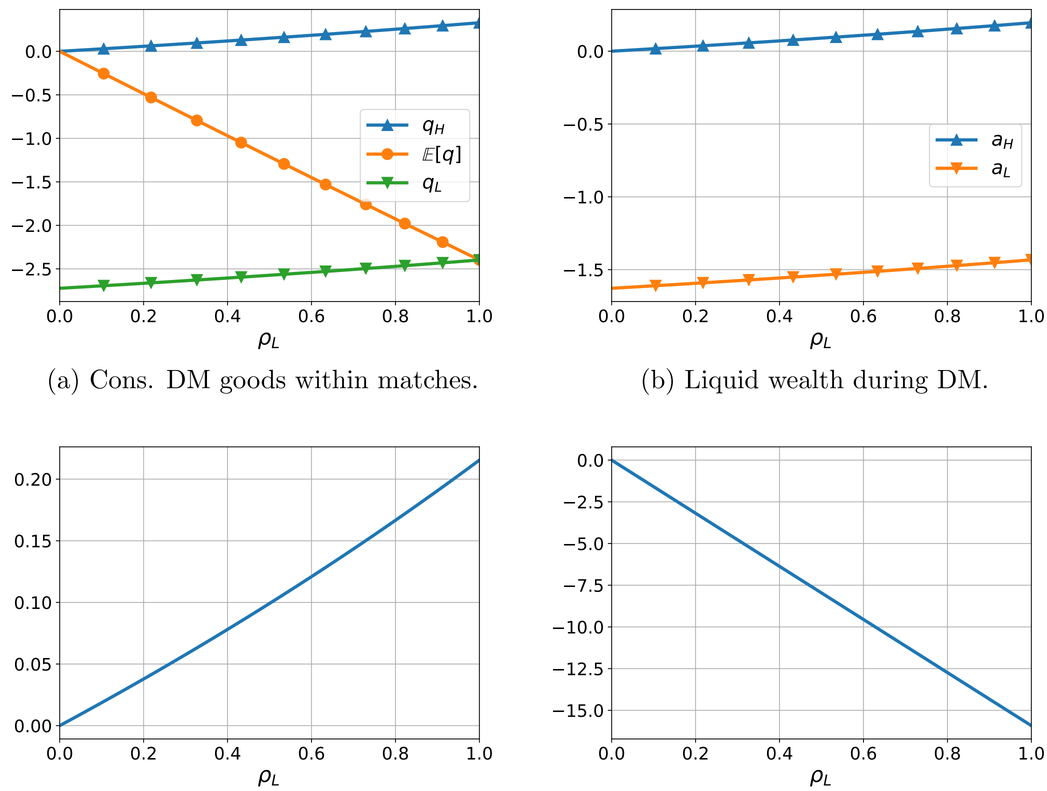
Welfare properties: What are the welfare implications of the coordination problem, i.e. the existence of SCEs? As a benchmark, consider flow welfare in the DCE and denote it with $\overline{\mathcal{W}}$. Using (13), we can express it as function of the Fisher rate: $\overline{\mathcal{W}} = \mathscr{W}(i)$. For expected flow welfare in an SCE, write $\widetilde{\mathcal{W}} = \mathbb{E}_{t-1}\{\mathcal{W}_t\}$. Again using (13), we find $\widetilde{\mathcal{W}} = \mathbb{E}_{t-1}\{\mathscr{W}(\iota_t)\}$.Finally, in SCEs the LOOP (20) links the stochastic liquidity premium to the Fisher rate according to $\mathbb{E}\{\iota_t\} = i$. Comparing welfare in a SCE to that in a DCE is therefore the same as comparing $\mathbb{E}_{t-1}\{\mathscr{W}(\iota_t)\}$ to $\mathscr{W}(\mathbb{E}_{t-1}\{\iota_t\})$.

Proposition 3. *Welfare in an SCE can be higher or lower than welfare in the DCE.*

Due to the non-linearity of the flow utility functions u and c, $\mathscr{W}$ is nonlinear and because it exhibits a jump at $\tilde{\iota}$, it can be locally convex or concave. As a result, Jensen's inequality implies $\mathbb{E}\{\mathscr{W}(\iota_t)\}$ can be greater or smaller than $\mathscr{W}(\mathbb{E}\{\iota_t\})$. The proof of Proposition 3 shows that on the one hand, if the Fisher rate is just below the threshold $\tilde{\iota}$, then SCEs attain less welfare than the DCE because with strictly positive probability, the buyers devote low search effort in the SCE. In turn, this generates a big drop in realized flow welfare compared to the DCE – characterized by high search effort – exactly because of the drop in welfare when switching from high to low search effort. On the other hand, when the Fisher rate is slightly above the threshold $\tilde{\iota}$, then SCEs attain more welfare than the DCE because with strictly positive probability, the buyers devote high search effort in the SCE. In turn, this generates a big increase in realized flow welfare compared to the DCE – characterized by low search effort – exactly because of the upwards jump in welfare when switching from low to high search effort.

Qualitative properties: I conclude the analysis of SCEs by discussing their qualitative properties. Figure 1a plots, for a toy calibration, the traded amount of special goods within matches for the two realization of the sunspot. These quantities are plotted against ρ_L – the probability that the realization of the sunspot is L or equivalently, as I shall use to explain the qualitative effects of SCEs, the probability of a bust. In a similar vein, I call the state in which the realization of the sunspot is H a boom. Paradoxically, both in a bust and a boom trade within matches is monotonically increasing in the anticipated probability of a bust. This relates to how the economy depends on private assets as means of payment and is clarified by Figure 1b, which shows the DM value of buyers' assets in a boom – a_H – and bust – a_L.

Intuitively, private assets are bad at providing liquidity services when a bust is likely to occur. When portfolio decisions are made, buyers will therefore demand more fiat money, as Figure 1c illustrates. When a bust then indeed hits, buyers' asset are worth more than when a bust was perceived unlikely. Similarly, when a boom occurs buyers are



(a) Cons. DM goods within matches.

(b) Liquid wealth during DM.

(c) Fiat money carried out of CM.

(d) Private assets carried out of CM.

Figure 1: Real quantities as functions of the probability the sunspot turns out to be L. Real quantities are expressed as a percentage deviation from their value in the DCE.

flush with assets, resulting in more trade within pairwise matches. In expectation, trade within matches is however decreasing in the probability of a bust since trade in a bust is always lower than trade in a boom.

Additionally, Panel 1d shows that the ex-ante value of private assets is decreasing in the probability of a bust. Intuitively, in busts few firms find matches and the buyers spend little. So the more likely a bust, the lower the price at which shares in newborn firms trade. The proposition below demonstrates that the patters in Figure 1 are not specific to the choice of parameters for the toy calibration.

Proposition 4. *For given parameters and a given Fisher rate, in SCEs:*

1. *Trade of special goods within DM matches weakly increases with ρ_L.*
2. *The value of fiat money, $\phi_t M_{t+1}$, strictly increases with ρ_L.*
3. *The DM value of assets, a_L and a_H, strictly increases with ρ_L.*
4. *The value of newborn firms satisfies $\lim_{\rho_L \to 0} \Upsilon > \lim_{\rho_L \to 1} \Upsilon$.*

Discussion: To conclude this section, the results show that a government can partially mitigate the destabilizing nature of the search coordination problem by means of issuing a risk-free fiat money. Specifically, instead of a multiplicity of steady states and a continuum of cyclical equilibria when only private assets are used in payment, with fiat money and a constant inflation rate, there is a unique deterministic equilibrium. In the deterministic equilibrium, allocations depend on the inflation target.

Nevertheless, because buyers cannot re-balance their asset portfolios in the DM, introducing a risk-free fiat money does not eliminate the existence of stochastic equilibria. Specifically, boom-bust dynamics can still occur when policy deviates from the Friedman rule. The reason is that away from the Friedman rule, holding fiat money invokes opportunity costs, so that buyers still find it attractive to rely on risky private assets as a means of payment.

6 Stabilization policies with fiat money

In this section, I consider how policy interventions in the DM can be used to eliminate the remaining stochastic equilibria that exist when fiat money is available as a risk-free but costly substitute for private assets. Specifically, I consider a second-best scenario in which the objective of policy is to implement a deterministic coexistence equilibrium (DCE) in which gross inflation equals $\pi > \beta$. The inflation target π is such that the Fisher rate satisfies $i < \tilde{\imath}$ – the DCE is characterized by high search effort. Government therefore wants to prevent stochastically occurring busts.

To model stabilization policies and the fiscal consequences thereof, I consider a government consisting of a central bank, which is in charge of monetary policy, and a treasury department, which is in charge of fiscal policy. At all times $s > t$, the economy is expected to be in the unique DCE, for instance because stabilization policies are expected to be successfully deployed in the future. This allows a focus on matters at time t.

Let I_t denote the nominal value of a liquidity injection conducted by the central bank in DM_t. Let $\widetilde{M}_t$ and M_t denote the nominal supply of fiat money at the end of DM_t and CM_t, respectively. By construction

$$\widetilde{M}_t = M_{t-1} + I_t.$$

In CM_t, the central bank earns real profits

$$p_t = \phi_t(M_t - \widetilde{M}_t) + d_t, \tag{31}$$

where $\phi_t(M_t - \widetilde{M}_t)$ is real seignorage and d_t is other income accruing to the central bank. This income arises, for example, when the central bank holds private assets.

The CM_t price of fiat money, ϕ_t, is determined by forward looking expectations. That means, because the economy is expected to be in the DCE at time $t+1$, the aggregate value of fiat money balances, measured at the end of time t, equals πm_D. Here, π is the central bank's gross inflation target and m_D is the DCE value of fiat money carried into DM_{t+1}, so that $m_D = \phi_{t+1} M_t$. We therefore have:

$$\phi_t = \pi m_D / M_t.$$

When agents coordinate on devoting high search effort with probability ρ_H in DM_t and without any government intervention, meaning $I_t = d_t = 0$, we have $\phi_t M_{t-1} = m_S(\rho_H)$, where m_S is the SCE value of fiat money when agents coordinate on devoting high search effort with probability ρ_H. Note that m_S is decreasing in ρ_H (see Proposition 4), and that $\lim_{\rho_H \to 1} m_S = m_D$. Real profits of the central bank in CM_t are then $p_S = \pi m_D - m_S$ and the CM_t price of fiat money satisfies $\phi_S = m_S / M_{t-1}$, where M_{t-1} has to be treated as a pre-determined variable.

Profits (or losses) of the central bank accrue to the treasury department. In turn, the treasury department levies a lump-sum tax to satisfy the budget constraint

$$\tau_t = -p_t. \tag{32}$$

I pay special attention to how the effectiveness of stabilization policies depends on whether the central bank takes τ_t as given, a regime I label as fiscal dominance, or whether the treasury takes p_t as given, a regime I label as monetary dominance. In the latter case, the central bank can freely set p_t, and the treasury department then adjusts taxation so that its budget constraint (32) is satisfied. In the former case, I let $\tau_t = \tau_S$ (where of course $\tau_S = -p_S$), representing a treasury department that keeps taxation constant. Then, the central bank is restricted in the sense that its CM_t profits (31) must satisfy $p_t = -\tau_S$.

Turning towards the stabilization policies, at the root of macroeconomic instability is the interaction between search effort and the value of assets. Revisiting from buyers' DM value functions, we know the buyers in the DM choose search effort to maximize

$$eL(q) - s(e), \quad \text{where} \quad q = \min\{v^{-1}(a), q^*\}.$$

The buyers therefore switch from devoting high search effort to devoting low search effort when the value of their assets falls below $v(\tilde{q})$, where $\tilde{q}$ solves $(h-l)L(\tilde{q}) = k$. In case the value of assets drops below $v(\tilde{q})$, for instance because of a financial panic due to a fear of coordination of low search effort, the central bank should intervene. Otherwise, the drop can become self-fulfilling due to the effect of lower search effort on the firms' profits.

Accounting for a potential intervention, the DM_t value of liquid assets held by the buyers is $\phi_t \widetilde{M}_t + \alpha_t F_t^e$, where F_t^e are the expected CM_t dividend payments by firms and α_t

is the measure of private assets held by the buyers. Here, we have $\alpha_t < 1$ when liquidity is injected by means of asset purchases. To ensure the value of assets remains above $v(\tilde{q})$, the real value of the DM_t money injection should therefore satisfy the feedback rule

$$\phi_t I_t = \max\{v(\tilde{q}) - \alpha_t F_t^e - \phi_t M_{t-1}, 0\}.$$

In what follows, I shall consider three empirically realistic ways in which the central bank can inject liquidity during the DM.

6.1 Helicopter money

Consider a monetary injection that takes the form of a lump-sum subsidy for the buyers, so that $\alpha_t = 1$ and $d_t = 0$. That means, the central bank acquires no assets and has no additional income in CM_t. The injection, commonly referred to as helicopter money, therefore has real effects only when the price of fiat money remains unchanged.

In a monetary dominance regime, the central bank can freely set $M_t = \pi m_D/\phi_S$ to ensure $\phi_t = \phi_S$ – the price of fiat money remains unaffected by a monetary injection undertaken in DM_t. Setting the nominal injection to

$$I_t = \max\{v(\tilde{q}) - F_t^e + m_S, 0\}/\phi_S$$

then keeps the total value of buyers' liquid assets above the threshold $v(\tilde{q})$. Such injections do have fiscal consequences off the equilibrium path, since the central bank's CM_t profits change due to the intervention:

$$p_t = p_S - \max\{v(\tilde{q}) - F_t^e + m_S, 0\}.$$

This is because the injected fiat money must be withdrawn from circulation by means of lump-sum taxation. In particular, the lump-sum tax levied by the treasury department increases by exactly the real value of the liquidity injection. However, the stabilization policy is never deployed on the equilibrium path: In a monetary dominance regime the private sector understands the ability of the central bank to prevent a drop in buyers' search effort, so it will always coordinate on high search effort.

In case of fiscal dominance, the central bank fails in stabilizing the economy. The central bank's real CM_t profits are

$$p_t = \phi_t M_t - \phi_t \widetilde{M}_t.$$

In a fiscal dominance regime, these profits are, through lump-sum taxation, fixed at $\pi m_D - m_S$. Combining this with the forward-looking expectations that set $\phi_t M_t = \pi m_D$, we find that $\phi_t\widetilde{M_t} = m_S$. That means, the real value of buyers' asset holdings remains unaffected by the injection, exactly because of the inflationary effects of a lump-sum monetary injection described by Friedman (1969). Hence, when the value of private assets drops because of a financial panic, the central bank can attempt to inject money, but its inability to create liquidity in real terms implies it is unable to prevent buyers from devoting low search effort. This implies the financial panic can turn into a bust.

6.2 Troubled-asset relief program

In a financial panic, the central bank can also inject money by buying private assets. Buying these assets at the prevailing market price F_t^e however fails to generate an increase the real value of liquid assets held by the buyers, the reason being that the value of the injected money equals the value of the purchased assets.

Therefore, consider that the central bank backs the value of private assets by purchasing a fraction θ_t of them at the price that would prevail when buyers coordinate on high search effort, the latter price being with denoted $\overline{F_S^e}$. The real value of the liquidity injection is then $\phi_t I_t = \theta_t\overline{F_S^e}$ and the value of the liquid assets held by the buyers becomes

$$\phi_t\widetilde{M_t} + \alpha_t F_t^e = \theta_t\overline{F_S^e} + (1-\theta_t)F_t^e + \phi_t M_{t-1}.$$

To prevent a drop in search effort, the fraction of private assets bought by the central bank should satisfy

$$\theta_t = \frac{\max\{v(\tilde{q}) - F_t^e - \phi_t M_{t-1}, 0\}}{\overline{F_S^e} - F_t^e} \quad \text{s.t.} \quad \theta_t \leq 1.$$

When the CM$_t$ price of fiat money remains unchanged by TARP, i.e. $\phi_t = \phi_S$, we have $\theta_t < 1$ since $\phi_S M_{t-1} = m_S \geq m_D$, $\overline{F_S^e} > \overline{F_D^e}$ (with $\overline{F_D^e}$ the value of private assets in the high search DCE) and $\overline{F_D^e} + m_D > v(\tilde{q})$ (this is required to have a high search DCE). Hence, buying sufficiently many private assets at the specified premium suffices to prevent a drop in search effort when such a policy is not inflationary. TARP will therefore succeed in a monetary dominance regime, since the central bank can then freely set $M_t = \pi m_D/\phi_S$.

Let F_t denote the actual aggregate dividends paid by the private assets. In a monetary dominance regime, CM$_t$ profits of the central bank become

$$p_t = p_S - \theta_t(\overline{F_S^e} - F_t).$$

TARP therefore has fiscal implications when the private sector's expectations regarding a low value for private assets are correct – the central bank has bought the private assets at a price exceeding the fundamental value ($\overline{F_S^e} > F_t$) and hence the central bank's dividend income falls short of its money injection. Nevertheless, this does not generate inflationary pressures as the central bank's losses are made up for by lump-sum taxation.

What makes TARP successful in a monetary dominance regime is exactly the private sector's understanding of the central bank being bailed-out by the treasury in case TARP produces losses. In particular, the private sector understands the ability of the central bank to purchase private assets at a real premium. TARP therefore has no fiscal implications, even off the equilibrium path: When TARP is conducted as a response to a panic driving down the DM value of private assets ($F_t^e < \overline{F_S^e}$), due to the central bank's ability to create liquidity in real terms, the buyers remain to devote high search effort and hence the aggregate dividends paid to the central bank equal the amount of money injected by the central bank ($\theta_t F_t = \theta_t \overline{F_S^e}$).[16]

Matters in the fiscal dominance regime are very different. CM$_t$ profits of the central bank are then given by

$$p_t = \phi_t M_t - \phi_t M_{t-1} + \theta_t(\overline{F_S^e} - F_t).$$

With lump-sum taxation fixing the central bank's CM$_t$ profits at $p_S = \pi m_D - m_S$ and with forward-looking expectations fixing $\phi_t M_t$ at πm_D, the CM$_t$ price of fiat money therefore becomes

$$\phi_t = \phi_S \frac{m_S - \theta_t(\overline{F_S^e} - F_t)}{m_S}. \tag{33}$$

Equation (33) shows that, without the promise of the treasury bailing-out the central bank, losses from TARP result in inflation.[17]

With TARP being conducted in a fiscal dominance regime, the value of liquid assets held by the buyers becomes

$$\phi_t M_{t-1} + \theta_t \overline{F_S^e} + (1-\theta_t)F_t^e = m_S + \theta_t F_t + (1-\theta_t)F_t^e.$$

When the private sector's expectations regarding a drop in the value of assets are correct, meaning that $F_t = F_t^e$, TARP therefore fails in creating real liquidity and thus in its objective to prevent a drop in search effort. In turn, through the effect of search effort on the firms' profits, this can rationalize the private sector's beliefs regarding the drop in the value of private assets – TARP cannot prevent a panic from turning into a bust.

[16] In fact, TARP conducted by the US in response to the 2007 financial crisis turned out to produce a significant profit for the US Treasury (Calomiris & Khan, 2015).

[17] The price of fiat money cannot become negative. Though a central bank can purchase the private assets at a nominal premium, it does not have unlimited control over the real premium in a fiscal dominance regime. This implies an upper bound on θ_t.

6.3 Emergency lending

Finally, suppose the central bank is, in contrast to the private sector, able to enforce the repayment of credit. Then, it can inject fiat money by providing zero-interest emergency loans to the buyers. Because the loans bear no interest, the buyers will fully exploit this opportunity to finance DM trade. To prevent real liquid asset holdings from falling below the threshold $v(\tilde{q})$, the central bank should therefore stand ready to provide each buyer with an emergency loan with a nominal face value

$$D_t = \max\{v(\tilde{q}) - F_t^e - \phi_t M_{t-1}, 0\}/\phi_t.$$

Because the central bank can enforce the repayment of the emergency loans, in CM_t it receives income $d_t = \phi_t D_t$ from the redemption of loans. Hence, the injected fiat money is withdrawn automatically and the emergency lending program has no inflationary implications – we have $\phi_t = \phi_S$ and $p_t = p_S$. In both a monetary and fiscal dominance regime, the emergency lending program is therefore effective in preventing a financial panic from turning into a bust. The central bank's ability to enforce repayment is crucial for this result. Otherwise, the buyers will default on emergency loans and the policy becomes equivalent to helicopter money.

7 Self-fulfilling inflation dynamics

To conclude the analysis, in this section I drop the assumption of inflation targeting. This allows me to study how the use private assets as payment instruments can interact with self-fulfilling inflation dynamics. For simplicity, I will focus on deterministic dynamics.

As is common in the money-search literature studying dynamics, suppose that monetary policy is passive, meaning that supply of fiat money follows the law of motion:

$$M_t = (1+\mu)M_{t-1}, \quad \text{with} \quad 1+\mu \geq \beta.$$

In steady state, real fiat money balances are constant and therefore, gross inflation satisfies $\pi = 1+\mu$ and the Fisher rate equals $i = (1+\mu-\beta)/\beta$.[18]

Write $m_t = M_{t-1}\phi_t$ for real fiat money balances available in DM_t. By construction

$$m_t = \frac{\beta(1+i_{t+1})}{1+\mu} m_{t+1}, \tag{34}$$

where $i_{t+1} = (\phi_t - \beta\phi_{t+1})/(\beta\phi_t)$ is the Fisher rate. In a deterministic environment, we

[18] The assumption $1+\mu \geq \beta$ ensures that the Fisher rate is non-negative.

have

$$i_t = \eta_t \frac{hL'(q_{h,t})}{v'(q_{h,t})} + (1-\eta_t)\frac{lL'(q_{l,t})}{v'(q_{l,t})}, \tag{35}$$

where η_t is the time t measure of buyers devoting high search effort and $q_{h,t}$ ($q_{l,t}$) is the time t trade within matches by buyers devoting high (resp. low) search effort.

When all buyers devote the same level of search effort $e \in \{l, h\}$, we have that

$$v(q_{e,t}) \leq m_t + e\Pi(q_{e,t}) + y, \quad \text{with} = \text{if } q_{e,t} < \hat{q}. \tag{36}$$

Recall that $v'(q) > \Pi'(q)$ on the relevant domain $[0, \hat{q}]$. Therefore, (36) uniquely fixes $q_{e,t}$ as a function of m_t. To capture this relationship, write $q_e(m_t)$ and to save on notation, write for the liquidity value of the marginal asset

$$\mathcal{L}_e(m_t) = \frac{eL' \circ q_e(m_t)}{v' \circ q_e(m_t)}. \tag{37}$$

To rationalize search effort levels h and l, we need $\mathcal{L}_h(m_t) \leq \tilde{\imath}$ and $\mathcal{L}_l(m_t) \geq \tilde{\imath}$, respectively. In turn, this requires

$$m_t \geq \underline{m} \equiv v(\underline{q}_H) - h\Pi(\underline{q}_H) - y \quad \text{if } \eta_t = 1,$$
$$\text{and} \quad m_t \leq \overline{m} \equiv v(\overline{q}_L) - l\Pi(\overline{q}_L) - y \quad \text{if } \eta_t = 0,$$

where $\underline{q}_H$ and $\overline{q}_L$ are as defined in Section 3.3.

Given the parametrization of the model, we have $\underline{m} \leq \overline{m}$ if and only if the set of Fisher rates for which an SCE exists in an inflation targeting regime (as defined in Section 5), is non-empty. The reason is that in an SCE, we need $q_L \leq \overline{q}_L$ and $q_H \geq \underline{q}_H$ for (24) to be satisfied. At the same time, in an SCE we have that m_t is independent of the sunspot's realization. With the search-contingent demand for fiat money increasing in q, (25) therefore implies $\underline{m} \leq \overline{m}$. Furthermore, we have $\underline{m} < \overline{m}$ if and only if the set of Fisher rates for which SCEs exists has positive measure.

In a situation in which some buyers devote high search effort and others devote low search effort – $\eta_t \in (0,1)$ – we must have $i_t = \tilde{\imath}$. This implies $q_{l,t} = \overline{q}_L$ and $q_{h,t} = \underline{q}_H$. The asset market clearance condition then requires

$$\eta_t v(\underline{q}_H) + (1-\eta_t)v(\overline{q}_L) \leq m_t + \eta_t h\Pi(\underline{q}_L) + (1-\eta_t)\Pi(\overline{q}_H) + y, \quad \text{with} = \text{if } \tilde{\imath} > 0,$$

which fixes η_t as a function of m_t when $\tilde{\imath} > 0$. In what follows, I will ignore the knife-edge cases $\tilde{\imath} = 0$ and $\underline{m} = \overline{m}$. For $\eta_t \in (0, 1)$ we then need

$$m_t \in \begin{cases} (\underline{m}, \overline{m}) & \text{if } \underline{m} < \overline{m} \\ (\overline{m}, \underline{m}) & \text{if } \underline{m} > \overline{m} \end{cases}.$$

In what follows, I consider the two cases $\overline{m} < \underline{m}$ and $\overline{m} > \underline{m}$ separately, as they have very different implications for the existence of deterministic cycles.

7.1 Interaction with private money creation

Suppose $\overline{m} < \underline{m}$ so that given the supply of real money balances, the fraction of buyers devoting high search effort is uniquely determined. In this sense, there is no coordination problem in search effort. Using the relationship between the Fisher rate and real fiat money balances, implied by (34), (35), and (37), we can write m_t as a function of m_{t+1}:

$$m_t = f(m_{t+1}) \equiv \begin{cases} \frac{\beta m_{t+1}}{1+\mu}\left[1 + \mathcal{L}_l(m_{t+1})\right] & \text{if } m_{t+1} \leq \overline{m} \\ \frac{\beta m_{t+1}}{1+\mu}\left[1 + \tilde{\imath}\right] & \text{if } \overline{m} < m_{t+1} < \underline{m} \\ \frac{\beta m_{t+1}}{1+\mu}\left[1 + \mathcal{L}_h(m_{t+1})\right] & \text{if } m_{t+1} \geq \underline{m} \end{cases}.$$

The monetary steady state, in which $m_t = m_{ss} > 0$ and $m_{ss} = f(m_{ss})$, is the same as a DCE with inflation target $\pi = (1+\mu)$ and except for the knife-edge case $1+\mu = \beta(1+\tilde{\imath})$, there is a unique monetary steady state.

The monetary nature of the model implies that $f(0) = 0$ – a non-monetary steady state, in which fiat money has zero value, always exists. Furthermore, $f'(0) > 1$, and $f'(m) = \beta(1+\mu) < 1$ for $m > \hat{m} \equiv v(\hat{q}) - h\Pi(\hat{q}) - y$. We therefore have $f'(m_{ss}) < 1$. From the method of flip-bifurcations, it is known that if and only if $f'(m_{ss}) < -1$, cyclical equilibria exist (Azariadis, 1993). Ignoring the knife-edge case in which $\beta(1+\mu) = 1+\tilde{\imath}$, the analysis is the same as in a model with fixed search effort e and we need to evaluate

$$f'(m_{ss}) = 1 + \frac{\beta m_{ss}}{1+\mu}\mathcal{L}_e'(m_{ss}). \tag{38}$$

Using (36) and (37), we can rewrite (38) as

$$f'(m_{ss}) = 1 - \frac{\beta[v(q_{ss}) - e\Pi(q_{ss}) - y]}{(1+\mu)[v'(q_{ss}) - e\Pi'(q_{ss})]}\frac{L'(q_{ss})v''(q_{ss}) - L''(q_{ss})v'(q_{ss})}{v'(q_{ss})^2}, \tag{39}$$

where Assumption (1) implies that $L'(q_{ss})v''(q_{ss}) - L''(q_{ss})v'(q_{ss})$ is positive. To understand how $f'(m_{ss})$ is affected by private assets, note that in an economy with the same payment protocol but only fiat money being accepted in payment (subscript *fiat*), one obtains

$$f'_{fiat}(m_{ss,fiat}) = 1 - \frac{\beta v(q_{ss})}{(1+\mu)v'(q_{ss})} \frac{L'(q_{ss})v''(q_{ss}) - L''(q_{ss})v'(q_{ss})}{v'(q_{ss})^2}, \tag{40}$$

with q_{ss} the same as in the economy with private assets accepted in payment. Similarly, in an economy where also one-period lived assets with a fixed real dividend equal to $y + e\Pi(q_{ss})$ are accepted in payment (subscript *fixdiv*), one obtains

$$f'_{fixdiv}(m_{ss,fixdiv}) = 1 - \frac{\beta[v(q_{ss}) - e\Pi(q_{ss}) - y]}{(1+\mu)v'(q_{ss})} \frac{L'(q_{ss})v''(q_{ss}) - L''(q_{ss})v'(q_{ss})}{v'(q_{ss})^2}, \tag{41}$$

with again the same q_{ss}. Comparison of (39), (40), and (41) suggests it is useful to rewrite (39) as

$$f'(m_{ss}) = 1 - \frac{\beta v(q_{ss})[L'(q_{ss})v''(q_{ss}) - L''(q_{ss})v'(q_{ss})]}{(1+\mu)v'(q_{ss})v'(q_{ss})^2} \frac{v(q_{ss}) - e\Pi(q_{ss}) - y}{v(q_{ss})} \frac{v'(q_{ss})}{v'(q_{ss}) - e\Pi'(q_{ss})}. \tag{42}$$

The first fraction in (42) is a standard term, capturing the effects when only fiat money is accepted in payment. If and only if this term is larger than 2, cyclical equilibria would arise in a pure fiat economy. The second fraction in (42) captures the effect of letting a one-period lived asset, paying a fixed dividend equal to that paid by the private asset in steady state, compete with fiat money as a means of payment. Because this fraction is smaller than one, for a given parametrization of the model, the existence of cyclical equilibria becomes less likely. Intuitively, the fact that the competing asset pays a fixed dividend acts as a stabilizing force. The third fraction in (42) captures the effect of letting the dividend of the one-period lived asset depend on the profits of the firms. It is greater than one and therefore, makes cyclical equilibria more likely to exist.

Discussion: The decomposition in (42) demonstrates that deterministic cycles become more likely due to the interaction between the use of claims on economic activity as a payment instrument and the role of expectations in determining the value of fiat money. The reason is a multiplier effect – when the value of fiat money increases, this loosens the liquidity constraint and increases the profits of the firms, which in turn increases the real value of private assets and thus further loosens the liquidity constraint. When the future value of fiat money balances increases, this multiplier effect leads to a stronger reduction in the Fisher rate and, through (34), to a potentially lower value of fiat money balances today. However, the existence of deterministic cycles remains dependent on the specific choice of parameters. In particular, the following term should be sufficiently large:

$$\frac{\beta v(q_{ss})}{(1+\mu)v'(q_{ss})} \frac{L'(q_{ss})v''(q_{ss}) - L''(q_{ss})v'(q_{ss})}{v'(q_{ss})^2}.$$

It is well-known that this requires specific parameterizations for the utility functions (Altermatt et al., 2021; Lagos & Wright, 2003; Rocheteau & Wright, 2013).

7.2 Interaction with the coordination of search effort

Now suppose $\underline{m} < \overline{m}$. The coordination of search effort then plays a role, because the supply of real fiat money balances no longer pins down search effort. Using again the relationship between the Fisher rate and real fiat money balances, we can write m_t as a correspondence of m_{t+1}:

$$m_t \in g(m_{t+1}) \equiv \begin{cases} \left\{ \frac{\beta m_{t+1}}{1+\mu} \left[1 + \mathcal{L}_l(m_{t+1})\right] \right\} & \text{if } m_{t+1} \leq \underline{m} \\ \left\{ \begin{array}{l} \dfrac{\beta m_{t+1}}{1+\mu} \left[1 + \mathcal{L}_l(m_{t+1})\right], \ \dfrac{\beta m_{t+1}}{1+\mu} \left[1 + \tilde{\imath}\right], \\ \quad \dfrac{\beta m_{t+1}}{1+\mu} \left[1 + \mathcal{L}_h(m_{t+1})\right] \end{array} \right\} & \text{if } \underline{m} < m_{t+1} < \overline{m} \ . \\ \left\{ \frac{\beta m_{t+1}}{1+\mu} \left[1 + \mathcal{L}_h(m_{t+1})\right] \right\} & \text{if } m_{t+1} \geq \overline{m}. \end{cases} \quad (43)$$

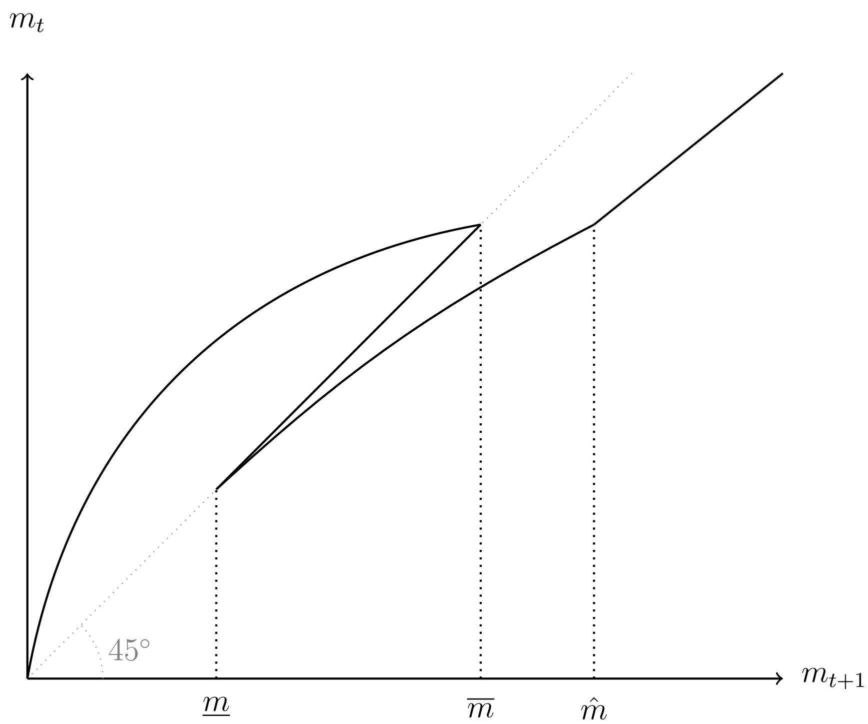
Here, g is continuous and the graph of g is a continuous line in the (m_{t+1}, m_t)-space.

Just as in the case in which the coordination of search effort plays no role, the monetary steady state is the same as a DCE with inflation target $\pi = 1 + \mu$ and except for the knife-edge case in which $1 + \mu = \beta(1 + \tilde{\imath})$, the steady state is unique. From the method of flip-bifurcations, it follows that the condition $g'_{ss} < -1$, where g'_{ss} is the slope of the graph of g in the steady state, is sufficient but no longer necessary to have the existence of deterministic cycles. In fact, if the steady state Fisher rate is sufficiently close to the critical threshold $\tilde{\imath}$ triggering a change in search effort, a deterministic two-cycle exists.

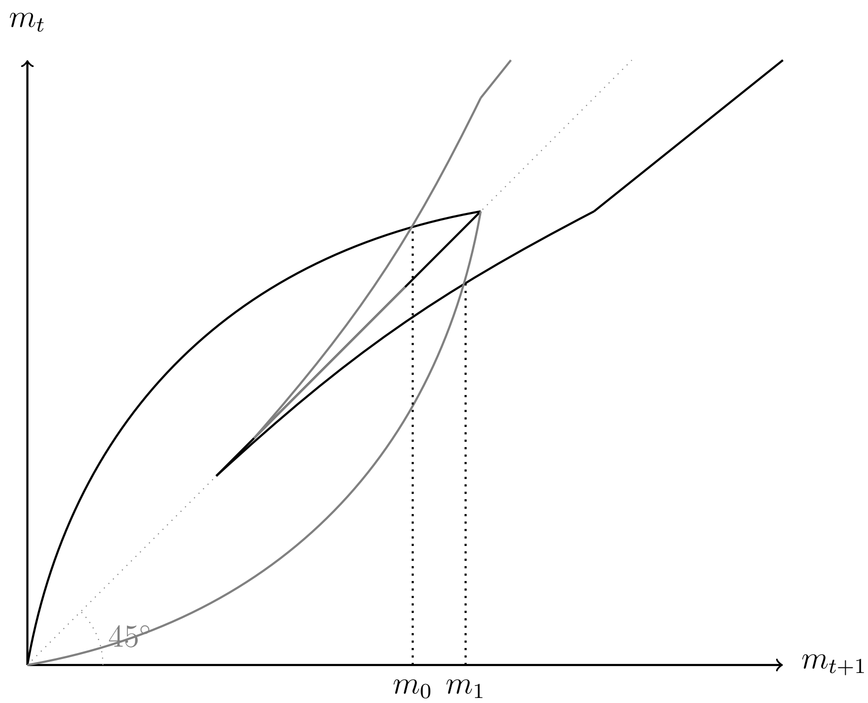
Proposition 5. *When $\underline{m} < \overline{m}$, there exists a two-cycle for $\mu \in [\underline{\mu}, \overline{\mu}]$. This two-cycle involves symmetric behavior by buyers that alternate between exerting high and low search effort.*

The proof of Proposition 5 is illustrated by Figure 2. It plots a hypothetical graph for g in the (m_{t+1}, m_t)-space when we set $1 + \mu = \beta(1 + \tilde{\imath})$. By construction, all points on the graph above the 45-degree line feature $\eta_{t+1} = 0$, all points on the graph and on the 45-degree line feature $\eta_{t+1} \in [0, 1]$, and all points on the graph and below the 45-degree line feature $\eta_{t+1} = 1$. These properties hold true for all parameterizations as long as $1 + \mu = \beta(1 + \tilde{\imath})$. Note there are two steady states in symmetric behavior, namely at $\underline{m}$ ($\eta = 1$) and at $\overline{m}$ ($\eta = 0$). Furthermore, there is a continuum of steady states in asymmetric behavior – all $\eta \in (0, 1)$ are a steady state.

Two-cycles can be identified by plotting the inverse of g, which boils down to mirroring the graph of g along the 45-degree line. Intersections of g and g^{-1} (plotted in gray) that do not lie on the 45-degree line are part of a two-cycle. Due to the properties of g, at least



(a) The graph of g. Steady states in symmetric behavior are $\underline{m}$ ($e = h$) and $\overline{m}$ ($e = l$). Buyers face a slack liquidity constraint for $m \geq \hat{m}$.



(b) The graph of g and its inverse. Together, m_0 and m_1 constitute a two-cycle.

Figure 2: Illustrated example of the graph of g when $\beta(1 + \mu) = 1 + \tilde{\iota}$ and $\underline{m} < \overline{m}$.

two of such points – one above and one below the 45-degree line – exist and together, they constitute a two-cycle. By construction these are points in which the buyers behave symmetrically – they either all devote high search effort at time t (the point above the 45-degree line) or they all devote low search effort at time t (the point below the 45-degree line). In Figure 2, there is a unique two-cycle in which buyers behave symmetrically and the value of fiat money alternates between m_0 (with $e = l$, i.e. a bust) and m_1 (with $e = h$, i.e. a boom). By continuity, the existence of such a two-cycle survives when $\beta(1 + \mu)$ is sufficiently close to $1 + \tilde{\imath}$.

Discussion: Intuitively, the existence of the two-cycle relies on the fact that search effort and the value of fiat money balances can move in the same direction very easily when there is a coordination problem in search effort. An anticipated expansion of both real fiat money supply and search effort at time $t + 1$ has a strong negative effect on the Fisher rate, which implies through the Fisher equation that inflation has to be low in between time t and $t + 1$, so that fiat money balances should have low value at time t. In turn, this allows to rationalize low search effort at time t. Also, when money supply and search effort move back to their time t level at time $t + 2$, this has a strong positive effect on the Fisher rate, which implies high inflation in between time $t + 1$ and $t + 2$. In turn, this rationalizes the high value for fiat money and high search effort at time $t + 1$.

As mentioned before, with only a fiat currency accepted in payment, the existence of cycles requires specific specifications for the utility functions. As long as there is no coordination problem in search effort, this results holds true if a one-period lived real asset is added to the model, even if it bears a dividend that depends on activity in the DM. With a coordination problem in search effort, this insight changes completely – deterministic cycles can arise for any parametrization which also gives rise to the coordination problem in search effort. Furthermore, the existence of the coordination problem in search effort can be guarantied by specifying search costs and the firms' endowments appropriately. This finding starkly contrasts existing models from the new monetarist literature.

8 Conclusion

This paper examines how the use of private assets as payment instruments can make a monetary economy prone to self-fulfilling boom-bust dynamics. A key feature of the analysis, is to let the fundamental value of privately issued means of payment be determined endogenously. This perspective isolates the role of private assets in generating booms and busts. In particular, the use of private assets gives rise to a coordination problem in search effort because (i) these assets are accepted as a means of payment and (ii) the fundamental value of private assets is determined in markets where search effort and payment instruments are essential.

In an economy without government intervention, this coordination problem generates boom-bust dynamics. Furthermore, when government intervenes by supplying fiat money but at the same time implements a passive monetary policy, the coordination problem also makes self-fulfilling inflation dynamics more likely. This finding confirms commonly held views that the private creation of money-like assets is a source of macroeconomic instability, which in turn calls for policy intervention.

In this respect, the model prescribes a simple objective for monetary policy: ensure that inflation is stable and that the total value of liquid asset remains unchanged in case of a financial panic. In combination with an inflation targeting regime, three policies are shown to be effective in combating financial panics: helicopter money, TARP, and emergency lending. The effectiveness of helicopter money and TARP hinge on fiscal backing for the central bank. The reason is that without fiscal backing, these policies fail to create liquidity in real terms due to the inflationary effects arising from the injection of fiat money. This result points towards the importance of coordination between monetary and fiscal policy.

A Two Sided Search Effort

A.1 Model

Besides the unit measure of buyers, there is also a measure one of workers. Workers value the net consumption of general goods and they can devote search effort on behalf of the firms. The flow utility function for a worker is given by

$$\mathcal{U}(e, y) = y - s(e)$$

and workers discount utility between periods at rate β. Here, $s(e)$ has the same properties as for the buyers. The CM operates exactly as in the baseline model and workers have no incentive to accumulate assets since they do not consume special goods.

A.1.1 Decentralized Markets

In the DM workers and firms form worker-firm pairs which disband after the DM has convened. Matching between firms and workers is frictionless – every worker is matched to a firm and *vice versa*. The workers devote search effort $e^w \in E \subseteq [0, 1]$ on behalf of the worker-firm pair. The measure of matches between buyers and workers in DM_t is given by a constant returns-to-scale matching function

$$\mathcal{N}(\tilde{e}_t^b, \tilde{e}_t^s),$$

where $\tilde{e}_t^b$ ($\tilde{e}_t^b$) is average search effort across buyers (resp. workers).

In DM_t a buyer devoting search effort e^b finds a match with a worker with probability $e^b\mathcal{N}(1, 1/\kappa_t)$, where $\kappa_t = \tilde{e}_t^b/\tilde{e}_t^w$ denotes market tightness. Similarly, a worker devoting search effort e^w finds a match with a buyer with probability $e^w\mathcal{N}(\kappa_t, 1)$. Once matched with a buyer, the worker can connect the buyer to the firm.

Assumption A.1. *The amount of search effort devoted by the worker is private information and the firm cannot incentive the worker to exert search effort. Moreover, the worker's decision to connect the buyer to the firm is not contractible.*

Assumption A.1 implies the firm negotiates with a worker after the matching process between buyers and workers has taken place. In particular, a worker matched to a buyer negotiates a payment w from the firm in return for connecting the buyer with the firm. In this negotiation process, the buyer's money holdings are observable to both the worker and the firm.[19] The firm can settle the payment w instantaneously with shares in its profits and I assume the payment follows from a protocol $\omega : \Pi \to w$, mapping the firm's

[19]This assumption is irrelevant in symmetric equilibria.

profitability of being connected with the buyer into a payment for the worker. Workers therefore choose search effort according to

$$\max_{e^w \in E} e^w \iint \frac{e' \mathcal{N}(\kappa_t, 1)}{\tilde{e}_{b,t}} \left[\omega \circ \Pi \left(\min\{\nu^{-1}(a'), \hat{q}\}\right)\right] \mathrm{d}G_t(a', e') - s(e^w)$$

and the buyer's value function of entering DM_t is

$$V_t(a_t) = \max_{e^b \in E} \left\{ e^b \mathcal{N}(1, 1/\kappa_t) L \left(\min\{v^{-1}(a_t), \hat{q}\}\right) - s(e^b) \right\} + a_t + W_t(0).$$

Finally, let $O_t(e'')$ be the probability a randomly drawn worker devotes search effort level $e''' \leq e''$. Expected dividend payments by a firm in CM_t then become:

$$F_t^e = \iiint \frac{2e'e'' \mathcal{N}(\kappa_t, 1)}{\tilde{e}_{b,t}} \left[(1-w) \circ \Pi \left(\min\{\nu^{-1}(a), \hat{q}\}\right)\right] \mathrm{d}G_t(a', e') \mathrm{d}O_t(e'').$$

A.2 Equivalence of symmetric equilibria

To obtain results equivalent to those in the baseline model, impose:

Assumption A.2. *Buyers and workers obtain the same share $\theta < 1/2$ of total match surplus $u(q) - c(q)$. That means, $v(q) = (1-\theta)u(q) + \theta c(q)$ and $\omega \circ \Pi(q) = \theta[u(q) - c(q)]$.*

Given Assumption A.2, we obtain $\kappa = 1$ in a symmetric equilibrium. To see this, note all buyers carry assets worth a into the DM. Workers then anticipate each buyer will consume $q = \min\{v^{-1}(a), \hat{q}\}$ special goods and workers therefore choose e^w to maximize $e^w \mathcal{N}(\kappa, 1)\theta[u(q) - c(q)] - s(e^w)$. Similarly, a buyer carrying assets worth a chooses e_b to maximize $e^b \mathcal{N}(1, 1/\kappa)\theta[u(q) - c(q)] - s(e^b)$. The optimal level of search effort for both the buyer and the worker is unique, except for knife-edge cases.[20]

When $\kappa = 1$ the buyers and the workers choose the same search effort, rationalizing $\kappa = 1$ as an equilibrium outcome. When $\kappa > 1$ the buyers must exert more search effort than the workers. However, high market tightness is especially beneficial for the workers – they get matched to a buyer with a high probability. Therefore, the workers are unwilling to devote strictly less search effort than the buyers. Similarly, when $\kappa < 1$ the workers must exert more search effort than the buyers but a low market tightness is especially beneficial for the buyers. Therefore, the buyers are unwilling to devote strictly less search effort than the workers.

Summarizing, in symmetric equilibria a buyer is matched to a worker with probability $e\mathcal{N}(1, 1)$ when exerting search effort e. Because one can normalize $\mathcal{N}(1, 1) = 1$, we obtain the same value functions for the buyer as in the model with one sided search effort. The only difference arises when calculating welfare and asset supply. The reason is that both

[20] In particular, multiple solutions only occur if $(e''-e')\theta[u(q)-c(q)] = s(e'')-s(e')$ and $(e', e'') \cap E = \emptyset$.

buyers and workers exert search effort, and that firms now earn lower profits due to the payment w to workers. However, this does not affect the other main properties of the model.

A.3 Strategic instability of asymmetric equilibria

Only symmetric equilibria have been considered in the baseline economy with $E = \{l, h\}$. The reason is that buyers are indifferent between search effort $e = l$ and $e = h$ only when $\iota_t = \tilde{\iota}$. In the baseline economy with one sided search effort, asymmetric equilibria therefore only exist for a knife-edge case.

With two sided search effort, market tightness need not equal one, so more asymmetric equilibria can arise. In particular, when $E = \{l, h\}$ the existence of asymmetric equilibria is no longer restricted to a knife-edge case. Nevertheless, these asymmetric equilibria turn out to be strategically unstable.

In an asymmetric equilibrium, the buyers must be indifferent between searching at $e = h$ and $e = l$. This implies

$$\max_q \{h\mathcal{N}(1, 1/\kappa)L(q) - \iota_t v(q)\} - \max_q \{l\mathcal{N}(1, 1/\kappa)L(q) - \iota_t v(q)\} = k.$$

Consider $\iota_t > \tilde{\iota}$. This implies $\kappa < 1$ in an asymmetric equilibrium as $\tilde{\iota}$ triggers indifference for $\kappa = 1$ and $\max_q\{h\mathcal{N}(1, 1/\kappa)L(q) - \iota_t v(q)\} - \max_q\{l\mathcal{N}(1, 1/\kappa)L(q) - \iota_t v(q)\}$ is strictly decreasing in κ. Suppose an infinitesimally small measure of buyers decides to search at $e = h$ instead of $e = l$, which they are willing to do because of indifference. Market tightness increases and it follows all buyers strictly prefer to search at $e = l$. The asymmetric equilibrium is therefore strategically unstable.

With $\iota_t > \tilde{\iota}$, the symmetric equilibrium implies the buyers strictly prefer to search at $e = l$ because $\kappa = 1$. If a small measure of buyers would instead decide to search at $e = h$, market tightness increases but all other buyers still strictly prefer to search at $e = l$ instead of $e = h$ – the symmetric equilibrium is strategically stable.

Now consider $\iota_t < \tilde{\iota}$, which implies $\kappa > 1$ in an asymmetric equilibrium. Suppose an infinitesimally small measure of buyers now decides to search at $e = l$ instead of $e = h$, which they are willing to do because of indifference. Market tightness decreases and all the buyers now strictly prefer to search at $e = h$. The asymmetric equilibrium is therefore strategically unstable.

With $\iota_t < \tilde{\iota}$, the symmetric equilibrium implies the buyers strictly prefer to search at $e = h$ because $\kappa = 1$. If a small measure of the buyers would instead decide to search at $e = l$, market tightness increases but all other buyers still strictly prefer to search at $e = h$ instead of $e = l$ – the symmetric equilibrium is strategically stable.

B Proofs

B.1 Proof of Lemma 1

By construction, welfare as of time t is given by

$$U_t = \iint V_t(a')\mathrm{d}G_t(a', e'). \tag{B.1}$$

Using (8) to replace $V_t(a')$ in (B.1), we obtain

$$U_t = \iint \max_{e\in\{l,h\}} \left\{eL\left(\min\{\nu^{-1}(a'), \hat{q}\}\right) - s(e)\right\} \mathrm{d}G_t(a', e') \\ + \iint a'\mathrm{d}G_t(a', e') + W_t(0). \tag{B.2}$$

Condition 2 in Definition 1 and Equation (9) imply

$$\iint a'\mathrm{d}G_t(a', e') = \phi_t M_{t-1} + \iint e'\Pi\left(\min\{\nu^{-1}(a'), \hat{q}\}\right) \mathrm{d}G_t(a', e') + y. \tag{B.3}$$

Condition 3 in Definition 1 implies that for all (a', e') on the support of CDF G_t we have

$$e' = \arg\max_{e\in\{l,h\}} \left\{eL\left(\min\{\nu^{-1}(a'), \hat{q}\}\right) - s(e)\right\}. \tag{B.4}$$

Using (B.3), (B.4), and $L(q) + \Pi(q) = u(q) - c(q)$ in (B.2) yields

$$U_t = \iint \left[e'(u-c)\circ\min\{\nu^{-1}(a'), \hat{q}\} - s(e')\right] \mathrm{d}G_t(a', e') + \phi_t M_{t-1} + y + W_t(0). \tag{B.5}$$

Using (5) substitute out $W_t(0)$ in (B.5), yields

$$U_t = \iint \left[e'(u-c)\circ\min\{\nu^{-1}(a'), \hat{q}\} - s(e')\right] \mathrm{d}G_t(a', e') + \phi_t M_{t-1} + y \\ + \Upsilon_t - \tau_t + \beta\mathbb{E}_t\left\{\max_{a_{t+1}\geq 0}\left\{V_{t+1}(a_{t+1}) - a_{t+1}(1+\iota_{t+1})\right\}\right\}. \tag{B.6}$$

Condition 3 in Definition 1 implies that for all a' on the support of CDF G_{t+1} we have

$$a' = \arg\max_{a\geq 0}\left\{V_{t+1}(a) - (1+\iota_{t+1})a\right\}. \tag{B.7}$$

Using (B.7) in (B.6)

$$U_t = \iint \left[e'(u-c)\circ\min\{\nu^{-1}(a'),\hat{q}\} - s(e')\right] \mathrm{d}G_t(a',e')$$
$$+\phi_t M_{t-1} + y + \Upsilon_t - \tau_t + \beta\mathbb{E}_t\left\{\iint [V_t(a') - (1+\iota_{t+1})a'] \,\mathrm{d}G_{t+1}(a',e')\right\}. \quad \text{(B.8)}$$

Using (7), (11), and $U_{t+1} = \iint V_{t+1}(a')\mathrm{d}G_{t+1}(a',e')$ in (B.8) yields

$$U_t = \mathcal{W}_t + \phi_t M_t + \Upsilon_t + \beta\mathbb{E}_t\left\{U_{t+1} - (1+\iota_{t+1})\iint a'\mathrm{d}G_{t+1}(a',e')\right\}. \quad \text{(B.9)}$$

Condition 2 in Definition 1 implies $\iint a'\mathrm{d}G_{t+1}(a',e') = \phi_{t+1}M_t + F^e_{t+1}$. Condition 1 implies $\Upsilon_t = \mathbb{E}_t\{\beta(1+\iota_{t+1})F^e_{t+1}\}$ and $\phi_t = \mathbb{E}_t\{\beta(1+\iota_{t+1})\phi_{t+1}\}$. Using this in (B.9) implies

$$U_t = \mathcal{W}_t + \beta\mathbb{E}_t\{U_{t+1}\},$$

with $\mathcal{W}_t$ given by (11). Q.E.D.

B.2 Proof of Proposition 1

Because fiat money is intrinsically useless, there always exists a non-monetary equilibrium. Furthermore, there always exists at least one pair (e_t, q_t) satisfying the system (16)-(17).

Regarding the multiplicity of non-monetary equilibria, we have that (16) uniquely maps levels of search effort into values for $q \in [0,\hat{q}]$. Therefore, q_h and q_l as defined in Section 4 solve

$$e\Pi(q_e) + y \geq v(q_e), \quad \text{with} = \text{if } q_e < \hat{q}, \quad e \in \{l,h\}. \quad \text{(B.10)}$$

From (17) it follows we have a multiplicity of non-monetary equilibria if and only if

$$\frac{hL'(q_h)}{v'(q_h)} \leq \tilde{\imath} \leq \frac{lL'(q_l)}{v'(q_l)}.$$

Implicitly, y determines q_l and q_h through (B.10), so write $q_l(y)$ and $q_h(y)$. Then let

$$\mathcal{Y} = \left\{y: \quad \frac{hL'\circ q_h(y)}{v'\circ q_h(y)} \leq \frac{lL'\circ q_l(y)}{v'\circ q_l(y)}\right\}$$

and note continuity implies

$$\mathrm{int}(\mathcal{Y}) = \left\{y: \quad \frac{hL'\circ q_h(y)}{v'\circ q_h(y)} < \frac{lL'\circ q_l(y)}{v'\circ q_l(y)}\right\}.$$

To ensure slack capacity constraints we imposed $y \geq c(\hat{q})$, so we have that there is a multiplicity of non-monetary equilibria for all

$$\tilde{\imath} \in \mathcal{I}(y) = \left[\frac{hL' \circ q_h(y)}{v' \circ q_h(y)}, \frac{lL' \circ q_l(y)}{v' \circ q_l(y)}\right] \quad \text{with} \quad y \in \mathcal{Y} \cap [c(\hat{q}), \infty).$$

Note that from $\tilde{\imath}$ one can back out k according to the formula

$$k = \kappa(\tilde{\imath}) \equiv \max_{\tilde{q}_h} \{hL(\tilde{q}_h) - \tilde{\imath}v(\tilde{q}_h)\} - \max_{\tilde{q}_l} \{lL(\tilde{q}_l) - \tilde{\imath}v(\tilde{q}_l)\},$$

which implies k is a decreasing function of $\tilde{\imath}$. Hence, there is a multiplicity of non-monetary equilibria for all

$$k \in \mathcal{K}(y) = \left[\kappa\left(\frac{hL' \circ q_h(y)}{v' \circ q_h(y)}\right), \kappa\left(\frac{lL' \circ q_l(y)}{v' \circ q_l(y)}\right)\right], \quad \text{with} \quad y \in \mathcal{Y} \cap [c(\hat{q}), \infty).$$

Furthermore, the set $\mathcal{K}(y)$ has positive measure for $y \in \text{int}(\mathcal{Y})$.

It remains to prove the set

$$\{(k, y) : \quad y \in \mathcal{Y} \cap [c(\hat{q}), \infty) \quad \text{and} \quad k \in \mathcal{K}(y)\}$$

has positive measure, for which it is suffices to show that $\text{int}(\mathcal{Y}) \cap [c(\hat{q}), \infty)$ has positive measure. Note that for

$$y \in \mathcal{Y}' = (v(\hat{q}) - h\Pi(\hat{q}), v(\hat{q}) - l\Pi(\hat{q}))$$

we have $q_l < q_h = \hat{q}$. Since $L'(q)/v'(q) \geq 0$ with equality if and only if $q = \hat{q}$, we have by construction that $\mathcal{Y}' \subseteq \text{int}(\mathcal{Y})$. Finally, note

$$\begin{aligned} v(\hat{q}) - h\Pi(\hat{q}) &= v(\hat{q}) - h[v(\hat{q}) - c(\hat{q})] \\ &= (1 - h)v(\hat{q}) + hc(\hat{q}) \\ &> c(\hat{q}), \end{aligned}$$

where the first line uses that $\Pi(q) = v(q) - c(q)$ and the second line uses that for all $q \in (0, \hat{q}]$ we have $v(q) > c(q)$. It follows $\mathcal{Y}' \subseteq \text{int}(\mathcal{Y}) \cap [c(\hat{q}), \infty)$, with $\mathcal{Y}'$ having positive measure. Q.E.D.

B.3 Proof of Proposition 2

I start with an implicit characterization of the set of all sunspot equilibria. Define

$$\overline{a} = y + \phi M,$$

where ϕM is the DM value of fiat money balances. This value is independent of the realization of the sunspot and can therefore be treated as given in the DM. Equation (25) is satisfied if and only if $q_L = q_l(\overline{a})$ and $q_H = q_h(\overline{a})$, with $q_l(\overline{a})$ and $q_h(\overline{a})$ determined, as in the proof of Proposition 1, by

$$e\Pi(q_e) + \overline{a} \geq v(q_e), \quad \text{with} = \text{if } q_e < \hat{q}, \quad e \in \{l, h\}.$$

Define

$$\mathcal{A} = \left\{ \overline{a} : \quad \frac{hL' \circ q_h(\overline{a})}{v' \circ q_h(\overline{a})} \leq \frac{lL' \circ q_l(\overline{a})}{v' \circ q_l(\overline{a})} \right\}.$$

From the proof of Proposition 1 it is immediate that (24) is satisfied if and only if $\overline{a} \in \mathcal{A}$ and $\tilde{\imath} \in \mathcal{I}(\overline{a})$. Moreover, since $q_l = q_l(y)$ and $q_h = q_h(y)$, we have $q_L \geq q_l$ and $q_H \geq q_h$ if and only if $y \leq \overline{a}$. To ensure we also have $y \geq c(\hat{q})$ satisfied, so that the capacity constraint is always slack, we therefore need

$$y \in [c(\hat{q}), \overline{a}].$$

Clearly, this is a non-empty set for all $\overline{a} \in \mathcal{A} \cap [c(\hat{q}), \infty)$. Finally, for all $\overline{a} \in \mathcal{A} \cap [c(\hat{q}), \infty)$, we have existence of a pair $(\rho_L, \rho_H) \in \Delta^1$ such that Equation (23) is satisfied for all

$$i \in \mathcal{I}(\overline{a}).$$

Summarizing, the set of Fisher rates, search costs, and endowments for which SCEs exist is implicitly defined by the set

$$\{(i, k, y, \overline{a}) : \quad \overline{a} \in \mathcal{A} \cap [c(\hat{q}), \infty), y \in [c(\hat{q}), \overline{a}], \quad k \in \mathcal{K}(\overline{a}), \quad \text{and} \quad i \in \mathcal{I}(\overline{a})\}.$$

From the proof of Proposition 1, it follows this set has positive measure. To see why, note $\mathcal{Y}' \subseteq \text{int}(\mathcal{A}) \cap [c(\hat{(q)}), \infty)$ and that $\mathcal{Y}'$ has positive measure. Hence, for all $\overline{a} \in \mathcal{Y}'$, $\mathcal{K}(\overline{a})$ and $\mathcal{I}(\overline{a})$ have positive measure. Furthermore, for all $a' \in \mathcal{Y}'$ we have $c(\hat{q}) < \overline{a}$, so the set $[c(\hat{q}), \overline{a}]$ also has positive measure.

Next, given parameters (k, y) for which an SCE exits, consider the particular Fisher rates for which this is indeed the case. First, focus on the generic case in which $k < (h - l)L(\hat{q})$ so that $\tilde{\imath} > 0$. Equation (25) implicitly pins down q_L as a function of q_H and *vice versa*. Therefore, let $\tilde{q}_H(q_L)$ and $\tilde{q}_L(q_H)$ capture these implicit relationships. Importantly, according to (25) $\tilde{q}_L$ is increasing in q_H and similarly, $\tilde{q}_H$ is increasing in q_L.

As k pins down $\tilde{\imath}$, (25) implies a lower bound on q_H and an upper bound on q_L:

$$\underline{q}_H : \quad hL'(\underline{q}_H)/v'(\underline{q}_H) = \tilde{\imath} \quad \text{and} \quad \overline{q}_L : \quad lL'(\overline{q}_L)/v'(\overline{q}_L) = \tilde{\imath}.$$

Furthermore, we need $q_H \geq q_h$ and $q_L \geq q_l$ to ensure fiat money balances are non-negative.

This implies additional lower bounds on q_L and q_H

$$q_L \geq q_l \quad \text{and} \quad q_H \geq q_h.$$

It follows there exist $(\rho_H, \rho_L) \in \text{int}(\Delta^1)$ and a pair (q_L, q_H) satisfying (23)-(25) if and only if

$$i \in \left(\frac{hL' \circ \tilde{q}_H(\overline{q}_L)}{v' \circ \tilde{q}_H(\overline{q}_L)}, \frac{lL' \circ \max\{\tilde{q}_L(\underline{q}_H), \tilde{q}_L(q_h), q_l\}}{v' \circ \max\{\tilde{q}_L(\underline{q}_L), \tilde{q}_L(q_h), q_l\}} \right).$$

For the knife edge case $k = (h-l)L(\hat{q})$, we have $\tilde{\imath} = 0$. It follows immediately from (24) that we need $q_H = \hat{q}$. In turn, (25) then holds for all $q_L \in [\underline{q}_L, \hat{q}]$, where $\underline{q}_L$ solves

$$(1-l)\Pi(\underline{q}_L) + c(\underline{q}_L) = (1-h)\Pi(\hat{q}) + c(\hat{q}).$$

With $q_L \in [\underline{q}_L, \hat{q}]$ and $q_H = \hat{q}$, (24) is satisfied. We also need $q_L \geq q_l$ to ensure fiat money balances are non-negative ($q_h \leq \hat{q}$ by construction, so $q_H \geq q_h$ is already satisfied). It follows there exist $(\rho_H, \rho_L) \in \text{int}(\Delta)$ and a pair (q_L, q_H) satisfying (23)-(25) if and only if

$$i = \left[0, \frac{lL' \circ \max\{\underline{q}_L, q_l\}}{v' \circ \max\{\underline{q}_L, q_l\}} \right).$$

Finally, given the Fisher rate, search costs, and endowments, I characterize the $\boldsymbol{\rho}$ for which an SCE exist. From (25) we know that q_L and q_H can be related to each other according to the implicit functions $q_L = \tilde{q}_L(q_H)$ and $q_H = \tilde{q}_H(q_L)$. Furthermore, (24) implies $hL'(q_H)/v'(q_H) \leq lL'(q_L)/v'(q_L)$. Combing (24) and (25) with (23) and $(\rho_L, \rho_H) \in \Delta^1$, implies that ρ_L (ρ_H) increases (resp. decreases) with q_L. In particular, using (23) and (25) we can characterize ρ_L as a function of q_L:

$$\tilde{\rho}_L(q_L) = \frac{i - h[L' \circ \tilde{q}_H(q_L)]/[v' \circ \tilde{q}_H(q_L)]}{lL'(q_L)/v(q_L) - h[L' \circ \tilde{q}_H(q_L)]/[v' \circ \tilde{q}_H(q_L)]}.$$

Suppose first $i < \tilde{\imath}$. With $q_L \uparrow \overline{q}_L$, we find

$$\lim \tilde{\rho}_L(q_L)_{q_L \uparrow \overline{q}_L} = \frac{i - h[L' \circ \tilde{q}_H(\overline{q}_L)]/[v' \circ \tilde{q}_H(\overline{q}_L)]}{\tilde{\imath} - h[L' \circ \tilde{q}_H(\overline{q}_L)]/[v' \circ \tilde{q}_H(\overline{q}_L)]} < 1,$$

providing a strictly positive upper bound on ρ_L since existence of an SCE requires $\overline{q}_L \geq q_l$ and $\tilde{q}_H(\overline{q}_L) \geq q_h$, as well as (24) being satisfied for $(q_L, q_H) = (\overline{q}_L, \tilde{q}_H(\overline{q}_L))$. With $q_L \downarrow \tilde{q}_L(\underline{q}_H)$ we find

$$\lim \tilde{\rho}_L(q_L)_{q_L \downarrow \tilde{q}_L(\underline{q}_H)} = \frac{i - \tilde{\imath}}{l[L' \circ \tilde{q}_L(\underline{q}_H)]/[v' \circ \tilde{q}_L(\underline{q}_H)] - \tilde{\imath}} < 0,$$

where the denominator being positive follows from the existence of an SCE, suggesting the relevant lower bound for ρ_L is zero. However, as $(q_L, q_H) > (q_l, q_h)$, we need to account for the fact that $q_L \geq \max\{q_l, \tilde{q}_L(q_h)\}$ as well. Hence the relevant lower bound on ρ_L can be written as $\max\{0, \tilde{\rho}_L(q_l), \tilde{\rho}_L \circ \tilde{q}_L(q_h)\}$. Concluding,

$$\rho_L \in [\max\{0, \tilde{\rho}_L(q_l), \tilde{\rho}_L \circ \tilde{q}_L(q_h)\}, \tilde{\rho}_L(\overline{q}_L)] \quad \text{if} \quad i < \tilde{\imath}. \tag{B.11}$$

Suppose next $i > \tilde{\imath}$. With $q_L \downarrow \tilde{q}_L(\underline{q}_H)$, we find

$$\lim \tilde{\rho}_L(q_L)_{q_L \downarrow \tilde{q}_L(\underline{q}_H)} = \frac{i - \tilde{\imath}}{l[L' \circ \tilde{q}_L(\underline{q}_H)]/[v' \circ \tilde{q}_L(\underline{q}_H)] - \tilde{\imath}} > 0,$$

where the denominator being positive follows from the existence of an SCE, providing a strictly positive lower bound on ρ_L. For a pair (q_L, q_H) to be a sunspot equilibrium, we do not only need $q_L \geq \tilde{q}_L(\underline{q}_H)$ but also $q_L \geq q_l$ and $q_H \geq q_h$. This implies the relevant lower bound on ρ_L is given by $\max\{\tilde{\rho}_L \circ \tilde{q}_L(\underline{q}_H), \tilde{\rho}_L \circ \tilde{q}_L(q_h), \tilde{\rho}_L(q_l)\}$. With $q_L \uparrow \overline{q}_L$ we find

$$\lim \tilde{\rho}_L(q_L)_{q_L \uparrow \overline{q}_L} = \frac{i - h[L' \circ \tilde{q}_H(\overline{q}_L)]/[v' \circ \tilde{q}_H(\overline{q}_L)]}{\tilde{\imath} - h[L' \circ \tilde{q}_H(\overline{q}_L)]/[v' \circ \tilde{q}_H(\overline{q}_L)]} > 1,$$

where the denominator being positive follows from the existence of an SCE, indicating the relevant lower bound for ρ_L is one. Concluding,

$$\rho_L \in [\max\{\tilde{\rho}_L \circ \tilde{q}_L(\underline{q}_H), \tilde{\rho}_L \circ \tilde{q}_L(q_h), \tilde{\rho}_L(q_l)\}, 1] \quad \text{if} \quad i > \tilde{\imath}. \tag{B.12}$$

Finally, suppose $i = \tilde{\imath}$. It follows we have $i = lL'(\underline{q}_L)/v'(\underline{q}_L) = hL'(\underline{q}_H)/v'(\underline{q}_H)$. Therefore,

$$\lim \tilde{\rho}_L(q_L)_{q_L \downarrow \tilde{q}_L(\underline{q}_H)} = \frac{i - \tilde{\imath}}{l[L' \circ \tilde{q}_L(\underline{q}_H)]/[v' \circ \tilde{q}_L(\underline{q}_H)] - \tilde{\imath}} = 0$$

and

$$\lim \tilde{\rho}_L(q_L)_{q_L \uparrow \overline{q}_L} = \frac{i - h[L' \circ \tilde{q}_H(\overline{q}_L)]/[v' \circ \tilde{q}_H(\overline{q}_L)]}{\tilde{\imath} - h[L' \circ \tilde{q}_H(\overline{q}_L)]/[v' \circ \tilde{q}_H(\overline{q}_L)]} = 1,$$

indicating that if an SCE exists, then for all $\rho_L \in [0, 1]$ there exists a pair (q_L, q_H) satisfying the system (23)-(25). Because we also need $(q_L, q_H) \geq (q_l, q_h)$, we obtain

$$\rho_L \in [\max\{0, \tilde{\rho}_L(q_l), \tilde{\rho}_L \circ \tilde{q}_L(q_h)\}, 1] \quad \text{if} \quad i = \tilde{\imath}. \tag{B.13}$$

Summarizing, (B.11), (B.12), and (B.13) characterize for a given Fisher rate, search costs, and endowment, the ρ_L for which an SCE exists. The relevant set for ρ_H follows from the fact that $\rho_H = 1 - \rho_L$.

Given the Fisher rate i and the sunspot process $\boldsymbol{\rho}$, the uniqueness of the SCE follows from combining (23) and (25). In particular, (25) implies that q_L and q_H move in the same direction and by assumption $L'(q)/v'(q)$ is decreasing in q. For a fixed $\boldsymbol{\rho}$ and i, this implies there can only be a unique pair (q_L, q_H) satisfying (23) and (25) simultaneously. Q.E.D.

B.4 Proof of Proposition 3

I consider a constructive proof. Consider a constellation in which we have $q_H = \hat{q}$ and $q_L = \underline{q} \equiv \max\{\underline{q}', \underline{q}''\}$ where,

$$(1-l)\Pi(\underline{q}') + c(\underline{q}') = (1-h)\Pi(\hat{q}) + c(\hat{q})$$

and

$$\frac{lL'(\underline{q}'')}{v'(\underline{q}'')} = \frac{hL'(\underline{q}_h)}{v'(\underline{q}_h)} \quad \text{with} \quad v(\underline{q}_h) - h\Pi(\underline{q}_h) = c(\hat{q}).$$

By construction $\underline{q}_h < \hat{q}$. To see this, observe the converse yields a contradiction:

$$\begin{aligned} c(\hat{q}) &= v(\underline{q}_h) - h\Pi(\underline{q}_h) \\ &= (1-h)v(\underline{q}_h) + hc(\underline{q}_h) \\ &> (1-h)v(\hat{q}) + hc(\hat{q}) \\ &> c(\hat{q}). \end{aligned}$$

In turn, $\underline{q}_h < \hat{q}$ implies $\underline{q}'' < \hat{q}$ and we also have $\underline{q}' < \hat{q}$. These properties imply the pair $(q_L, q_H) = (\underline{q}, \hat{q})$ satisfies (25) by construction. To ensure $q_L \geq q_l$, it follows from $\underline{q} \geq \underline{q}'$ and (16) that it suffices to have

$$y \leq v(\underline{q}') - l\Pi(\underline{q}').$$

We have $q_H \geq q_h$ satisfied by construction, since $q_h \leq \hat{q}$ and $q_H = \hat{q}$. To ensure the capacity constraint is always slack, we have imposed $y \geq c(\hat{q})$. Hence, we need y in the set

$$\left[c(\hat{q}), v(\underline{q}') - l\Pi(\underline{q}')\right] \tag{B.14}$$

This is a set with positive measure, which follows, as in the proof of Proposition 2, from showing that the converse yields a contradiction:

$$\begin{aligned} c(\hat{q}) &\geq v(\underline{q}') - l\Pi(\underline{q}') \\ &= (1-l)\Pi(\underline{q}') + c(\underline{q}') \\ &= (1-h)\Pi(\hat{q}) + c(\hat{q}) \\ &= (1-h)v(\hat{q}) + hc(\hat{q}) \\ &> c(\hat{q}). \end{aligned}$$

Then, define $\iota_L = lL'(\underline{q})/v'(\underline{q})$, $\overline{k} = (h-l)L(\hat{q})$, and $\underline{k} = \max_q\{hL(q) - \iota_L v(q)\} - \max_q\{lL(q)/ - \iota_L v(q)\}$. By construction $\overline{k} > \underline{k}$ and $k \in [\underline{k}, \overline{k}]$ implies $\tilde{\iota} \in [0, \iota_L]$.

First, I demonstrate that there exists an i and a parameter configuration so that there is an SCE attaining less welfare than the unique DCE (supposing a DCE exists). Consider that $k = \underline{k} + \delta$ and $i = \iota_L - \varepsilon$, with $\varepsilon > 0$ and $\delta > 0$ both arbitrarily small but such that $\varepsilon > \gamma$, where γ is such that

$$\underline{k} + \delta = \max_q\{hL(q) - (\iota_L - \gamma)v(q)\} - \max_q\{lL(q) - (\iota_L - \gamma)v(q)\}.$$

Such a parameter configuration is feasible because $\delta > 0 \Rightarrow \gamma > 0$ and by continuity, $\delta \downarrow 0 \Rightarrow \gamma \downarrow 0$. Because $i < \iota_L - \gamma = \tilde{\iota}$, the DCE is unique and characterized by the pair $(e, q) = (h, q_{DCE})$, where

$$\frac{hL'(q_{DCE})}{v'(q_{DCE})} = \iota_L - \varepsilon.$$

To ensure existence of the DCE, we need $q_{DCE} \geq q_h$, which, according to (16), requires

$$y \leq v(q_{DCE}) - h\Pi(q_{DCE}).$$

To ensure the capacity constraint is always slack, we have imposed $y \geq c(\hat{q})$. Note

$$\frac{hL'(q_{DCE})}{v'(q_{DCE})} = i = \iota_L - \varepsilon < \iota_L = \frac{lL'(\underline{q})}{v'(\underline{q})} \leq \frac{lL'(\underline{q}'')}{v'(\underline{q}'')} = \frac{hL'(\underline{q}_h)}{v'(\underline{q}_h)},$$

which implies $q_{DCE} > \underline{q}_h$. In turn, $q_{DCE} > \underline{q}_h$ implies

$$v(q_{DCE}) - h\Pi(q_{DCE}) > v(\underline{q}_h) - h\Pi(\underline{q}_h = c(\hat{q}).$$

Hence, there are y for which we have existence of the DCE and the SCE, as we have a non-empty intersection between (B.14) and

$$[c(\hat{q}), v(q_{DCE}) - h\Pi(q_{DCE})].$$

Flow welfare in the DCE is given by $\overline{\mathcal{W}} = h[u(q_{DCE}) - c(q_{DCE})] - \underline{k} - \delta + y$. Now, consider an SCE with the pair (q_L, q_H) described above. To ensure (23) is satisfied we need $\iota_L - \varepsilon = \rho\iota_L$. Clearly, for $\varepsilon > 0$ but small we have $\rho \in (0, 1)$ and with $\varepsilon \downarrow 0$ we have $\rho \uparrow 1$. Expected flow welfare in an SCE is given by

$$\widetilde{\mathcal{W}} = \frac{\iota_L - \varepsilon}{\iota_L} l\left[u(\underline{q}) - c(\underline{q})\right] + \frac{\varepsilon}{\iota_L}\left[h[u(\hat{q}) - c(\hat{q})] - \underline{k} - \delta\right] + y.$$

With $\varepsilon \downarrow 0$ and $\delta \downarrow 0$, we have

$$\overline{\mathcal{W}} - \widetilde{\mathcal{W}} \to h[u(q_{DCE}) - c(q_{DCE})] - \underline{k} - l[u(\underline{q}) - c(\underline{q})].$$

Because $(h-l)L(q_{DCE}) > \underline{k} > (h-l)L(\underline{q})$ and $L(q)$ is strictly increasing in q for $q < \hat{q}$, there exists a $\tilde{q} \in (\underline{q}, q_{DCE})$ such that $\underline{k} = (h-l)L(\tilde{q})$. Because $u(q) - c(q)$ is strictly increasing for all $q < \hat{q}$, we have

$$\begin{aligned} h[u(q_{DCE}) - c(q_{DCE})] - \underline{k} - l[u(\underline{q}) - c(\underline{q})] &> h[u(\tilde{q}) - c(\tilde{q})] - \underline{k} - l[u(\tilde{q}) - c(\tilde{q})] \\ &= (h-l)[L(\tilde{q}) + \Pi(\tilde{q})] - \underline{k} \\ &= (h-l)\Pi(\tilde{q}) \\ &> 0. \end{aligned}$$

Welfare in the DCE is therefore larger than welfare in the SCE.

Second, I demonstrate that there exists an i and a parameter configuration so that there is an SCE attaining more welfare than the DCE. Let $k = \overline{k} - \delta$ and $i = \varepsilon$, with $\varepsilon > 0$ and $\delta > 0$ both arbitrarily small but such that $\varepsilon > \gamma$, where γ is such that

$$\overline{k} - \delta = \max_q\{hL(q) - \gamma v(q)\} - \max_q\{lL(q) - \gamma v(q)\}.$$

Such a parameter configuration is feasible because $\delta > 0 \Rightarrow \gamma > 0$ and by continuity, $\delta \downarrow 0 \Rightarrow \gamma \downarrow 0$. Because $i > \gamma = \tilde{\iota}$, the DCE is unique and characterized by a pair $(e, q) = (l, q_{DCE})$, where

$$\frac{lL'(q_{DCE})}{v'(q_{DCE})} = \varepsilon.$$

By construction, the DCE exists since

$$\frac{lL'(q_{DCE})}{v'(q_{DEM})} = \varepsilon < \iota_L = \frac{lL'(\underline{q})}{v'(\underline{q})} \leq \frac{lL'(\underline{q}')}{v'(\underline{q}')}$$

implies $q_{DCE} > \underline{q}'_L$. Therefore, all y in the set (B.14) are in line with a slack capacity constraint as well as the existence the DCE and SCE.

Flow welfare in the DCE satisfies $\overline{\mathcal{W}} = l[u(q_{DCE}) - c(q_{DCE})] + y$ and observe that $\varepsilon \downarrow 0 \Rightarrow q_{DCE} \uparrow \hat{q}$. Now, consider an SCE with the pair (q_L, q_H) described before. With $k = \overline{k} - \delta$ and $\delta > 0$ but small, (24) is satisfied. To ensure (23) is satisfied we need $\varepsilon = \rho\iota_L$. Clearly, for $\varepsilon > 0$ but small we have $\rho \in (0,1)$ and with $\varepsilon \downarrow 0$ we have $\rho \downarrow 0$. Expected flow welfare in the SCE is given by

$$\widetilde{\mathcal{W}} = \frac{\varepsilon}{\iota_L} l[u(\underline{q}) - c(\underline{q})] + \frac{\iota_L - \varepsilon}{\iota_L}\{h[u(\hat{q}) - c(\hat{q})] - \overline{k} + \delta\} + y.$$

With $\varepsilon \downarrow 0$ and $\delta \downarrow 0$, we obtain

$$\widetilde{\mathcal{W}} - \overline{\mathcal{W}} \to h[u(\hat{q}) - c(\hat{q})] - \overline{k} - l[u(\hat{q}) - c(\hat{q})].$$

Using that $\overline{k} = (h-l)L(\hat{q})$, we find

$$\begin{aligned} h[u(\hat{q}) - c(\hat{q})] - \overline{k} - l[u(\hat{q}) - c(\hat{q})] &= (h-l)[L(\hat{q}) + \Pi(\hat{q})] - \overline{k} \\ &= (h-l)\Pi(\hat{q}) \\ &> 0. \end{aligned}$$

Welfare in the SCE is therefore larger than welfare in the DCE. Q.E.D.

B.5 Proof of Proposition 4

Given the Fisher rate and $\boldsymbol{\rho}$, the pair (q_L, q_H) is pinned down uniquely by the system (23)-(25). Define

$$\iota_L(q_H) = \frac{lL'(q_L)}{v'(q_L)} \quad \text{and} \quad \iota_H(q_H) = \frac{hL'(q_H)}{v'(q_H)}. \tag{B.15}$$

Excluding knife-edge cases, $\iota_L(q_L) > \iota_H(q_H)$ in an SCE and $\iota_L(\hat{q}) = \iota_H(\hat{q}) = 0$. Moreover, $\iota'_L(q_L) < 0$ and $\iota'_H(q_H) < 0$. Totally differentiating (23) and using $0 = \mathrm{d}\rho_L + \mathrm{d}\rho_H$, we obtain

$$0 = \mathrm{d}\rho_L(\iota_L - \iota_H) + \rho_L \mathrm{d}\iota_L + (1-\rho_L)\mathrm{d}\iota_H, \tag{B.16}$$

with $\mathrm{d}\iota_L = \iota'_H(q_L)\mathrm{d}q_L$ and $\mathrm{d}\iota_H = \iota'_H(q_H)\mathrm{d}q_H$. Suppose that for a given $\boldsymbol{\rho}$, $q_H = \hat{q}$. Small changes in $\boldsymbol{\rho}$ preserve this property, so $\mathrm{d}q_H = \mathrm{d}\iota_H = 0$. Using this in (B.16) yields $\mathrm{d}\iota_L = -(\iota_L/\rho_L)\mathrm{d}\rho_L$. In turn, using this in (B.15) yields $\mathrm{d}q_L = -[\iota_L/(\rho_L \iota'_L(q_L))]\mathrm{d}\rho_L$. Hence, $\mathrm{d}q_L/\mathrm{d}\rho_L > 0$ and $\mathrm{d}\iota_L/\mathrm{d}\rho_L < 0$. Then suppose that for a given $\boldsymbol{\rho}$, $q_H < \hat{q}$. Small changes in $\boldsymbol{\rho}$ preserve this property and $\iota_H(q_H) > 0$. Equation (25) holds with equality when $q_H < \hat{q}$ and totally differentiating it yields

$$[(1-h)\Pi'(q_H) + c'(q_H)]\,\mathrm{d}q_H = [(1-l)\Pi'(q_L) + c'(q_L)]\mathrm{d}q_L. \tag{B.17}$$

Since $v(q) = \Pi(q) + c(q)$, $v'(q) > 0$, and $c'(q) > 0$, (B.17) implies $\mathrm{d}q_H/\mathrm{d}q_L > 0$. Using this in (B.16), we find

$$\rho_L \iota'_L(q_L)\frac{\mathrm{d}q_L}{\mathrm{d}\rho_L} + (1-\rho_L)\iota'_H(q_H)\frac{\mathrm{d}q_H}{\mathrm{d}q_L}\frac{\mathrm{d}q_L}{\mathrm{d}\rho_L} = \iota_L - \iota_H. \tag{B.18}$$

Here, (24) implies $\iota_L - \iota_H > 0$. Therefore, using $\mathrm{d}q_H/\mathrm{d}q_L > 0$ in (B.18) implies $\mathrm{d}q_L/\mathrm{d}\rho_L > 0$ and $\mathrm{d}q_H/\mathrm{d}\rho_L > 0$. Using this in (B.15) yields $\mathrm{d}\iota_L/\mathrm{d}\rho_L < 0$ and $\mathrm{d}\iota_H/\mathrm{d}\rho_L < 0$. Observe that when $q_H = \hat{q}$, (25) imposes

$$(1-h)\Pi(\hat{q}) + c(\hat{q}) \le (1-l)\Pi(q_L) + c(q_L).$$

The RHS of this inequality is strictly increasing in q_L. Since $\iota_L(q_L)$ is strictly decreasing in q_L, it immediately follows from (23) that in an SCE, $q_H = \hat{q}$ if and only if

$$\rho_L \ge \frac{iv'(\underline{q})}{lL'(\underline{q})}, \quad \text{where} \quad (1-l)\Pi(\underline{q}) + c(\underline{q}) = (1-h)\Pi(\hat{q}) + c(\hat{q}).$$

This proves statement 1 in Proposition 4.

Excluding a knife-edge case in which $k = (h-l)L(\hat{q})$, in an SCE we have that $\iota_L > 0$ since $k > (h-l)L(\hat{q})$ implies $\tilde{\iota} > 0$. Combining (14), (15), and $v(q) = \Pi(q) + c(q)$ we have that

$$\phi_t M_{t-1} = (1-l)\Pi(q_L) + c(q_L) - y.$$

The RHS of this equation is strictly increasing in q_L. Because q_L is strictly increasing in ρ_L, the DM$_t$ value of fiat money is strictly increasing in ρ_L. The value of fiat money carried out of CM$_{t-1}$ satisfies $\phi_{t-1}M_{t-1} = \phi_t M_{t-1}/\pi$. This is also strictly decreasing in ρ_L, proving statement 2 in Proposition 4.

Because $\iota_L > 0$, the supply of assets in the DM when the realization of the sunspot is L, satisfies $a_L = v(q_L)$. By construction, $v(q_L)$ is strictly increasing in q_L, so a_L is strictly increasing in ρ_L. Next, when $q_H = \hat{q}$, then (15) implies $a_H = \phi_t M_{t-1} + h\Pi(\hat{q}) + y$. Because the DM$_t$ value of fiat money is strictly increasing in ρ_L, it follows that a_H is strictly increasing in ρ_L. When $q_H < \hat{q}$ then $\iota_H > 0$, so combining (14) and (15) implies $a_H = v(q_H)$. Because q_H is then strictly increasing in ρ_L, it follows that a_H is strictly increasing in ρ_L. This proves statement 3 in Proposition 4.

The CM$_t$ value of a newborn firm satisfies

$$\Upsilon = \beta\rho_L(1+\iota_L)\left[l\Pi(q_L) + y\right] + \beta(1-\rho_L)(1+\iota_H)\left[h\Pi(q_H) + y\right],$$

Equation (23) implies $\lim_{\rho_L \to 0} \iota_H = i$ and turn a unique value for q_H. Moreover, $\lim_{\rho_L \to 0} q_L$ and $\lim_{\rho_L \to 0} \iota_L$ are then bounded through (25), where (25) fixes q_L as a function of q_H. Similarly, $\lim_{\rho_L \to 1} \iota_L = i$, and through (25) $\lim_{\rho_L \to 1} q_H$ and $\lim_{\rho_L \to 1} \iota_H$ are bounded. We obtain

$$\lim_{\rho_L \to 0} \Upsilon = \beta(1+i)\left[h\Pi(q_H') + y\right] \quad \text{and} \quad \lim_{\rho_L \to 1} \Upsilon = \beta(1+i)\left[l\Pi(q_L') + y\right],$$

where $lL'(q_L')/v'(q_L') = i$ and $hL'(q_H')/v'(q_H') = i$, so $q_L' < q_H'$. In turn this implies $l\Pi(q_L') < h\Pi(q_H')$, proving statement 4 in proposition 4.

B.6 Proof of Proposition 5

Consider a two-cycle in which we alternate between $\eta = 0$ and $\eta = 1$. Index quantities with η and focus on a cycle in in which $(m_0, m_1) \in (\underline{m}, \overline{m})^2$. From (43), it is clear that such a two-cycle exists if and only if

$$m_0 = f_h(m_1) \equiv \frac{\beta m_1 \left[1 + \mathcal{L}_h(m_1)\right]}{1+\mu} \quad \text{and} \quad m_1 = f_l(m_1) \equiv \frac{\beta m_0 \left[1 + \mathcal{L}_l(m_0)\right]}{1+\mu}. \tag{B.19}$$

To show the existence, let $1 + \mu = \beta(1 + \tilde{\imath})$ and ignore the knife edge case in which $\tilde{\imath} = 0$. Then, by construction, we have that

$$f_h(m) \begin{cases} > m & \text{if } m < \underline{m} \\ = m & \text{if } m = \underline{m} \\ < m & \text{if } m > \underline{m} \end{cases} \quad \text{and} \quad f_l(m) \begin{cases} > m & \text{if } m < \overline{m} \\ = m & \text{if } m = \overline{m} \\ < m & \text{if } m > \overline{m} \end{cases}.$$

It follows that there exists at least one pair $(m_0, m_1) \in (\underline{m}, \overline{m})^2$ satisfying (B.19). Because $f_h(m)$ and $f_l(m)$ are continuous in μ, it follows that a pair $(m_0, m_1) \in (\underline{m}, \overline{m})^2$ satisfying (B.19) also exists for $1 + \mu = \beta(1 + \tilde{\imath}) - \varepsilon'$ and $1 + \mu = \beta(1 + \tilde{\imath}) + \varepsilon'$, with $\varepsilon' > 0$ but sufficiently small. Q.E.D.

References

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, *109*(3), 659–684.

Altermatt, L. (2017). *Inside money, investment, and unconventional monetary policy* (ECON - Working Papers No. 247). Department of Economics - University of Zurich.

Altermatt, L., Iwasaki, K., & Wright, R. (2021). Asset pricing in monetary economies. *Journal of International Money and Finance*, *115*, 1023–1052.

Andolfatto, D., Berentsen, A., & Waller, C. J. (2016). Monetary policy with asset-backed money. *Journal of Economic Theory*, *164*, 166–186.

Azariadis, C. (1981). Self-fulfilling prophecies. *Journal of Economic Theory*, *25*(3), 380–396.

Azariadis, C. (1993). *Intertemporal macroeconomics.* Oxford, England: Blackwell.

Benigno, P. (2020). A central bank theory of price level determination. *American Economic Journal: Macroeconomics*, *12*(3), 258–283.

Berentsen, A., Menzio, G., & Wright, R. (2011). Inflation and unemployment in the long run. *American Economic Review*, *101*(1), 371–398.

Berentsen, A., & Waller, C. J. (2011). Price-level targeting and stabilization policy. *Journal of Money, Credit and Banking*, *43*, 559–580.

Berentsen, A., & Waller, C. J. (2015). Optimal stabilization policy with search externalities. *Macroeconomic Dynamics*, *19*(3), 669–700.

Branch, W. A., & Silva, M. (2019). *Unemployment and the stock market when households lack commitment.* Mimeo.

van Buggenum, H. (2021). *Risk, inside money, and the real economy* (CentER Discussion Paper No. 2021-020). Tilburg University, Center for Economic Research.

Calomiris, C. W., & Khan, U. (2015). An assessment of TARP assistance to financial institutions. *Journal of Economic Perspectives*, *29*(2), 53–80.

Del Negro, M., & Sims, C. A. (2015). When does a central bank's balance sheet require fiscal support? *Journal of Monetary Economics*, *73*, 1–19.

Demertzis, M., Marcellino, M., & Viegi, N. (2009). *Anchors for inflation expectations* (DNB Working Papers No. 229). Netherlands Central Bank, Research Department.

Diamond, P. A. (1982). Aggregate demand management in search equilibrium. *Journal of Political Economy*, *90*(5), 881–894.

Fisher, I. (1936). 100% money and the public debt. *Economic Forum*(Spring Numer, April-June 1936), 406–420.

Friedman, M. (1969). *The optimal quantity of money and other essays.* Chicago, IL: Aldine.

Geromichalos, A., & Herrenbrueck, L. (2016). Monetary policy, asset prices, and liquidity in over-the-counter markets. *Journal of Money, Credit and Banking*, *48*(1), 35–79.

Geromichalos, A., Licari, J. M., & Suárez-Lledó, J. (2007). Monetary policy and asset prices. *Review of Economic Dynamics*, *10*(4), 761–779.

Gorton, G., & Ordoñez, G. (2014). Collateral crises. *American Economic Review*, *104*(2), 343–378.

Gu, C., Mattesini, F., Monnet, C., & Wright, R. (2013). Banking: A new monetarist approach. *The Review of Economic Studies*, *80*(2), 636–662.

Gu, C., Monnet, C., Nosal, E., & Wright, R. (2020). *On the instability of banking and other financial intermediation* (BIS Working Papers No. 862). Bank for International Settlements.

Gu, C., & Wright, R. (2016). Monetary mechanisms. *Journal of Economic Theory*, *163*, 644–657.

Gürkaynak, R. S., Swanson, E., & Levin, A. (2010). Does inflation targeting anchor long-run inflation expectations? Evidence from the U.S., U.K., and Sweden. *Journal of the European Economic Association*, *8*(6), 1208–1242.

Herrenbrueck, L., & Geromichalos, A. (2017). A tractable model of indirect asset liquidity. *Journal of Economic Theory*, *168*, 252–260.

Howitt, P., & McAfee, R. P. (1987). Costly search and recruiting. *International Economic Review*, *28*(1), 89–107.

Howitt, P., & McAfee, R. P. (1992). Animal spirits. *The American Economic Review*, *82*(3), 493–507.

Kalai, E. (1977). Proportional solutions to bargaining situations: Interpersonal utility comparisons. *Econometrica*, *45*(7), 1623–1630.

Kaplan, G., & Menzio, G. (2016). Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy*, *124*(3), 771–825.

Kocherlakota, N. R. (1998). Money is memory. *Journal of Economic Theory*, *81*(2), 232–251.

Lagos, R. (2010). Asset prices, liquidity, and monetary policy in the search theory of money. *Federal Reserve Bank of Minneapolis Quarterly Review*, *33*(1), 14–20.

Lagos, R., & Rocheteau, G. (2008). Money and capital as competing media of exchange. *Journal of Economic Theory*, *142*(1), 247–258.

Lagos, R., Rocheteau, G., & Wright, R. (2017). Liquidity: A new monetarist perspective. *Journal of Economic Literature*, *55*(2), 371–440.

Lagos, R., & Wright, R. (2003). Dynamics, cycles, and sunspot equilibria in "genuinely dynamic, fundamentally disaggregative" models of money. *Journal of Economic Theory*, *109*(2), 156–171.

Lagos, R., & Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, *113*(3), 463–484.

Lettau, M., & Madhavan, A. (2018). Exchange-traded funds 101 for economists. *Journal of Economic Perspectives*, *32*(1), 135–154.

Levin, A. T., Natalucci, F. M., & Piger, J. M. (2004). The macroeconomic effects of inflation targeting. *Federal Reserve Bank of St. Louis Review*, *86*(4), 51–80.

Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica*, *46*(6), 1429–1445.

Mortensen, D. T., & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, *61*(3), 397–415.

Nash, J. F. (1950). The bargaining problem. *Econometrica*, *18*(2), 155–162.

Nosal, E., & Rocheteau, G. (2011). *Money, payments, and liquidity.* The MIT Press.

Peck, J., & Shell, K. (2003). Equilibrium bank runs. *Journal of Political Economy*, *111*(1), 103–123.

Pissarides, C. A. (1984). Search intensity, job advertising, and efficiency. *Journal of Labor Economics*, *2*(1), 128–143.

Reis, R. (2015). *Different Types of Central Bank Insolvency and the Central Role of Seignorage* (NBER Working Papers No. 21226). National Bureau of Economic Research, Inc.

Rocheteau, G., Hu, T.-W., Lebeau, L., & In, Y. (2021). Gradual bargaining in decentralized asset markets. *Review of Economic Dynamics*, *42*, 72–109.

Rocheteau, G., & Wright, R. (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, *73*(1), 175–202.

Rocheteau, G., & Wright, R. (2013). Liquidity and asset-market dynamics. *Journal of Monetary Economics*, *60*(2), 275–294.

Rubinstein, A., & Wolinsky, A. (1987). Middlemen. *The Quarterly Journal of Economics*, *102*(3), 581–593.

Sargent, T. J., & Wallace, N. (1981). Some unpleasant monetarist arithmetic. *Quarterly Review*, *5*(3), 1–17.

Tanaka, A. (2021). Central bank capital and credibility: A literature survey. *Comparative Economic Studies*, *63*(2), 249–262.