The Great Gatsby Goes to College: Tuition, Inequality and Intergenerational Mobility in the U.S.

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Abstract

This paper analyzes the role of higher education in shaping income inequality and intergenerational mobility. I introduce a model where overlapping generations of heterogeneous households make college choices subject to a borrowing constraint and with heterogeneous colleges that maximize quality. First, in response to the observed rise in the returns to human capital in the U.S. since 1980, the model predicts an increase in income inequality, tuition, the dispersion of spending per-student across colleges, the exclusion of low-income students from top colleges, and the intergenerational elasticity of earnings (IGE), all consistent with the data. Second, I use the model to run counterfactuals. If all students received the same higher education, the income Gini and the IGE would decrease by up to 9% and 33%, respectively. Current government interventions—financial aid and transfers to colleges—decrease the Gini coefficient by 3% and the IGE by 12% compared to the *laissez-faire*.

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1 Introduction

To what extent does the higher education system in the U.S. accentuate or dampen income inequality and reduce or enhance intergenerational mobility? College has traditionally been viewed as one of the main pathways to upward mobility. However, access remains extremely selective and unequal, especially at top-quality universities. For example, Chetty, Friedman, Saez, Turner, and Yagan (2019) report that children whose parents are in the top 1% of the income distribution are 77 times more likely to attend an Ivy League college than those whose parents are in the bottom income quintile. What are the forces determining the sorting of students and financial resources across colleges of different quality? To what extent does parental income matter relative to ability? How does this sorting in turn shapes inequality at the next generation and intergenerational mobility?

These questions regarding the contributions of colleges to income inequality and intergenerational mobility are all the more important as trends over the past forty years show that (a) the market returns to education and income inequality have increased (Piketty and Saez, 2003; Autor, Katz, and Kearney, 2008); (b) the dispersion of expenditures per students across colleges has increased (Capelle, 2019); (c) the share of students from the lowest income quintile in top colleges has stagnated (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2019); (d) tuition fees before financial aid have increased by a factor of four in real terms since 1980; and (e) the intergenerational elasticity of income (hereafter IGE) has slightly increased, corresponding to a decline in intergenerational mobility (Davis and Mazumder, 2017).

In this paper, I provide a framework to understand the interaction between the allocation of students and financial resources across heterogeneous colleges, income inequality and intergenerational mobility. I analyze how the higher education system—the endogenous response of colleges and government policies—accentuates or dampens income inequality and reduces or enhances intergenerational mobility. I then offer a unified explanation for the stylized facts (a) to (e): an increase in the returns to human capital. Finally, I use the model to analyze how higher education propagates the increase in income inequality and run policy counterfactuals.

The household side of the model builds on a large theoretical literature that formalizes how human capital transmission across generations perpetuates inequality (e.g., Benabou (2002)). A continuum of heterogeneous households characterized by their human capital transmit—with some randomness—ability to their children and make an educational investment choice subject to a borrowing constraint. The supply side of the market for higher education is a continuum of colleges that differ in quality. Households face an equilibrium tuition schedule that depends on college quality, student ability and parental income.[1] After college, each child becomes an adult, sells their human capital—a combination of their ability, college quality and some labor market shock—in a competitive labor market and has a child.

A key novelty of my framework is to embed into this overlapping generation general equilibrium model a distribution of heterogeneous colleges that is endogenous. Colleges seek to maximize the quality they provide to their students. Their quality depends not only on the amount of educational resources spent per student but also on the average ability of the student body, what will be referred to as the "peer-effect." Colleges have an incentive to attract high-ability students because of the peer-effect, as well as students from rich families who bring in additional resources to finance educational spending. This microfoundation of the college sector is borrowed from a literature that estimates equilibrium models of higher education (e.g., Rothschild and White (1995); Epple, Romano, and Sieg (2006); Cai and Heathcote (2018)). As in Cai and Heathcote (2018), colleges are price-takers and the tuition schedule clears each segment of the higher education market. Finally, I close the model with an educational sector that produces educational services and a government that implements non-linear merit and need-based financial aid to students and non-linear transfers to colleges.

The first contribution of the paper is to provide an analytical characterization of the equilibrium allocation and a unified explanation for the stylized facts (a) to (e): under weak conditions, an increase in the returns to education—a primitive of the model—is shown to lead to an increase in income inequality, an increase in the inequality of resources across colleges, a decrease in the share of low income students at top colleges, a decline in intergenerational mobility and an increase in tuition. Intuitively, the rising returns to education increase the dispersion of labor earnings for a given distribution of human capital, thereby increasing income inequality. This leads richer households to demand higher quality of education, incentivizing top colleges to raise tuition, increasing the dispersion of revenues and educational spending across colleges. This in turn feeds back into more inequality in human capital at the following

[1] In Benabou (2002), households buy an educational "good" traded at a constant unit price, independent on the households/students' characteristics and there is no notion of quality ladder.

generation. Individuals from low-income background are priced out of top colleges, hence the stagnation of their shares and the decline in mobility. Higher education thus contributes to the gradual shift of the U.S. society to the right side of the Great Gatsby curve.[2]

I then estimate a more general version of the model that I use to run policy counterfactuals as well as to quantify the above effects. I allow for some degree of intergenerational transfers of financial wealth, for individuals to not go to college, for some degree of noise in the sorting of students and for the educational services to be intensive in high skilled labor. The model is estimated using several microdata sources: (i) the National Longitudinal Survey of Youth of 1997, a representative panel of high-schoolers, with detailed information on parental background, the children's abilities, their journey through the higher education system and their income in their early thirties; (ii) the NCES-NPSAS, a detailed student-level dataset on net tuition and financial aid; and (iii) the NCES-IPEDS, a panel of the universe of colleges. The estimation proceeds in two steps: first I estimate the closed-form model which I show is, under some assumptions, exactly identified. I then use the resulting estimates as initial values in the estimation of the richer quantitative version. The closed-form expressions for the targeted moments in the first step make it transparent which moments are important to pin down each parameter. They also make this first step computationally quick, which allows me to estimate a large set of parameters. I finally conduct several validation exercises.

The second contribution is to quantify the extent to which the higher education system shapes income inequality and intergenerational (im)mobility. I run six policy counterfactuals. The first one aims at quantifying the total contribution of higher education to inequality and mobility. It consists in randomly allocating students to colleges and equalizing spending across institutions. This leads to a decrease in the income Gini by 9% (4.3 p.p.) and a decrease in the IGE by 24%.[3] With the second counterfactual, I am interested in isolating the contribution of the peer-effect. To do so, I increase the degree of progressivity of transfers to colleges to the point where

[2]The Great Gatsby curve is the negative empirical correlation between cross-sectional income inequality and intergenerational mobility. It has been documented in the cross-section of countries and over time in the U.S.

[3]To give a sense of the magnitudes, the Gini coefficient of household gross income in the last decade has been around .45. And it has increased by 10 p.p. since 1980. Estimates for the IGE range from .36 to .55. My favorite estimate, which is also the value targeted in the calibration, is .4.

spending is equalized across all colleges. In equilibrium, students perfectly sort across colleges by ability.[4] This policy experiment leads to a decrease in the income Gini by 3% (1.4 p.p.) and in the IGE by 15%.

The third policy counterfactual studies the implications of removing current government interventions in higher education. In a *laissez-faire* allocation, where all government interventions are removed, I find that the Gini coefficient would be 3% higher, and the IGE 10% higher. The most powerful dimension of current policy is the subsidy to colleges, which contributes to a reduction of the income Gini by 2%, followed by need-based financial aid, which decreases the income Gini by 1%. Merit-based financial aid has virtually no effect, because it is very small in the *status quo*. I document elsewhere that government transfers to colleges have become significantly less redistributive over the past forty years (Capelle, 2019), bringing the system closer to a *laissez-faire* allocation. In the fourth counterfactual, I set the parameters of the transfers schedule to what they were at the beginning of the 1980s. I find that the Gini and the IGE would be 1% and 3.4% lower than in the *status quo*, respectively.

The last two counterfactuals analyze the implications of a sharp increase in the progressivity of need-based aid provided by colleges and of a version of "College for All", two proposals widely discussed in policy circles. To implement the former, I modify the strength of the social objective of colleges to the point where the tuition schedule at a given institution and for a given child ability is linear in parental income. Like in the first two experiments, the effect of parental income on the sorting of students is neutralized. However college spending remains strongly increasing in college quality because child ability and parental income are strongly positively correlated. This leads to an increase in the income Gini by 3% and in GDP by 22% because of the increase in the positive assortative matching of students across colleges. The increase in the Gini occurs despite the decrease in the IGE by 6%. Finally I evaluate the implications of a version of the "College for All" proposal. I implement the latter by fixing the dispersion of spending across colleges to its level in the *status quo* and by sorting students across colleges by ability. Perfect assortative matching is not an assumption but an equilibrium outcome. Surprisingly, I find that the income Gini would increase by 2.5% and the IGE would decrease by 7%. GDP would increase by 2.8%, thanks to the improvement in the allocation of students.

[4] A very progressive need-based financial aid schedule would generate the same equalization of resources and assortative matching by abilities.

The last contribution is to assess the quantitative effects of a rise in the returns to human capital and to decompose the rise in inequality into a direct effect and the endogenous propagation through the higher education sector. The increase in the returns to human capital is calibrated to match the increase in the returns to college. Following this increase, the model generates an increase in the income Gini coefficient by 13 p.p., which corresponds to 130% of the empirical change, an increase in the expenditure per student Gini by 5 p.p. corresponding to 100% of the empirical change and an increase in the IGE by 6%. In a counterfactual world in which the returns to human capital increase but do not propagate through the higher education sector, the increase in the Gini coefficient of income would have been 6% lower. There are two sources of amplification: the allocation of resources and the allocation of student ability across colleges. More than 100% of the total amplification is coming from the former. The latter actually dampens the increase in inequality because the positive assortative matching of students along the quality ladder of colleges worsens as relatively richer and less able children buy their way to top colleges.

Literature. The present paper relates most directly to the literature that models and quantifies the transmission of human capital, educational choice and inequality in an intergenerational framework (Benabou, 1996; Fernandez and Rogerson, 1996; Kotera and Seshadri, 2017; Caucutt and Lochner, 2017; Guerrieri and Fogli, 2017; Durlauf and Seshadri, 2018; Blandin, Herrington, et al., 2018; Lee and Seshadri, 2019; Eckert and Kleineberg, 2019). Restuccia and Urrutia (2004) focuses on the role of higher education and shows that half of the intergenerational correlation of earnings is accounted for by parental investment in education. The college sector is simple: there is a representative college with an exogenously parametrized tuition fees. It abstracts from the facts that the higher education sector is extremely stratified, and that the degree of stratification and tuition are endogenous to the economic conditions.

In contrast, another stream of the literature models in detail but in static frameworks the admission and tuition decisions of colleges and the rich heterogeneity in colleges and student types. It focuses on the impact of financial aid policies (Epple, Romano, and Sieg, 2006; Fillmore, 2016), of a change in the supply of seats in public colleges (Fu, 2014) and affirmative action policies (Kapor, 2015) on the sorting by itself, while my focus is the study of equilibrium inequality and mobility. More closely related, Cai and Heathcote (2018) shows that income inequality can fully account for the rise in average tuition since 1990 in a static model where households choose a quality of college and price-taking colleges maximize profit. My model has a flavor of theirs. There are three main differences. First, my model is dynamic, to be able to analyze the role of higher education in shaping inequality and mobility. To the best of my knowledge, it is the first one to embed a sorting problem of heterogeneous students across heterogeneous clubs into an intergenerational setting.[5] Second, I assume that there is a continuum of student ability instead of two types, which simplifies the analysis. Thirdly, higher education is not a consumption good like in theirs but matters for the accumulation of human capital. Incidentally, the equilibrium allocation is fully efficient in their model, but inefficient in mine because of the financial frictions. Finally, in contrast with the literature, the model can be solved analytically.[6]

Another related stream of the literature studies the macroeconomic determinants of tuition and the effects of financial aid. Lucca, Nadauld, and Shen (2015) stresses the role of the expansion of credit supply, Gordon and Hedlund (2017) the importance of financial aid and Jones and Yang (2016) the rising cost of universities input and professors—the Baumol's disease, a mechanism our model accommodates—implied by the rise in the skill premium, to explain the rise in tuition. In this paper, I stress the role of the increase in the returns to education to explain the rise in the average and the dispersion of tuition fees, due to the increase in demand by households for higher quality of higher education, especially at the top of the distribution. This mechanism is akin to the revenue theory of cost by Bowen (1980), but applied to a framework with a ladder of colleges, whereby universities raise all the money they can through tuition fees and then spend it on projects that enhance quality. Martin, Hill, and Waters (2017) have estimated that this mechanism accounts for two third of the increase in the average real tuition fees. Regarding financial aid policy, Abbott, Gallipoli, Meghir, and Violante (2013) shows that current financial aid policy improves

[5] Jovanovic (2014) studies an economy where long-term growth depends on the quality of assignment between workers and managers. In my model, (i) the assignment problem is instead of students to colleges, which are a bundle of educational services and other students; (ii) students are heterogeneous in two dimensions (abilities and parental income) and not just one (ability), and (iii) the source of the misallocation is a financial friction, not an exogenous noise in the assignment process. In section 6, I also allow for some noise in the sorting.

[6] This has three advantages. First, one can analyze in a transparent manner how technology and policy parameters shape the sorting of students, inequality and mobility. Second, the identification of structural parameters in the estimation is also very transparent. Thirdly, existence and uniqueness properties of the equilibrium—two issues that have plagued the theoretical and quantitative literature on clubs—can be characterized.

efficiency and increases GDP. Although we share the same deep source of inefficiency—a borrowing constraint—I focus on the misallocation of heterogeneous students across heterogeneous colleges rather than on the enrollment rate.[7]

The rest of the paper is organized as follows. Section 2 presents the model and section 3 explains the closed-form equilibrium expressions. Section 4 generalizes the model to include public transfers to colleges and public and institutional financial aid to students. Section 5 derives the key analytical comparative statics: an increase in the return to human capital generates facts (a) to (e). Section 6 explains the estimation procedure and derives the quantitative results regarding the role of the higher education sector for the amplification of inequality and the reproduction of economic status over generation. Section 7 concludes.

2 Human Capital Transmission with a Hierarchy of Colleges

The economy is populated by two types of agents: dynastic households and colleges. At each generation, households imperfectly transmit human capital to their child and decide which college to send them to after high school. Colleges choose their pool of students as well as educational spending to maximize the quality they deliver.

2.1 Households

There is an infinity of dynasties, indexed by $i \in \mathscr{I}$. Individuals live for two periods: one as a child and one as an adult. Each adult has one child. A generation $t \in \mathbb{N}$ household of dynasty i is characterized by its level of human capital h_{it} and the child's human capital at the end of high school z_{it}. They choose consumption c_{it}, labor supply ℓ_{it} and college quality q_{it} for their child. When no confusion results, I drop

[7]A large reduced-form literature provides evidence on the returns to college quality and/or selectivity. While most of them find significant returns on the labor and marriage markets as well as for children' achievements (Black and Smith, 2006; Long, 2010; Hoekstra, 2009; Zimmerman, 2014; Bleemer, 2019), some influential papers have cast doubt on these findings and the debate is still on-going (Dale and Krueger, 2011; Hickman and Mountjoy, 2019). My results suggest moderate amplification effects of higher education. Another literature has shown that parental background matters a lot for achievements and access in top colleges (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2019; Hoxby and Turner, 2019) and that financial aid policy has a significant impact on college decisions (Dynarski, 2003; Angrist, Autor, Hudson, and Pallais, 2016).

the generation and dynasty subscripts and denote the state variables of the next generation with a prime. The current generation value $\mathscr{U}(h, z)$ is solution to

$$\mathscr{U}(h, z) = \max_{c,\ell,q} \left\{ (1-\beta)\left[\ln c - \ell^{\eta}\right] + \beta E\left[\mathscr{U}(h', z')\right] \right\} \tag{1}$$

where β denotes the intergenerational discount factor. A child's human capital at the end of high school is modeled as a log-linear combination of parents' human capital h and the birth shock ξ_b, capturing the randomness of the transmission process.

$$z = (\xi_b h)^{\alpha_1} \qquad \text{Child's High School Ability} \tag{2}$$

A household's lifetime earnings denoted y is a function of their level of human capital h, their supply of raw labor ℓ and the tax schedule. I describe the earning function in section 2.3.

Households are subject to a lifetime budget constraint. Their income y can be spent on consumption and on tuition fees. The tuition schedule is an equilibrium object which depends on college quality q, household income y and the child ability z. Normalizing the price of the final good to one, it is given by

$$y = c + e(q, y, z) \qquad \text{Household Lifetime Budget Constraint.} \tag{3}$$

This budget constraint implies that households face an intergenerational borrowing constraint, *i.e.* the current adults cannot leave bequest or pass-on debt along to their offspring. This assumption draws on a large set of evidence that borrowing constraints do matter for college choices. Lochner and Monge-Naranjo (2012) review the evidence on borrowing constraint in education. Although this specification rules out net financial transfers across generation, the quantitative version I introduce later partially relaxes this assumption.[8] The adulthood human capital of the child after college is a log-linear combination of its pre-college ability, the quality of the college

[8]Ruling out net financial transfers across generations doesn't prevent gross flows to occur within a lifetime. For example, children are allowed to borrow from their parents early in life and repay later. It doesn't rule out student loans as long as they are exactly offset by a parental transfer of the same amount, *i.e.* student debt is possible as long as it is paid by parents.

they went to and a labor market shock.[9] It is given by

$$h' = zq^{\alpha_2}\xi_y \qquad \text{Child's Post-College Human Capital} \tag{4}$$

There are two sources of randomness in the accumulation process of human capital. The birth shock ξ_b is known before the college quality decision has to be made, while the labor market shock ξ_y is realized once the child enters the labor market. It is assumed that the birth and labor market shocks are i.i.d across generations and households and log-normally distributed.[10]

$$\ln \xi_b \sim \text{i.i.d.}\mathcal{N}\left(\mu_b, \sigma_b^2\right) \qquad \text{Birth Shock} \tag{5}$$

$$\ln \xi_y \sim \text{i.i.d.}\mathcal{N}\left(\mu_y, \sigma_y^2\right) \qquad \text{Labor Market Shock} \tag{6}$$

2.2 Colleges

Technology. There is a mass one of *ex-ante* identical colleges indexed by $j \in [0,1]$. They are all of a fixed size, $\mathfrak{c}$.[11] A college is a technology that delivers to its students a quality that depends on educational services per student I_j and the average of student ability $\bar{z}_j$, which will be referred to as the "peer effect." Furthermore, I assume that quality depends negatively on the degree of dispersion of abilities and parental income within the college, σ_u^2, which I define later. The production function of quality is given by

$$\ln q_j = \ln I_j^{\omega_1} \bar{z}_j^{\omega_2} - \sigma_{u,j}^2$$

where $\omega_1, \omega_2 > 0$.

Colleges are clubs because who belongs to the college matters for the quality

[9]There is empirical evidence that the law of accumulation of human capital is characterized by complementarities between pre-college ability and college quality. Dillon and Smith (2018) finds evidence of such complementarities for long-term earnings. Lee and Seshadri (2019) estimate that the elasticity of substitution across periods of the human capital accumulation process is one, which amounts to a Cobb-Douglas functional form.

[10]This formulation of the household problem draws from and extends Benabou (2002). The latter can be seen as the case where there is no birth shock, $\sigma_b^2 = 0$ and a constant unitary price for education $e(q, y, z) = q$.

[11]As I explained in appendix A.7, it is natural to set the size of a college to the cardinality of the continuum $\mathfrak{c} = \aleph_1$ as the paper analyzes equilibria in which there is a continuum of heterogeneous students in each college.

delivered to all members, through $\bar{z}$. There is empirical evidence that peers enter the production function of college quality. For example, Sacerdote (2011), Smith and Stange (2016) and Mehta, Stinebrickner, and Stinebrickner (2018) find evidence of peer-effects, especially from roommates, for achievements while in college. Zimmerman (2019) finds evidence that the network and social capital built while in college matters for labor market outcomes. Peer effects are also supported by the fact that colleges compete for the best students and seek to maximize $\bar{z}$ (Hoxby, 2009, 2013).

I make two assumptions about the negative impact of student heterogeneity on quality. First I assume that the peer-effects are a geometric average of student abilities which therefore punishes heterogeneity relative to an arithmetic average:

$$\ln \bar{z}_j = E_{\phi_j(.)}[\ln(z)]$$

where $\phi_j(.)$ denotes the distribution of student abilities within college j. Secondly, through $\sigma_{u,j}$, I explicitly assume that the more heterogeneous the class in terms of student ability and economic background the more difficult it is for a college to deliver a given quality to its students. I define $\sigma^2_{u,j}$ as the within-college variance of a weighted average of (log) ability and parental background, $\log z^{\frac{\omega_2}{\omega_1}} y^{-\frac{\omega_3}{\omega_1}}$:

$$\sigma^2_{u,j} = \frac{\omega_1}{2} V_{\phi(.)} \left(\log z^{\frac{\omega_2}{\omega_1}} y^{-\frac{\omega_3}{\omega_1}} \right) \tag{7}$$

Defining $\sigma^2_{u,j}$ in this manner ensures tractability by making $I_j \times e^{-\sigma^2_{u,j}}$ a geometric average of tuition fees. The solution to this problem would therefore be the same if colleges maximized a weighted geometric average of tuition and student ability[12]

Educational services I_j are financed through the collection of tuition fees from all students. If p_I denotes the price of educational services, the static budget constraint of a college is

$$p_I I_j = E_{\phi_j(.)}[e_u(q, z, y)]$$

Objective and Problem. Taking the tuition schedule $e(q, z, y)$ and the price of educational services p_I as given, a college chooses the amount of educational services per student I_j and the composition of the student body $\phi_j(z, y)$—a density over (z, y),

[12] From this perspective, the college's problem has a flavor of Fu (2014), where colleges maximize a weighted average of average student ability and a quadratic function of net tuition.

which determines the average student ability $\bar{z}_j$—to maximize the quality q_j they deliver to their students. For simplicity, asymmetries of information are assumed away and clubs have perfect information about the type of applicants (z, y). Dropping the college subscript when no confusion results, the problem of a college is:

$$\max_{I,\bar{z},\phi(.)} \mathscr{V} = q \tag{8}$$

$$\text{subject to:} \quad \ln q = \ln I^{\omega_1} \bar{z}^{\omega_2} - \sigma_u^2 \qquad \text{College Technology} \tag{9}$$

$$p_I I = E_{\phi(.)}[e_u(q, z, y)] \qquad \text{Budget Constraint} \tag{10}$$

$$\ln \bar{z} = E_{\phi(.)}[\ln(z)] \qquad \text{Average Student Ability} \tag{11}$$

The formulation of the college problem follows the literature that studies the behavior of universities (Fu, 2014; Epple, Romano, and Sieg, 2006). I depart from it by assuming that they behave competitively as in Cai and Heathcote (2018). While the latter paper assume colleges maximize profits, I maintain the assumption that colleges maximize quality. It can be shown that the dual problem of maximizing profit subject to a constraint on $\mathscr{V}$ leads to the same first order conditions. However, when I allow colleges to have a social objective and implement need-based aid to student, it is not easy to interpret the nature of such a problem. Like this paper, I assume a constant returns to scale technology which implies that the size of a college is irrelevant.[13]

Entry and Positioning Game. At each generation, before operating, colleges play a positioning game on the line of qualities.[14] Taking the position of all other colleges as given, each college sequentially chooses which quality to offer, $q \in \mathbb{R}_+$. The order in which they choose is exogenous. Since colleges are otherwise identical, the order is arbitrary and inconsequential. The payoff for operating a given quality is given by (8) and is assumed to be $\mathscr{V} = 0$ if the college is not operating. A subgame perfect Nash equilibrium of this positioning game is a mapping from the set of colleges $j \in [0, 1]$ to

[13] This formulation for the college problem abstracts from several potentially relevant issues, such as the heterogeneity of tracks, colleges and fields of study within the same institution, the existence of congestion forces, and the choice of a size, and the existence of a fixed factor of production (e.g. endowments). Allowing for within-college heterogeneity would require detailed data about the exact peer-group of a student within a college, and expenditures by field of study—a set of data that is currently not available. Regarding the issues of size, congestion and the existence of inelastic factors of production of higher education, although it matters in the short-term, the paper takes a long-term view where these concerns are arguably less relevant.

[14] In appendix A.7 I give a game-theoretic formalization of the positioning game.

the set of qualities $\mathbb{R}^+$ such that given the positioning of all other colleges, no college wants to change its position.

This structure for entry ensures that all positive qualities are offered in equilibrium. The assumption that a non-operating college gets $\mathscr{V} = 0$ sets the lower bound of the support of qualities offered in equilibrium, $q = 0$. The assumption that all colleges must be of size $\mathfrak{c}$ ensures that colleges do not agglomerate at the highest quality level with each one of them operating with an infinitely small mass of students.[15,16]

2.3 Final Good and Educational Service Technologies

Apart from the college sector, there are two other sectors in the economy: the consumption good sector and the educational services sector. The consumption good is produced by households who operate their own production function and who sell their output on a competitive market at a price normalized to 1. The market earnings function is

$$y_m = Ah^{\lambda}\ell^{\mu} \qquad \text{Household Market Income} \qquad (12)$$

where A is an aggregate constant and $\lambda, \mu > 0$. The elasticity of income to human capital, λ will be called the "returns to human capital." This parameter plays an important role in the rest of the paper. I argue in section 5 that an increase in λ is able to rationalize the trends observed in higher education and explained in introduction.

Although simple, this functional form is also the reduced-form expression of a more sophisticated production function with physical capital and/or the payoff to a household involved in an aggregate production process with some degree of complementarity across heterogeneous tasks. Educational services are produced using

[15] Intuitively, if there were no lower bound to their size, all colleges would locate at the highest quality level and operate with virtually no students. In other words, all colleges would like to be Harvard but there is only one Harvard. To formalize this notion, I assume that colleges can't be too small and can't steal students from colleges that are already located. This forces colleges lower in the order to locate at qualities for which there are available students. In equilibrium this will imply that colleges will locate in decreasing order on the line of quality, with the first mover operating at the highest quality. More details are given A.7.

[16] One can see the positioning game with quality-maximizing colleges as the equivalent of the free-entry/non-profit condition with profit-maximizing colleges, like in Cai and Heathcote (2018). A key difference, however, is that contrary to a free-entry condition that equalizes payoff for all colleges, in a subgame perfect Nash equilibrium of the positioning game in my setup, colleges receive heterogeneous payoffs if (and only if) they offer different qualities. Again, all colleges would like to be Harvard, but there is room for only one Harvard (or maybe Princeton?).

the final good as input. Each unit of final good gives A_I/A units of educational services: $y_I = \frac{A_I}{A} y_m$. Colleges buy services from the educational sector at relative price p_I. In equilibrium, it will be the case that $p_I = A/A_I$. Given that higher education is intensive in its use of high skilled labor, it is of interest to extend the analysis to this case. This would make the price of educational services endogenous to economy-wide conditions. In particular it would imply a positive relationship between the returns to human capital, λ, and the price of education services p_I, through the increase in the relative wages of faculty. I provide a generalization along these lines in section 6.[17]

2.4 Equilibrium

An equilibrium path is a sequence of tuition schedules, prices of educational services, household's policy functions, colleges' policy functions, a sorting rule, a distribution of human capital $\{e_t(q,z,y), p_{I,t}, c_t(h,\xi_b), \ell_t(h,\xi_b), \phi_t(q,y,z), I_t(q), q_t(j), q_t(h,\xi_b), f_t(h)\}_{t=0}^{\infty}$ such that i) given the sequence of prices, the household's policy functions $c_t(h,\xi_b), \ell_t(h,\xi_b)$ are solution to (1), ii) the college's policy functions $\phi_t(q,y,z), I_t(q)$ are solution to (8), and the allocation of colleges along the quality line $q_t(j)$ is a subgame perfect Nash equilibrium of the positioning game, iii) the demand for quality q from students of type (z,y) is matched by a supply for this type at that quality, iv) the final good market as well as the educational services market clear, v) the evolution of the distribution of human capital, $f_t(h)$, is consistent with the intergenerational law of motion of human capital and the sorting rule, $q_t(h,\xi_b)$.

3 Properties of the Decentralized Equilibrium

I construct an equilibrium in which the distribution of human capital is log-normal. A necessary and sufficient condition for this distribution to remain log-normal over generations is for the tuition schedule to be a log-linear function of college quality q, student ability z and parental income y. Given the assumptions laid out in the previous section, the unique tuition schedule compatible with the equilibrium conditions and colleges being in an interior solution is log-linear. These two restrictions—log-normality

[17] By itself this generalization preserves the tractability of the model. See appendix A.3.2 for a detailed treatment. None of the key mechanisms and analytical results emphasized in this paper depends on this generalization.

of human capital and interior solutions for colleges—ensure the tractability of the equilibrium expressions.[18]

3.1 Equilibrium Tuition Schedule

Consider a college that decides to supply quality q. It then has to choose the optimal combination of inputs—educational services I_j and the distribution of students' quality that are consistent with q. Given the substitutability between educational resources and student ability, a college will trade off lower tuition for higher student ability. The first-order conditions with respect to the density over student types and to the level of spending in the college's problem reflect this trade-off. The following proposition gives the unique equilibrium tuition schedule that is compatible with all colleges being at an interior solution. It takes a log-linear form and, incidentally, implies that all colleges are indifferent between all student types.[19]

Proposition 3.1. *The equilibrium before-financial-aid tuition schedule is given by*

$$e_{u,t}(q, z) = p_I q^{\frac{1}{\omega_1}} z^{-\frac{\omega_2}{\omega_1}} \tag{13}$$

and all colleges are indifferent between all types.

Tuition fees are increasing in quality q and decreasing in student ability z with respective elasticities of $\frac{1}{\omega_1}, \frac{\omega_2}{\omega_1}$. These elasticities are intuitive. Colleges of higher quality need to finance higher expenses, hence require higher tuition. If educational services are important for the production of college quality, ω_1 is high hence $\frac{1}{\omega_1}$ is low, tuition will not be very elastic to quality, because a small increase in revenues implies a large increase in quality. The elasticity $-\frac{\omega_2}{\omega_1}$ captures the importance of the peer-effect relative to educational spending: if peers significantly matter, colleges have strong incentives to subsidize high ability students to attract them.

[18] I cannot however rule out the existence of equilibria outside of this class.

[19] Although it is natural to focus on interior solutions, I cannot rule out the existence of other equilibria where tuition fees deviate from this log-linear expression and some colleges are at corner solutions for some student types. The real world tuition schedule does display kinks and a log-linear tuition schedule should be interpreted as a smooth approximation of reality. Although looking at a more general class of equilibria is potentially interesting, it is beyond the scope of this analysis and would defeat a key purpose of this paper, as all tractability would be lost.

3.2 Household Policy Functions

Given the equilibrium tuition schedule (26), households choose where to send their offspring. Since the tuition schedule is monotonic in q, this decision amounts to choosing how much of their income to spend on higher education. Define the spending rate of a household of type (z, y) going to college of quality q:

$$s_t(q, z, y) = \frac{e_t(q, z)}{y}.$$

An attractive feature of the class of models with unitary elasticity of intergenerational substitution, log-normal innovations and log-linear technologies is the possibility to obtain analytic expressions for the optimal spending rate and labor supply.[20] The following proposition characterizes the solution to the F.O.Cs associated with the households' problem.

Proposition 3.2. *Defining $U = \frac{\partial \ln \mathscr{U}}{\partial \ln h}$, the elasticity of the value function to human capital, one has that, in equilibrium, for all households, the households' spending rate, labor supply and marginal value of human capital U are given by:*

$$s_t = \frac{\beta \alpha_2 \omega_1 U_{t+1}}{1 - \beta + \beta \alpha_2 \omega_1 U_{t+1}} \tag{14}$$

$$\ell_t = \left[\frac{\mu}{\eta} \left(1 + \frac{\beta}{1-\beta} \alpha_2 \omega_1 U_{t+1} \right) \right]^{\frac{1}{\eta}} \tag{15}$$

$$\textit{with} \quad U_t = (1-\beta) \sum_{k=0}^{\infty} \beta^k \lambda_{t+k} \prod_{m=0}^{k-1} \alpha_{h,t+m} \tag{16}$$

$$\textit{and} \quad \alpha_{h,t} = \alpha_1 + \alpha_2 \left[\omega_2 \alpha_1 + \omega_1 \lambda_t \right]$$

where $\alpha_{h,t}$ is the Intergenerational Elasticity (IGE) of human capital at generation t. The spending rate and labor supply are independent of the household type and depends positively on U_{t+1} which is also common to all households. The latter depends positively on all future α_h's, which is the IGE. The higher the future IGEs the more incentive the current generation has to invest in human capital and work. Importantly U_t—thus s_t—is also increasing in both the current and future returns to education—λ_t. It will play a key role in the dynamics of human capital afterwards.

[20]This paper draws on a long tradition that uses log preferences and lognormal distributions in dynastic frameworks to derive analytically tractable expressions, *e.g.* Glomm and Ravikumar (1992).

3.3 Equilibrium Sorting Rule

By combining the equilibrium tuition schedule and the equilibrium positioning of colleges on the quality line—the "supply side"—with the household spending rule—the "demand side"—one obtains the equilibrium sorting rule, a mapping from the set of household and student types into the set of qualities of higher education.

Proposition 3.3. *In equilibrium, the sorting rule is given by*

$$q_t(y, z) = \left(\frac{s_t y}{p_I}\right)^{\omega_1} z^{\omega_2} \tag{17}$$

Equation (17) tells us which quality of higher education a student from family background y and ability z gets. The elasticity of quality to income and ability capture the strength of what I call, respectively, the income-sorting and ability-sorting channel. The income-sorting channel captures the fact that richer households are able to buy a higher quality of college, not only because they are richer but also because colleges are ready to trade-off financial resources for ability. The ability-sorting channel captures the desire of colleges to attract high ability students because of the peer effect.

3.4 Educational Sector and Market Clearing

In the simple case considered here, the price of educational services is pinned down by the relative productivity parameter $p_I = A/A_I$. Market clearing then simply pins down the share of final good allocated to consumption and to the production of educational services.

3.5 Law of Motion of Human Capital

Having described the static equilibrium conditions, I now derive the law of motion for the distribution of human capital. Since the first two moments of this distribution are the only aggregate states, it also describes the dynamics of the aggregate economy. I start with the law of motion of human capital at the individual level.

Intergenerational Transmission of Status. Plugging the expression for the equilibrium sorting rule (17) into the law of accumulation of human capital (4) and gathering all terms in $\ln h$ gives the following intergenerational law of motion of human capital: $\ln h_{t+1} = \alpha_{h,t} \ln h_t + \ln \xi_y + (\alpha_1 + \alpha_2\omega_2) \ln \xi_b + \ln \kappa + \alpha_2 x_t$ with $\alpha_{h,t}$ the intergenerational elasticity of human capital (IGE) and $x_t = \omega_1 \ln\left(\frac{s_t A_t \ell^\mu}{p_{I,t}}\right)$.

The IGE is a linear combination of the before, during and after college transmission of human capital. This paper focuses on and opens the box of the transmission of economic status through college. The transmission during college decomposes itself into the two sub-channels introduced in the previous paragraph: the income-sorting channel that emphasizes the role of parental income and the ability-sorting channel that emphasizes the role of ability in the sorting of students across the ladder of college quality.

$$\alpha_{h,t} = \underbrace{\underbrace{\alpha_1}_{\text{Before Coll.}}}_{\text{Direct Transmission}} + \underbrace{\alpha_2 (\underbrace{\alpha_1\omega_2}_{\text{Ability-Sorting Channel}} + \underbrace{\omega_1\lambda_t}_{\text{Income-Sorting Channel}})}_{\text{College}}$$

Aggregate Law of Motion of Human Capital. Using the assumption of log-normality of both shocks, (5) and (6), if the economy starts from a log-normal distribution then human capital stays log-normally distributed along the equilibrium path:

Proposition 3.4. *If* $\ln h_t \sim \mathcal{N}\left(m_{h,t}, \Sigma^2_{h,t}\right)$ *then*

$$\ln h_{t+1} \sim \mathcal{N}\left(m_{h,t+1}, \Sigma^2_{h,t+1}\right) \tag{18}$$

$$m_{h,t+1} = \rho_t m_{h,t} + X_{1,t} \qquad \text{Mean of (log) Human Capital} \tag{19}$$

$$\Sigma^2_{h,t+1} = \alpha^2_{h,t}\Sigma^2_{h,t} + X_2 \qquad \text{Variance of (log) Human Capital} \tag{20}$$

$$\begin{aligned} \textit{where} \quad \rho_t &= \alpha_1 + \alpha_1\alpha_2\omega_2 + \alpha_2\omega_1\lambda_t \\ X_{1,t} &= -\frac{\sigma_y^2}{2} + \ln\kappa - \alpha_1(\alpha_2\omega_2 + 1)\frac{\sigma_b^2}{2} + \alpha_2\omega_1 \ln\left(A\ell_t^\mu \frac{s_t}{p_I}\right) \\ X_2 &= \sigma_y^2 + (\alpha_1[1 + \alpha_2\omega_2])^2 \sigma_b^2. \end{aligned}$$

It is intuitive that the shifter in the law of motion of the mean of the distribution (19) is increasing in the spending rate s_t, labor supply ℓ_t, but decreasing in the price of educational services p_I. The law of motion of the variance (20) is the mathematical expression of the Great Gatsby curve: the positive relationship between the level of inequality Σ_h and the strength of the intergenerational transmission of status, α_h.

The law of motion of Σ_h, given by (20), is an auto-regressive process of order 1.

The expression (19) is also auto-regressive of order 1 since the paths of s and ℓ are only functions of the parameters and all the future λ's. The full system, (19) and (20), is therefore recursive which allows us to characterize the existence and uniqueness of the equilibrium path.

Proposition 3.5 (Existence and Uniqueness of Equilibrium Path)**.** *Within the class of equilibria with an initial log-normal distribution of human capital, there exists a unique globally stable steady-state and a unique equilibrium path.*

3.6 Distribution of Students along the Quality Ladder and Within-College Distribution of Students

Recall facts (b) and (c) noted in introduction: the dispersion of expenditures per student across colleges has increased and the share of low-income students at top colleges has stagnated. One can actually derive analytical expressions for the distribution of students across college qualities (and the implied distribution of expenditures) and for the within-college distributions of parental income and student ability. These closed-form solutions enable us to shed light on the forces that determine these two objects and will prove useful for the derivation of comparative statics in the next section. These three distributions are log-normal and their first and second moments depend on the aggregate states, directly and indirectly through the income-sorting and ability-sorting elasticities,

$$\varepsilon_{I,t} = \omega_1 \lambda_t \quad \text{and} \quad \varepsilon_A = \omega_2 \alpha_1.$$

As the following proposition establishes, the dispersion of qualities is an increasing function of both of these variables. But the dispersion of parental income and ability within a college is a function of their ratio. The former is increasing with the ratio $\varepsilon_A/\varepsilon_I$ while the latter is decreasing: the more students sort into colleges based on parental income, the less economic diversity there is in a college and the more students sort into colleges based on abilities, the lower the dispersion of abilities.

Proposition 3.6. *1. The distribution of college quality is given by*

$$\ln q \sim \mathcal{N}\left(\mu_{1,t}(m_{h,t}, \Sigma_{h,t}), \sigma^2_{1,t}(\Sigma_{h,t}, \varepsilon_{I,t}, \varepsilon_A)\right)$$

with $\sigma_{1,t}$ increasing in ε_A, $\varepsilon_{I,t}$ and $\Sigma_{h,t}$.

2. *Within a college of quality q, the distribution of parents' (log) human capital is:*

$$\ln h|q \sim \mathscr{N}\left(\mu_{2,t}(m_{h,t}, \Sigma_{h,t}), \sigma^2_{2,t}(\Sigma_{h,t}, \varepsilon_{I,t}, \varepsilon_A)\right)$$

with $\mu_{2,t}$ increasing in q; $\sigma_{2,t}$ increasing in ε_A and $\Sigma_{h,t}$ and decreasing in $\varepsilon_{I,t}$.

3. *Within a college of quality q, the distribution of students' (log) abilities is:*

$$\ln z|q \sim\sim \mathscr{N}\left(\mu_{3,t}(m_{h,t}, \Sigma_{h,t}), \sigma^2_{3,t}(\Sigma_{h,t}, \varepsilon_{I,t}, \varepsilon_A)\right)$$

with $\mu_{3,t}$ increasing in q; $\sigma_{3,t}$ increasing in $\varepsilon_{I,t}$ and $\Sigma_{h,t}$ but decreasing in ε_A.

4 Taxes, Transfers and Financial Aid in Higher Education

In this section, I introduce a government which implements non-linear transfers of income across households and provides merit and need-based financial aid to students as well as subsidies to colleges. I also allow colleges to provide need-based aid by assuming they have a social objective. I use log-linear tax and transfer schedules as introduced by Persson (1983) and Benabou (2002) and estimated more recently by Heathcote, Storesletten, and Violante (2017). They fit well the empirical schedules and they preserve the tractability of the framework introduced in the previous section.

4.1 Government

The government implements four kind of taxes: two are specific to higher education (non-linear merit-based and need-based financial aid to college students and non-linear transfers to colleges) and two that are more standard (a linear consumption tax and a progressive income tax).

Progressive Income Tax Schedule The household labor income is subject to a progressive tax schedule with a_y the average tax rate and τ_y its progressivity. The after-tax and transfers lifetime earnings is given by

$$y = (1 - a_y) y_m^{1-\tau_y} T_y \qquad \text{Household After-Tax Income} \tag{21}$$

where T_y is a normalizing aggregate endogenous factor ensuring that a_y parametrizes the average income tax rate. The non-linear schedules for financial aid and the college subsidy are in the same spirit as this income tax schedule.

Merit and Need-Based Financial Aid Financial aid is allowed to be progressive with income and regressive with abilities:

$$e(q, z, y) = T_e z^{-\tau_m} y^{\tau_n} \frac{e_u(q, z, y)}{(1 + a_h)} \qquad \text{Net Tuition} \tag{22}$$

where $e(q, z, y)$ is the after financial net tuition faced by households, as specified in (3) and $e_u(q, z, y)$ is the before financial aid price, commonly referred to as the sticker price. τ_m is the rate of progressivity (or rather regressivity) of the merit-based subsidy, τ_n is the rate of progressivity of the need-based subsidy and T_e ensures that a_h is the average financial aid to students.

Transfers to Colleges Financial transfers to colleges by states and the federal government are large and highly progressive, as is documented in a companion paper Capelle (2019). Colleges that spend less per student receive relatively more subsidies.[21] This progressivity is closely related to the location of public and private colleges in the distribution of quality. Most papers modeling the higher education sector differentiate between public and private colleges. In contrast, I do not specify any *ex ante* differences across colleges.[22] In my model, the bottom and middle of the distribution of qualities,

[21] The notion of progressivity used here doesn't refer to the progressivity with respect to the average students parental income that populate these colleges. Even if students from richer families are more likely to be in high revenue colleges, it might be that overall these transfers are regressive since many children from low-income background do not enroll in college—a mechanism I abstract from in this version of the model but allow for in the quantitative version of the model presented in section 6.

[22] There is little agreement in the literature about what really differentiates public colleges' objectives and constraints from non-profit private ones, apart from the fact that the former receive public subsidies but not the latter. Most papers assume that tuition fees at public universities are subject to specific constraints. For example, Epple, Romano, and Sieg (2006) and Cai and Heathcote (2018) assume that tuition fees at public colleges are exogenous. This corresponds to the notion that tuition fees are fixed by States' legislatures. But most States have been decentralizing and deregulating tuition policies. Public colleges now have significant autonomy in their tuition and hiring policies

i.e. the colleges that receive relatively more transfers from the government, can be interpreted as public colleges. This way of modeling government transfers allows me to keep the model tractable while capturing most of the heterogeneity in government transfers along the quality distribution. Taking into account these transfers, the budget constraint of a college is:

$$p_I I = T_u(1 + a_u) \left(E_{z,y}[e_u(q, z, y)]\right)^{1-\tau_u} \qquad \text{Colleges After-Transfer Revenues} \tag{23}$$

where τ_u is the degree of progressivity of subsidies to universities and T_u ensures that a_u is the average amount of transfers per student received by colleges. The budget constraint presented in the college problem, (10), is the special case when $\tau_u = 0$. I show in Capelle (2019) that the functional form assumption in (23) is a good approximation of the data.

Government Budget Constraints. There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any period. The other three constraints pin down T_u, T_y, T_e such that a_y, a_h, a_u are respectively the average rate of income tax, financial aid and transfers to college. I give more details in appendix A.3.1.

4.2 College Need-based Aid and Social Objective

I now allow colleges to give need-based aid. There is indeed evidence that colleges do discriminate tuition fees based on parental income (Epple, Romano, and Sieg, 2006). To do so, I assume that colleges have a social objective. There is direct evidence for this social objective: the claimed and (growing) public effort of private and public universities to recruit low-income students.[23] The social objective is modeled as follows. A college's payoff is increasing in the quality of higher education, as in the previous section, and decreasing in the (geometric) average of parental incomes, $\bar{y}_j$, and this

(Mc Guinness, 2011). (And even in the States where legislatures still have a lot of power, it is not clear that their objective would be radically different than maximizing the quality delivered.) For-profit colleges do display different behavior, but they make up a very small part of total enrollment.

[23] Colleges give need-based to students in Epple, Romano, and Sieg (2006) not because of a social objective but because of parents with higher income are less elastic to prices and therefore higher mark-up. Colleges do not discriminate by parental income in Cai and Heathcote (2018).

penalty is parametrized by $\omega_3 > 0$:[24]

$$\ln \mathscr{V}_j = \ln q_j - \omega_3 \ln \bar{y}_j \tag{24}$$

where $\bar{y}_j$ is the geometric average parental income of students:

$$\ln \bar{y}_j = E_{\phi_j(.)}[\ln(y)] \qquad \text{Average Parental Income} \tag{25}$$

A college maximizes (24) subject to the technology for quality (9), the definition of the peer-effect (11), the after-subsidy budget constraint (23) and the definition of average parental income (25).

4.3 Properties of the Decentralized Equilibrium

Equilibrium Tuition Schedule. In this generalized framework, the log-linear form of the tuition schedule is preserved.

Proposition 4.1. *The equilibrium before-financial-aid tuition schedule is given by*

$$e_{u,t}(q,z,y) = \left(\frac{p_{I,t}}{(1+a_{u,t})T_{u,t}} q^{\frac{1}{\varepsilon_{1,t}}} z^{-\frac{\varepsilon_{2,t}}{\varepsilon_{1,t}}} \left(\frac{y}{\kappa_{2,t}} \right)^{\frac{\varepsilon_{3,t}}{\varepsilon_{1,t}}} \right)^{\frac{1}{1-\tau_{u,t}}} \tag{26}$$

$$\textit{where} \quad \varepsilon_{l,t} = \frac{\omega_l}{1-\nu_t(\Sigma_{h,t})\omega_3} \quad \forall\, l = 1,2,3$$

with $\nu_t(\Sigma_t)$ the elasticity of mean parental income within a college to quality

$$\bar{y}_t(q) = \kappa_{2,t} q^{\nu_t(\Sigma_{h,t})}$$

and all colleges are indifferent between all types.

There are three new elements relative to the previous section. First, tuition are increasing in parental income because of the social objective. The elasticity $\frac{\varepsilon_3}{\varepsilon_1(1-\tau_u)} = \frac{\omega_3}{\omega_1(1-\tau_u)}$, depends on the strength of the social objective: the larger ω_3, the more progressive tuition fees are.[25] Secondly, tuition decreases with the average

[24] There is no *a priori* restrictions on ω_3. But it will become clear in the next paragraphs that to rationalize the strong sorting on parental income, it cannot be too large.

[25] The equilibrium tuition function turns out to be similar to the one in Epple, Romano, and Sieg (2006). While the progressivity of tuition fees with parental income originates from market power in their framework, it comes from the social objective in this paper.

subsidies to colleges a_u, and the elasticity of tuition fees with quality is increasing in the degree of progressivity of the college subsidy schedule, τ_u.

Thirdly, the elasticities of tuition with respect to quality, student ability and parental income—$\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}$—are equilibrium objects that depend on current aggregate states, in particular the dispersion of human capital in the economy Σ_h, and the policy parameters. Mathematical expressions for ν_t and $\kappa_{2,t}$ are given in appendix A.4.1. $\kappa_{2,t}$ depends not only on current states but also on all future states through the labor supply decision ℓ_t. The notation $\nu_t(\Sigma_{h,t})$ makes explicit that the elasticity of mean parental income to quality depends on the dispersion of human capital in the economy. As I show in appendix A.9, it is increasing in the latter. It also depends on the current policy parameters and λ the returns to human capital. Note that when colleges have no social objective and only maximize quality, $\omega_3 = 0$, then $\varepsilon_l = \omega_l$ for all $l = 1, 2, 3$ and the ω's are independent of the state of the economy.

The term $\frac{1}{1-\nu_t(\Sigma_{h,t})\omega_3}$ that transforms ω_1 into ε_1 reflects the cross-subsidization from high-income to low-income families within a college implied by the social objective. Tuition fees for a family with a given income y increase with a lower elasticity with respect to quality when colleges have a social objective. This family becomes poorer and poorer relative to the within-college mean parental income as one climbs the college quality ladder (since parental income increases in equilibrium with quality). This effect is all the more pronounced as the social objective parameter ω_3 and the equilibrium elasticity of parental income to college quality ν are large.[26]

Household Policy Functions and Sorting Rule For conciseness, and because it is very similar to its expression in the previous section, the equilibrium spending rate of households is given by equation (38) in appendix A.1. Combining the household decision with the equilibrium tuition schedule gives the equilibrium sorting rule.

Proposition 4.2. *In equilibrium, the sorting rule is given by*

$$q_t = \left(\frac{s_t y_t^{1-\tau_{n,t}} z_t^{\tau_{m,t}} (1 + a_{h,t})}{T_{e,t}} \right)^{\varepsilon_{1,t}(1-\tau_{u,t})} \left(\frac{(1 + a_{u,t}) T_{u,t}}{p_{I,t}} \right)^{\varepsilon_{1,t}} z_t^{\varepsilon_{2,t}} \left(\frac{y_t}{\kappa_{2,t}} \right)^{-\varepsilon_{3,t}} \tag{27}$$

The elasticity of quality to income and ability which capture the strength of the

[26] If inequality increases for exogenous reasons—as will be the case in our comparative statics with respect to the returns to education λ—the endogenous increase in ν provides a partial mitigating force by making colleges willing to endogenously redistribute more across students, provided $\omega_3 > 0$.

income-sorting and ability-sorting channel now captures the progressivity of taxes and financial aid:

$$\frac{\varepsilon_{I,t}}{(1-\tau_y)\lambda_t} = \varepsilon_{1,t}(1-\tau_u)(1-\tau_n) - \varepsilon_{3,t} \qquad \text{Elasticity to Income}$$
$$\frac{\varepsilon_{A,t}}{\alpha_1} = \varepsilon_{2,t} + \tau_m(1-\tau_u)\varepsilon_{1,t} \qquad \text{Elasticity to Ability}$$

Relative to the framework without government intervention and a social objective for college, the income-sorting channel is tempered by government subsidies to colleges that are progressive with slope $(1-\tau_u)$, by need-based financial aid that are progressive with slope τ_n, financial aid by colleges that is progressive with slope ω_3. In theory, this elasticity, ε_I, could be negative, if the social objective parameter, ω_3, was large enough, such that $\varepsilon_{1,t}(1-\tau_u)(1-\tau_n) < \varepsilon_{3,t}$. The elasticity with respect to ability—associated with the ability-sorting channel—is amplified by the merit-based subsidy of the government, τ_m.

Intergenerational Transmission of Status. The IGE is now given by

$$\underbrace{\underbrace{\alpha_1}_{\text{Before Coll.}} + \underbrace{\alpha_3}_{\text{After Coll.}}}_{\text{Direct Transmission}} \underbrace{+\alpha_2\Big(\underbrace{\alpha_1(\varepsilon_{2,t}+\varepsilon_{1,t}(1-\tau_{u,t})\tau_{m,t})}_{\text{Ability-Sorting Channel}} + \underbrace{(\varepsilon_{1,t}(1-\tau_{u,t})(1-\tau_{n,t})-\varepsilon_{3,t})(1-\tau_{y,t})\lambda}_{\text{Income-Sorting Channel}}\Big)}_{\text{College}}$$

Aggregate Law of Motion of Human Capital. Like in the simpler version of the model, human capital remains log-normally distributed over the equilibrium path:

Proposition 4.3. *If* $\ln h_t \sim \mathcal{N}\left(m_{h,t}, \Sigma^2_{h,t}\right)$ *then*

$$\ln h_{t+1} \sim \mathcal{N}\left(m_{h,t+1}, \Sigma^2_{h,t+1}\right) \tag{28}$$
$$m_{h,t+1} = \rho_t m_{h,t} + X_1\Big(m_{h,t}, \{\Sigma_h\}_{s=t}^{\infty}\Big) \qquad \text{Mean of (log) Human Capital} \tag{29}$$
$$\Sigma^2_{h,t+1} = (\alpha_{h,t}(\Sigma_{h,t}))^2\,\Sigma^2_{h,t} + X_{2,t}(\Sigma_{h,t}) \qquad \text{Variance of (log) Human Capital} \tag{30}$$

where ρ_t *has the same expression as in the previous section. Expressions for* $X_1\Big(m_{h,t}, \{\Sigma_h\}_{s=t}^{\infty}\Big)$ *and* $X_{2,t}(\Sigma_{h,t})$ *can be found in appendix A.5.*

In general the expression (29) is not a linear recursive formulation for the law of motion of m_h because s and ℓ are forward looking variables that depend on all the future Σ_h's via the ε's. In contrast, the law of motion of Σ_h, given by (30), is still recursive—although in general not linear since both the autoregressive coefficient and the shifter depend on Σ_h. The full system, (29) and (30), is therefore block-recursive which allows us to characterize the existence and uniqueness of the equilibrium path after the exposition of the government budget constraints.

Government Budget, Educational Sector and Market Clearing The aggregate government budget constraint (42) imposes, in all periods, a restriction on the path of the consumption tax rate $a_{c,t}$ given an exogenous path of income tax $a_{y,t}$, higher education subsidies $a_{h,t}, a_{u,t}$ and endogenous spending rate s_t. Analytical expressions for this constraint as well as for equations (43),(44) and (45) defining respectively $T_{y,t}, T_{e,t}$ and $T_{u,t}$ are derived in appendix A.3.1.

Existence and Uniqueness of the Equilibrium Path Existence and uniqueness of the steady-state and of the macroeconomic equilibrium path are slightly harder to obtain than in the previous section. Although existence and local stability is obtained under an intuitive sufficient condition, a sufficient condition for global stability is that ω_3 be small enough.

Proposition 4.4. *Existence and Uniqueness of Equilibrium Path*

- *If* $\lim_{\Sigma_h \to \infty} \alpha_h(\Sigma_h) < 1$*, there exists at least one locally stable steady-state.*
- *For ω_3 small enough, there exists a unique globally stable steady-state and a unique equilibrium path.*

with $\lim_{\Sigma_h^2 \to \infty} \alpha_h(\Sigma_h) = \alpha_1 + \alpha_1\alpha_2(\omega_2 + \tau_m(1-\tau_u)\omega_1) + \alpha_2[\omega_1(1-\tau_u)(1-\tau_n) - \omega_3](1-\tau_y)\lambda$

A high ω_3 might lead to multiple equilibria by making inequality Σ_h potentially grow too fast in some parts of the state-space, *i.e.* by making the derivative of the right-hand-side of (30) higher than 1, thus failing to meet the crucial defining feature of a contraction mapping. This stems from the fact that ν is increasing in Σ_h, hence that ε_l for $l = 1, 2, 3$, α_h and X_2 are increasing in Σ_h.

5 Rationalizing Trends in Higher Education

This section derives what I consider to be the most important analytical result of the paper. The increase in the market returns to education λ is able to generate (a) the increase in income inequality (Piketty and Saez, 2003; Autor, Katz, and Kearney, 2008); (b) the increase in the dispersion of expenditures per students across colleges (Capelle, 2019); (c) the stagnation of the share of students from the lowest income quintile in top colleges despite the increase in financial aid (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2019); (d) the increase in real terms of tuition fees before and after financial aid; and (e) the slight increase in the intergenerational elasticity of income mobility (Davis and Mazumder, 2017). It is natural to focus on the increase in the returns to education as it is widely recognized to be one of the main sources of the increase in inequality (Katz and Murphy, 1992; Autor, Katz, and Kearney, 2008).[27] I also show that a decline in the progressivity of government subsidies to colleges—an observed feature of the data—is able to rationalize the same facts. However, the quantitative analysis in section 6 suggests that this change would fail to match the large increase in income inequality observed in the data.

5.1 An Increase in the Returns to Human Capital

The following proposition formally states the key comparative static result.

Proposition 5.1. *Assume the economy starts from a steady-state at $t = 0$. Consider a weakly increasing sequence $\{\lambda_t\}_0^{+\infty}$. If $\omega_1(1-\tau_n)(1-\tau_u) > \omega_3$, along the equilibrium path,*

a) The Gini coefficient of human capital and income increase.

b) The Gini coefficient of colleges' (log) expenditures per student (and quality) increase.

c) The ratio of variance of (log) income within a college over variance of (log) income in economy decreases.

d) The intergenerational elasticity increases.

e) The average expenditure for college as a share of income increases.

The formal proof of this proposition is contained in appendix A.9. Here I present intuition for the stated effects.

[27] I do not take a stand on the exact source of increase in the returns to human capital. Many factors have contributed to this rise: a skill-biased technological change, capital-skill complementarity, an improvement in the assortative matching of workers and firms, an increase in assortative mating and in the number of single households and an increase in the substitutability across skills due to international trade or due to better communication technology.

Intuitively, when the returns to human capital, λ, increase, the dispersion of households' income rises for a given distribution of human capital [fact (a)]. Given that households all spend the same share of their income for the higher education of their child, it implies an increase in the dispersion of desires to pay for college. Following this change on the demand side of the higher education market, colleges react: top colleges take advantage of the rising willingness to pay of their pool of students by increasing their fees and their spending relative to colleges at the bottom. Inequality of revenues and spending across colleges rise [fact (b)].

Poor but high ability students get priced out of top colleges for two reasons. First tuition fees at top colleges have increased relative to lower ranked colleges. Second their parents' income have decreased relative to the average parental income. More generally this rise in the dispersion of tuition for colleges implies that parental income matters even more to access a higher quality college than it used to, relative to ability. It corresponds to an increase in the elasticity of college quality to income, ε_I, what I described earlier as a strengthening of the income-sorting channel.

Consequently, top colleges become less diverse in terms of economic background because poor students are priced out and students from rich families are able buy their way to the top. More generally, colleges become more segregated and homogeneous in terms of parental income [fact (c)]. Another implication is that intergenerational mobility decreases, as parental income becomes increasingly determinant for the opportunities of children [fact (e)]. This is a direct manifestation of the Great Gatsby curve (Corak, 2013), whereby an increase in income inequality leads to a strengthening of the transmission of economic status, here through access to better quality higher education, which feeds back into higher inequality.

Overtime, the initial increase in inequality gets amplified through the higher education system. Students from richer backgrounds get relatively higher quality of higher education, which increases the dispersion of human capital and therefore of income once their generation become adults. The shock propagates over generations as this increased dispersion of human capital translates into a higher dispersion of children abilities which gets amplified by the increasingly unequal distribution of college quality.[28]

The amplification of the initial increase in the returns to human capital, λ, through

[28] The quantitative section provides a structural decomposition of the increase in the Gini coefficient and in the IGE into the direct impact and the amplification through the higher education system.

colleges happens through two channels: the reallocation of resources and the reallocation of students. As I have argued above, financial resources and expenditures become increasingly concentrated at the top of the college distribution. In contrast, high ability students become slightly less concentrated at the top of the college ladder, partially mitigating the amplification.

Why do colleges accommodate the increased dispersion in desires to pay for colleges? They are led to do so by their desire to maximize the quality they provide, despite their social objective. Even if an individual college at the top of the distribution didn't raise its tuition fees relative to, say, the median college, another college would fill up this gap, offering higher quality for higher tuition fees. This mechanism is akin to the revenue theory of cost by Bowen (1980), but now applied to a hierarchy of colleges.[29]

Finally, average tuition fees and the share of total income devoted to higher education increase because higher returns to human capital gives stronger incentives to households to accumulate human capital which drives their demand for higher education up [fact (d)]. It is therefore the same demand-driven mechanism that drives both the average increase in tuition and the rise in inequality across colleges.

5.2 A Decrease in Public Transfers Progressivity

In a companion paper Capelle (2019), I have documented the large decline in the average rate of government subsidy to colleges, a_u, as well as in its progressivity, τ_u.[30] In this section, I explain intuitively why a weakly decreasing sequence $\{\tau_{u,t}\}_0^\infty$ has the same qualitative effects as an increasing sequence $\{\lambda_t\}_0^\infty$ as stated in proposition 5.1.[31]

For a given distribution of tuition fees across colleges, a decrease in the progressivity of public subsidies, τ_u, leads to an increase in the dispersion of financial resources and therefore of quality across colleges [fact (b)]. Mechanically, because of this decline in the progressivity of public subsidies, the college quality ladder becomes steeper

[29] Bowen summarizes his theory page 19:

> 1) The dominant goals of institutions are educational excellence, prestige, and influence. 2) In quest of excellence, prestige, and influence, there is virtually no limit to the amount of money an institution could spend for seemingly fruitful educational ends. 3) Each institution raises all the money it can. 4) Each institution spends all it raises. 5) The cumulative effect of the preceding four laws is toward ever increasing expenditure.

[30] Government subsidies in this paper refer to appropriations, grants and contracts from the federal, state and local governments. It excludes Pell Grants.

[31] The formal proposition and proof are given in appendix A.9.

and the sensitivity of the quality of higher education received by a student to the income of their parents increases. Moreover, the decline in τ_u implies that, from the point of view of colleges, the marginal productivity of tuition fees in terms of quality has increased relative to students ability. This gives incentives to colleges to target students with a higher desire to pay, at the expenses of high ability students. Overall, this implies a strengthening of the income-sorting channel.[32]

Like in the case of an increase in λ, top colleges become less diverse in terms of family background and colleges become more segregated by parental income [fact (c)]. Intergenerational mobility decreases, as parental income become increasingly determinant for the opportunities of children [fact (d)]. The initial change in the allocation of students across colleges gets amplified over generations: because they experienced a more unequal distribution of higher education, the next generation of households is more unequal in terms of human capital and therefore income. This translates into higher inequality of abilities of their children, and so on and so forth... The economy gradually shifts rightwards (higher inequality, lower mobility) along the Great Gatsby curve.

Finally, the decrease in τ_u incentives households to invest more in higher education, since the elasticity of quality to tuition has risen, which leads to an increase in the average spending rate s and in average tuition.

6 The Role of Higher Education: Quantitative Results

In the previous sections I developed a tractable model of human capital accumulation with a ladder of colleges which allows for a sharp analytical characterization of the equilibrium sorting of students across colleges, income inequality, intergenerational mobility and aggregate production. I now relax some of the assumptions and extend the model to a richer quantitative environment. I then explain how I estimate this richer model. I finally use the calibrated model to assess the quantitative relevance of the higher education system in shaping inequality, intergenerational mobility and the efficiency of the accumulation of human capital. I first present a set of policy experiments that shed light on different aspects of the system and/or are of a specific

[32]Formally, it is easy to see that it also strengthens the ability-sorting channel—by increasing the impact of merit-based subsidy.

political or historical interest. Second I assess the quantitative effects of a rise in the return to human capital and decompose the rise in inequality into a direct effect and the endogenous amplification through the higher education system.

6.1 Quantitative Extension

I extend the model in four dimensions. First the restrictions on intergenerational financial transfers are partially relaxed: negative transfers up to a limit are allowed (student debt not offset by parental transfers) as well as positive transfers (bequests):

$$\mathscr{U}(h,z,a) = \max_{c,\ell,q,a'} \left\{(1-\beta)\left[\ln c - \ell^{\eta}\right] + \beta E\left(\mathscr{U}(h',z',a')\right)\right\} \tag{31}$$

$$y + e^{rH} a = c(1+a_c) + e(q,y,z) + a' \tag{32}$$

$$a' \geq \underline{a} \tag{33}$$

where e^{rH}, r, H denote respectively the "generational" rate of return, the annual rate of interest and generation length and $\underline{a}$ is the exogenous borrowing limit.

Households therefore face a portfolio problem: they have to decide upon the optimal combination of bequest and higher education for their offspring. High ability children from a poor background will take up loans and rich families with low ability children will choose to transmit financial wealth instead of buying a high quality college for their kid. Overall, allowing for financial transfers should weaken the link between parental income and the child's position on the college ladder.

Secondly, there is an outside option delivering $\underline{q}$ for free:

$$e(\underline{q},y,z) = 0 \quad \forall\,(z,y) \tag{34}$$

Some individuals will find it optimal not to go to college and take up the free outside option. This gives rise to a meaningful enrollment decision that was absent from the previous framework where all individuals got at least some arbitrarily low quality of higher education. A direct implication of equation (34) is that, if $\underline{q} > 0$, in equilibrium no individual ever chooses $q < \underline{q}$ and there is a Dirac peak at $\underline{q}$. It is natural to define the enrollment rate as the share of individuals with $q > \underline{q}$.

Thirdly, I allow higher education to be intensive in its use of high skilled labor. More specifically, the production function of educational services is given by $y_I = A_I h^{\bar{\lambda}} \ell^{\mu}$

with $\bar{\lambda} \geq \lambda$, so that high human capital individuals have a comparative advantage in the educational sector. This makes the price of educational services endogenous to economy-wide conditions. In particular it would imply a positive relationship between the returns to human capital, λ, and the price of education services p_I, through the increase in the relative wages of faculty. I provide a detailed treatment in appendix A.3.2.

Finally, I allow for some degree of noise in the matching between students and colleges. More specifically, I assume that colleges can only observe a noisy signal of students ability, $\ln s(z)|z \sim \mathcal{N}\left(\ln z - \sigma_n^2/2, \sigma_n^2\right)$. While ability and parental income play a major role in the sorting process, there is a fair amount of heterogeneity of ability within a college conditional on parental income.

The set of technological constraints faced by the household is otherwise similar to the original problem described in section 4. Formally, the problem of the household consists in maximizing (31) subject to (12), (21), (2)-(6) and the new constraints (32)-(34). The rest of the model remains the same.[33] The original problem is the special case when $a' = \underline{q} = 0$, $\lambda = \bar{\lambda}$ and $\sigma_n = 0$.

In this version of the model, the households policy functions and the distributions lose their closed-form expressions. This is an implication of either the outside option, by introducing a lower bound on the distribution of college qualities, or the intergenerational financial transfer, by making the share of tuition in household expenditure a general function of parental income and child ability compared. The noisy matching and the higher education sector being intensive in high skilled labor do not affect the tractability of the framework.

6.2 Data and Calibration

The core dataset is the restricted-use version of the NLSY-1997, a representative panel of individuals who were 12 to 17 years-old in 1997, whom I follow every year up to now. It features data on parental income, abilities measured by a common comprehensive test-score, the Armed Services Vocational Aptitude Battery or ASVAB, a detailed description of their journey through the higher education system—each college they attended, the time spent and the degree obtained—and their labor earnings.

[33] I explain how the problem of the colleges is kept tractable in this more complicated framework in appendix B.1.

To estimate the parameters related to financial aid, I use the restricted-use NCES-NPSAS in 2000, which is the closest survey to the average year when individuals in the NLSY go to college. It is a representative survey of students that features detailed information about parental income, out-of-pocket college costs and financial aid disaggregated by source—federal government, state, private and institutional.

The publicly available NCES-IPEDS annual surveys provide college-level information on expenditures, revenues, enrollment and the distribution of test scores within each college. I use the 2000 to 2004 surveys. Finally I complement these data with statistics on enrollments from the NCES and measures of aggregate spending for higher education from the OECD.

External Calibration The full list of the nineteen parameters that need to be calibrated is given in the first column of table 1. Seven of them I set without solving the model while the remaining twelve are calibrated solving it. I provide here a quick overview of the procedure for the former. See appendix B.2 provides more details.

The income tax schedule parameters a_y, τ_y are informed by the average income tax rate and the slope of the income tax schedule estimated by Heathcote, Storesletten, and Violante (2017). In a companion paper, I estimate the average per-student state transfers to college, a_u, and the degree of progressivity of these transfers τ_u (Capelle, 2019). I use the average financial aid received by students to calibrate a_h.

I use estimates of the Frisch elasticity of labor supply from the literature to calibrate η (Chetty, Guren, Manoli, and Weber, 2011). The elasticity of substitution in the educational services sector is set to $\bar{\lambda} = \lambda$ so that the price of educational services is given by $p_I = \frac{1}{A_I}$. The generation length is set to $H = 30$ years. The lower limit, $\underline{a}$ is set to match the official borrowing limit for student loan.

Internal Calibration The algorithm used to estimate the parameters is akin to a Simulated Method of Moments. The results are reported in table 1.

I now make a heuristic identification argument that justifies the choice of moments used in the estimation. Although no parameter can be identified out of a single moment, I stress in this section which moment is important for each parameter. Thanks to the closed-form expressions of these moments in terms of structural parameters in the model analyzed in sections 4 (model "M1" hereafter), it is possible to formalize this argument, which I do in appendix B.3.[34]

Assume for a moment that one perfectly observes child ability and college quality $\{z_i, q_i\}_i$. I first estimate the financial aid schedule (22), and use the elasticity of government financial aid to parental income and to students ability to inform the slopes of the financial aid schedule, which pins down τ_n and τ_m. I estimate the tuition schedule by running a regression of before-government-aid tuition fees on a college fixed-effect, ability and parental income. I use the elasticity with respect to parental income to inform the social objective parameter ω_3. I then estimate the sorting rule by running a regression of college quality on students ability and parental income. The elasticity of college quality to students ability has a first-order effect on the peer-effect parameter ω_2.

I then estimate the human capital accumulation function (4), and the market earnings function (12), to recover $\alpha_1, \alpha_2, \alpha_3$ and the returns to human capital λ. I thus run a regression of (log) child earning on (log) ability, (log) college quality and (log) parental income. The elasticity of a child's income to their ability identifies λ. Conditional on λ, the elasticity of child's income to college quality (resp. parental income) identifies α_2 (resp. α_3). Similarly, the elasticity of child's income to parental income identifies α_3. Finally, I use the IGE of income to inform α_1.

I have assumed so far that child ability and college quality were observable. However, they are not. I need to construct them by combining observable data and restrictions implied by the model. I first explain how I construct student ability. All children in the NLSY take the same test in high school. I assume that the resulting test scores are ranked in the same order as ability, z. Conditional on a (α_1, λ) it is possible to show that the correlation between the rank of the test scores and parental income $\text{corr}(\text{rank}(z), y)$ identifies the variance of the birth shock, σ_b^2. I then generate model-consistent abilities that have the following properties (i) they preserve the ranking of test scores, (ii) they are compatible with the distributional assumption for the birth shock (5) where σ_b^2 has just been estimated and (iii) they are compatible with the functional form for the transmission process (2). The construction of college quality is more direct. I use the information about which college each child has attended, average

[34] Another advantage of the closed-form expressions in M1 is the ability to investigate the invertibility of the model, given a set of targeted moments. It is possible to show that if abilities are directly observable and that there is no social objective $\omega_3 = 0$, then the parameters are exactly identified. Although child's ability z are non-observable, it helps build confidence in the identification of the parameters in my procedure. The proposition and more details are provided in appendix B.4.

test scores and educational spending in each college and the assumed production function for quality (9) to construct the model-consistent variable q_i.

The Gini coefficient of income is used to inform the variance of labor market shocks, σ_y^2.[35] The intergenerational rate of preference, β, is strongly related to the share of private spending in higher education in GDP. The outside option to college, $\underline{q}$, is directly related to the enrollment rate, the lower $\underline{q}$, the stronger the incentives to go to college. Recall that my model takes r as exogenous. Changing r has an impact on the incentives to accumulate the financial asset and, in steady-state, on the mass of households close to the borrowing constraint. r has consequently a first order effect on the elasticity of college quality to parental income.

[35] I assume that the economy is at steady-state in the 2000s. More details in appendix .

Table 1: Parameters and Moments

Parameter	Description	Value	Target/Source	Moments Data	Moments Model
η	(Inv.) elast. labor	2	Chetty, Guren, Manoli, and Weber (2011), Own Comput.		
τ_y	Income Tax Slope	.23	Heathcote, Storesletten, and Violante (2017), Own Comput.		
λ	Return to human capital	.67	Own Comput.[1]		
a_u	Av. Transfer to College	.4	Av. Transfer to College	.4	.4
a_y	Av. Income Tax Rate	.2	Av. Income Tax Rate	.2	.2
a_h	Av. Financial Aid	.2	Av. Financial Aid	.1	.1
τ_u	Elas. Transfers to Coll.	.35	Elas. Transfers to Coll. (Capelle, 2019)	.35	.35
$\underline{a}$	Borrowing Limit	.03	Borrowing Limit	.03	.03
$\bar{a}$	Bequest Limit	$+\infty$	Bequest Limit	$+\infty$	$+\infty$
τ_n	Elas. Gov. Fin. Aid to y	.195	Elas. Gov. Fin. Aid to y_m	.195	.195
τ_m	Elas. Gov. Financial Aid to z	.07	Elas. Gov. Financial Aid to z	.07	.07
ω_3	Social Obj. Param. of Coll.	0	Elas. Tuition to y	.13	.13
ω_2	Elas. q to Average Ability	.84	Elas. q to z in sorting rule	.94	.96
ω_1	Elas. q to I	1	Normalization	-	-
σ_b^2	Var. birth shock	6.6	$\rho(y_{m,i}, rank(z_i))$	.43	.43
α_1	Elas. h' to z	.21	InterGen. Elas. (Mazumder, 2015)	.5	.5
α_2	Elas. h' to q	.2	Elas. y'_m to q	.13	.13
α_3	Elas. h' to h	.2	Elas. y'_m to y_m	.2	.2
σ_y^2	Var. Lab. Mkt. shock	.74	Income Gini Coef. (Kopczuk, Saez, and Song, 2010)	.45	.45
β	Intergen. Preference	.27	% Priv. Spend. High. Ed. in GDP (OECD)	1.3%	1.3%
$\underline{q}$	Outside Option	.0278	Enrollment Rate (NCES)	70%	70%
r	Interest Rate	3.5%	Elas. q to y in sorting rule	.2	.21

[1] See appendix B for more details.

6.3 Policy Experiments in Higher Education

In table 2, I gather the results of the six policy experiments discussed below. I provide the percentage change from the status quo steady-state to counterfactual steady-state of the Gini coefficient of labor earnings, expenditures per students, the intergenerational elasticity, GDP and a measure of welfare. The social welfare function is a generalized mean of households values with constant elasticity of substitution across households, σ, in the range $[.2, 1]$.[36]

Total contribution of higher education. What is the effect of higher education on income inequality, intergenerational persistence and GDP? To address this question, the first policy experiment consists in randomly allocating students across colleges. As a result, spending per student and average student ability are equalized across all colleges and every student receives the same higher education. The common college quality, $\bar{q}$, is given by the production function of quality (9), the average children ability in society and average government transfers per student.[37] Because households optimal spending rate for higher education drops to zero, all the resources spent in the higher education system have to be financed through taxes and transfers to colleges. One therefore needs to take a stand on the level of government subsidies. I assume that there are such that the share of GDP going to higher education remains the same as in the *status quo* allocation.[38] I find that doing so would reduce the income Gini by 8.5% and the IGE by 24.3% (see line no. 1 in table 2). To get a sense of the magnitude, a reduction by 8.5% of the income Gini corresponds to a reduction of 4 p.p., which is half of the total increase since 1980. It is therefore a sizable effect. GDP however drops by 7% because of the increase in the misallocation of students and resources across colleges.

[36]Recall that the cases $\sigma \to 0$, $\sigma = 1$ correspond to a Rawlsian and a utilitarian social welfare functions, respectively and the case $\sigma \to +\infty$ to a social welfare function that is a monotonic transformation of GDP. In the context of my model, with missing insurance markets for birth and labor market shocks, a concern for equity also captures a concern for insurance against these shocks.

[37]The random allocation of students not only equalizes college experiences among college-goers but implies that everyone goes to college. It thus neutralizes both the extensive (going or not) and the intensive (quality) margin. More details in appendix C.

[38]The choice for the level of subsidies does not influence inequality or mobility, but it does have a first order effect on the aggregate level of production. This assumption allows to focus on the effect of misallocation on aggregate production. In practice it means that the level of government subsidies should increase to offset the decline in private spending for higher education.

Contribution of peer-effects. With the second policy experiment I am interested in isolating the contribution of the peer-effect. To do so, I equalize financial resources across all colleges. The most direct way to implement such a distribution of educational services is to redistribute and equalize resources across all colleges ($\tau_u = 1$). But one could also obtain the same allocation with a very progressive need-based financial aid schedule ($\tau_n = 1$). Incidentally such policies neutralize the effect that parental income has on the sorting of students across colleges conditional on child ability. The distribution of student abilities across colleges changes in the counterfactual. In particular, colleges become more homogeneous in terms of ability.[39] Like in the previous policy experiment, I assume that government policies exactly offset the decrease in average private spending, so that the aggregate spending rate in higher education remains constant in the two steady-states. I find that in the counterfactual allocation, the income Gini is reduced by 2.5% (or 1.2 p.p.) and the IGE by 14.7% (line no 2). On the one hand the mismatch of student abilities is reduced with the elimination of the income-sorting channel, but the equilibrium equalization of spending across colleges leads to a less efficient accumulation of human capital. Overall, GDP falls by 1.8%.

Institutional need-based aid. The third experiment requires colleges to propose more progressive institutional need-based aid. It is an interesting experiment because it is widely discussed in policy circles and, increasingly, colleges advertize their need-based financial aid program. To implement such aid, I raise the social objective parameter of colleges, ω_3, to the point where $\omega_3 = \omega_1(1 - \tau_n)(1 - \tau_u)$ so that all households pay the same share of their income to get into a given college conditional on ability. Recall that in M1, it perfectly shuts down the income-sorting channel. But in contrast with the previous policy experiment, it doesn't lead to an equalization of resources and spending across colleges. The segregation of colleges by abilities and the positive correlation between abilities and parental income leads to a positive sorting

[39] One could therefore argue that this counterfactual isn't capturing only the effect of equalizing resources across colleges. For example, Chetty, Friedman, Saez, Turner, and Yagan (2019) do a counterfactual exercise in which they reallocate students across colleges so that the distribution of abilities remain unchanged but so that conditional on ability, the allocation becomes independent of parental income. Like in our counterfactual, they neutralize the role of parental income on the allocation of students across colleges. However such a counterfactual is not compatible with a general equilibrium allocation of students and resources. Reallocating students entails a reallocation of financial resources, and therefore of value-added. The advantage of the counterfactual I propose is its implementability and compatibility with realistic policy tools and a decentralized equilibrium.

of resources across colleges. Colleges with higher ability students have more resources and spend more. This policy has, maybe surprisingly, a positive effect on income inequality (+3%) because it improves the matching of students, like the previous policy, but increases (by 13%) rather than reduces the dispersion of expenditures across colleges. For the same reason, the IGE falls by less than in the previous experiment (-6.4%). The perfect positive assortative matching of students and the positive sorting of resources lead to a large increase in GDP (+22%). As a result, the welfare gains of this policy are very large, around 8%, irrespective of the strength of the concern for equity, σ.

Laissez-faire. The fourth policy experiment consists in eliminating all current government interventions in higher education. It aims at quantifying the extent to which the current government interventions affect inequality and mobility. Formally, a *laissez-faire* equilibrium corresponds to the case $\tau_u = \tau_n = \tau_m = a_u = a_h = 0$. I find that doing so leads to an increase in the income Gini by 2%, an increase in the IGE by 12% and an increase in the Gini coefficient of college expenditures by 70% (line no. 4a). Across all measures, two-third of these changes are due to the transfers to colleges (line no. 4b) and one-third to need-based financial aid (line no. 4c). Merit-based aid plays virtually no role (line no. 4d). While most of the policy debates have been, in recent decades, centered around the issue of federal financial aid and income tax credit and as transfers to colleges have been significantly cut, these results highlights the importance, in the current system, of transfers to colleges.

College for all. The fifth policy experiment evaluates a (conservative) version of the recent proposal by democratic candidates to make college free for all. Although not fully specified as of now, the plan envisions (i) setting tuition to a minimum fee at public institutions (ii) offsetting the implied revenue losses with federal and state subsidies to colleges.[40] The proposal states that on aggregate the loss of tuition will be offset by subsidies to colleges, but it doesn't specify how much redistribution of resources across colleges shall occur. The most progressive option, where resources are fully equalized across colleges, corresponds to the second counterfactual (line no. 2) derived earlier. In contrast, I present here the most conservative option, where government transfers exactly offset the loss of tuition revenues at the college

[40]See here for a detailed version of the Act.

level. Formally, I assume (i) $\tau_n = 1$, (ii) a_h and a_u are such that the share of GDP going to higher education remains the same as in the benchmark allocation—as in counterfactual 1 and 2—and (iii) τ_m is such that the Gini of expenditures per students remain constant. The equilibrium allocation therefore features perfect stratification of colleges by student ability and unchanged sorting of expenditures per students across colleges. Such a policy would lead to an increase in the income Gini by 2.4% and an decrease in the IGE by 7.4% (line no. 5). GDP would increase by 2.8% thanks to the improvement in the matching of students.

Decline in public transfers since 1980. The sixth policy experiment sets the level and progressivity of government transfers to colleges to what they were in 1980. It aims at quantifying the impact on inequality and mobility of the sharp decline in both the average and progressivity of government transfers to colleges over the past forty years, documented in Capelle (2019). I have shown in section 5 that qualitatively such policy changes have very likely contributed to the trends (a)-(e). Quantitatively, I find that setting the parameters of the subsidies to colleges schedule to what they used to be in 1980 implies a decrease in the income Gini by .6% and a decrease in the IGE by 3.4% (line no. 5). The decline in public transfers to colleges can quantitatively account for a very small share (2.7%) of the total increase in the income Gini but a large share of the total increase in the Gini of expenditures per student (90%). Although qualitatively consistent with fact (a)-(e), the latter finding makes the decline in public transfers a less compelling explanation than the increase in the returns to education, to which I now turn.

6.4 Increase in the Returns to Education and Propagation through Higher Education

I now quantify the extent to which a reasonably parametrized increase in the returns to education, λ, can explain the stylized facts (a)-(e) presented in the introduction. Let's denote λ_{1980} and λ_{2010} the value of the returns to education in the original steady-state (1980) and in the final steady-state (2010). The value in 2010, λ_{2010}, is the one estimated in the previous section. In order to calibrate the value in the old steady-state, λ_{1980}, I target the change in the college premium across the two periods, keeping all

Table 2: Policy Counterfactuals

	Policy	Gini Earnings	Gini Exp./Stud.	Intergen. Elas.	GDP	Welfare $\sigma = [.2, 1]$
		% Change from Status quo				
1	Random Admission	-8.5	-100	-24.3	-7.1	[-9.6,-0.8]
2	Equal Resources	-2.5	-100	-14.7	-1.8	[4.1,0.5]
3	Progressive Aid by College	3.0	13	-6.4	22.0	[8.3,7.9]
4a	Laissez-faire	2.0	70.3	12	2.9	[-6.1,-1.9]
4b	No Transfer to College	1.5	48.2	7.9	.6	[-2.5,0.2]
4c	No Need-based Aid	.5	18.2	3.0	2.4	[-2.7,-1.2]
4d	No Merit-based Aid	-.02	-3.1	-0.03	.2	[0.0,2.1]
5	College for All (conservative)	2.4	0	-7.4	2.8	[-2.2,0.8]
6	Transfers to College, 1980	-.6	-21.6	-3.4	-3.0	[-1.0,-1.1]
	Status quo Levels	*.45*	*.22*	*.4*	-	-

other parameters constant.[41] Comparing the two steady-states corresponding to the two values, $\lambda_{1980} < \lambda_{2010}$, I find that the model generates an increase in the income Gini coefficient by 13 p.p., which corresponds to 130% of the empirical change, an increase in the expenditure per student Gini by 5 p.p. corresponding to 100% of the empirical change and an increase in the IGE by 6%.

I then isolate the contribution of the higher education system to the increase in income inequality. I compute the Gini coefficient of a counterfactual distribution of labor incomes where (i) the underlying distribution of human capital is the one in the 1980 steady-state, (ii) the returns to human capital parameter, λ, is set at its 2010 level, λ_{2010}, and (iii) the labor supply policy function is the one in the 2010-steady-state. This gives the level of inequality if there were no propagation through higher education. I provide more details in appendix C. I find that the higher education sector accounts for 6% of the seven percentage points increase generated by the model, which corresponds to a little bit more than half a percentage point increase in the Gini coefficient of income.[42]

[41] As I discuss in the previous section, the model overestimates the level of the college premium because of the fat tail of income in the data. It is therefore natural to target the relative increase in the college premium rather than its level in 1980. I use the value provided by Autor, Katz, and Kearney (2008) for the college premium: $\Delta \log(\frac{w_{college}}{w_{HS}}) = .65 - .45 = .2$, see figure 2 in their paper.

[42] In M1, the version of the model without intergenerational transfers and without outside option to colleges, the endogenous amplification through the higher education is even lower, 3% of the total increase in inequality. The additional amplification in the quantitative version of the model comes from the extensive margin: the fact that more students get enrolled into college when the returns to

I now decompose the propagation through the higher education system into two channels: the reallocation of resources and the reallocation of students quality across clubs. Formally, I compute a counterfactual steady-state in which (i) the tuition schedule and the expenditures per student by college rank is fixed at what they were in the initial steady-state, in 1980, (ii) the returns to education parameter λ is set at its 2010 value. Notice that this counterfactual is not compatible with a decentralized equilibrium, because colleges are no longer on their F.O.C. and their budget constraint don't hold, so that some have deficits and some have surpluses. See appendix C for more details. I find that more than 100% of the total effect stems from the increased dispersion of financial resources. The reallocation of students across the quality ladder of colleges dampens very slightly the effect.[43]

Intuitively, in the counterfactual where expenditures have been fixed, top colleges display surpluses, because the willingness to pay of their equilibrium pool of students has increased with the rise of inequality pushing tuition fees up, while bottom colleges have deficits. The dispersion of average student ability however declines as rich but not so smart children manage to buy their way to top colleges while smart but poor children are being priced-out. From the counterfactual allocation to the final steady-state in the year 2010, expenditures per students adjust according to revenues and tuition fees, thus keeping up with the willingness to spend of their respective pool of families, which increases revenues and thus quality at the top and decreases it at the bottom. In the counterfactual, the Gini coefficient of income barely moves and if anything slightly declines. This allows us to conclude that it is really the increase in the dispersion of revenues and therefore expenditures per students across colleges that is the root of the increase in the amplification by colleges. One natural test of this mechanism is to look at the evolution of the dispersion of tuition fees. Davies and Zarifa (2012) indeed find that the Gini coefficient for tuition fees has increased over the period 1971-2006.

human capital increases reinforces the feedback mechanism of higher education on inequality. That the extensive margin plays an amplifying role is not a qualitative feature of the model but depends on the initial enrollment rate.

[43]This dampening effect is stronger in the short-run. In the long-run, low ability kids from rich family "catch up."

7 Conclusion

This paper studies the extent to which the higher education system shapes economic inequality and intergenerational mobility in the U.S., how it has propagated and amplified macroeconomic shocks such as an increase in the returns to human capital and how government policies may affect these responses. An increase in the market returns to human capital increase inequalities directly but also indirectly over time through a more unequal sorting of resources across colleges and a (more unequal) accumulation of human capital.

The paper has the following policy implications. First, two seemingly very different policy proposals—"College for All" and incentivizing colleges to adopt more progressive need-based aid—lead to similar implications for inequality and intergenerational mobility. The former leads to a much higher GDP in the long-run, however, and may therefore be preferable. Second, both policy proposals have the unintended consequence of increasing income inequality in the long-run. If the goal of the reform is also to decrease income inequality, policy-makers should make sure that it is accompanied with some degree of equalization of financial resources across colleges.

The analysis relied on a number of simplifying assumptions. For a better understanding of the role of higher education in shaping inequality and intergenerational mobility, I see three critical areas of investigation for future research: (i) the allocation of students across colleges in the model works through a system of clearing markets, while the real world displays a mix of price mechanism and quantity restrictions, (ii) beyond the accumulation of human capital and labor market returns, higher education has non-pecuniary returns and there is evidence that households get a direct consumption value from going to college. The implications for welfare and policy analysis are likely to be far-reaching, (iii) the paper has focused on the role played by tuition and public subsidies for the shaping of inequality in higher education: the analysis should be extended to take into account donations and endowments as it is likely to be an additional source of inequality.

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A Analytical Model - Details

I solve the model using a guess and verify. I guess that the tuition function before government financial aid are given respectively by:

$$e_u(q,z,y) = \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_1}} z^{-\frac{\varepsilon_2}{\varepsilon_1}} \left(\frac{y}{\kappa_2}\right)^{\frac{\varepsilon_3}{\varepsilon_1}} \right)^{\frac{1}{1-\tau_u}} \tag{35}$$

A.1 Solution to the Household Problem

Using the guess (35) and the expression for financial aid (22), the problem of the Households is

$$\mathscr{U} = \max_{s,\ell} (1-\beta) \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + \ln \left(Ah^\lambda \ell^\mu\right)^{1-\tau_y} - \ell^\eta \right] + \beta E \mathscr{U}' \tag{36}$$

$$\begin{aligned}
&\ln h' = \ln \xi_y + \ln \kappa + \alpha_1(1+\alpha_2(\varepsilon_2 + \tau_m(1-\tau_u)\varepsilon_1)) \ln \xi_b + \alpha_h \ln h \\
&+ \alpha_2 \varepsilon_1 \left(\ln \left(s(1+a_h)/T_e\right)^{1-\tau_u} (1+a_u)T_u/p_I \right) + \alpha_2(\varepsilon_1(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_3) \ln \ell^{(1-\tau_y)\mu} \\
&+ \alpha_2 \left(\varepsilon_1(1-\tau_u)(1-\tau_n) - \varepsilon_3\right) \left((1-\tau_{y,t}) \ln A(1-a_y)T_y\right) + \alpha_2 \varepsilon_3 \ln \kappa_2
\end{aligned} \tag{37}$$

with $\alpha_h = \alpha_1 + \alpha_3 + \alpha_1\alpha_2(\varepsilon_2 + \tau_m(1-\tau_u)\varepsilon_1) + \alpha_2(\varepsilon_1(1-\tau_u)(1-\tau_n) - \varepsilon_3)(1-\tau_y)\lambda$ and s the spending rate, i.e. the amount of spending for college over income. I then guess that $\mathscr{U}_t = U_t \ln h_t + Z_t \ln \xi_{b,t} + B_t$. Replacing this guess into (36), then using (37) to substitute for $\ln h_{t+1}$ and using (5) and (6)

$$\begin{aligned}
&U_t \ln h_t + Z_t \ln \xi_{b,t} + B_t = \max_{s,\ell} (1-\beta) \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + (1-\tau_y) \ln h^\lambda \ell^\mu A - \ell^\eta \right] \\
&+ \beta \Bigg[U_{t+1} \Bigg(\mu_y + \ln \kappa + \alpha_1(1+\alpha_2(\varepsilon_2 + \tau_{m,t}(1-\tau_u)\varepsilon_1)) \ln \xi_{b,t} + \alpha_{h,t} \ln h_t \\
&+ \alpha_2 \varepsilon_1 \left(\ln \left(s(1+a_h)/T_e\right)^{1-\tau_u} (1+a_u)T_u/p_I \right) + \alpha_2(\varepsilon_1(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_3) \ln \ell^{(1-\tau_y)\mu} \\
&+ \alpha_2(\varepsilon_1(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_3) \left((1-\tau_{y,t}) \ln A(1-a_{y,t})T_{y,t}\right) + \alpha_2 \varepsilon_3 \ln \kappa_{2,t} \Bigg) + Z_{t+1}\mu_b + B_{t+1} \Bigg]
\end{aligned}$$

Gathering all the terms in $\ln h_t$ one gets that U_t has to verify
$U_t = (1-\beta) \sum_{k=0}^{\infty} \beta^k (1-\tau^y_{t+k}) \lambda_{t+k} \prod_{m=0}^{k-1} \alpha^h_{t+m}$

Gathering all the terms in $\ln \xi_{b,t}$, one gets $Z_t = \Big(U_t - (1-\beta)(1-\tau_{y,t})\lambda\Big)\frac{\alpha_1(1+\alpha_2(\varepsilon_2+\varepsilon_1(1-\tau_u)\tau_{m,t}))}{\alpha_t^h}$. Finally gathering the independent terms, the F.O.C for s and ℓ give

$$s_t = \frac{\beta\alpha_2\varepsilon_1(1-\tau_u)U_{t+1}}{1-\beta+\beta\alpha_2\varepsilon_1(1-\tau_u)U_{t+1}} \tag{38}$$

$$\ell = \left[(1-\tau_{y,t})\frac{\mu}{\eta}\left(1+\frac{\beta}{1-\beta}\alpha_2\left(\varepsilon_1(1-\tau_u)(1-\tau_{n,t})-\varepsilon_3\right)U_{t+1}\right)\right]^{\frac{1}{\eta}} \tag{39}$$

A.2 University problem

I first provide a generalized definition of σ_u that takes into account government policies

$$\sigma_u^2 = \frac{\omega_1(1-\tau_u)}{2}E\left(\left(\ln\left(\bar{z}^{\frac{\omega_2}{\omega_1(1-\tau_u)}}\bar{y}^{-\frac{\omega_3}{\omega_1(1-\tau_u)}}\right)-\ln z^{\frac{\omega_2}{\omega_1(1-\tau_u)}}y^{-\frac{\omega_3}{\omega_1(1-\tau_u)}}\right)^2\right) \tag{40}$$

Using this definition and our guess for tuitions (35), one gets the following

$$\sigma_u^2 = \frac{\omega_1(1-\tau_u)}{2}E\left(\left(\ln e_u(z,y) - \ln\left(\frac{p_I\tilde{I}}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}\right)^2\right) = \frac{\omega_1(1-\tau_u)}{2}\tilde{\sigma}_u^2$$

where I define $\ln\tilde{I} = \ln I - \frac{\tilde{\sigma}_u^2}{2}$. I now show that $\tilde{\sigma}_u^2$ is the within-university variance of log tuition. I guess that tuition fees are log-normally distributed within the university. Denoting $\mu_{e,q}, \sigma_{e,q}$ the mean and standard deviation of log tuition within the university of quality q, the budget constraint of the university -given by (23)-becomes

$$\begin{aligned} p_I I &= T_u(1+a_u)\left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} = T_u(1+a_u)e^{(1-\tau_u)\mu_{eq}+(1-\tau_u)\frac{\sigma_{eq}^2}{2}} \\ \iff \frac{1}{(1-\tau_u)}\ln\frac{p_I}{(1+a_u)T_u}\tilde{I} &+ \frac{\tilde{\sigma}_u^2}{2} - \frac{\sigma_{eq}^2}{2} = \mu_{e,q} = E\ln e_u(z,y) \end{aligned}$$

Substituting this last line into the expression of σ_u^2 above gives

$$\begin{aligned} \tilde{\sigma}_u^2 &= \int\phi(z,y)\left(\ln e_u(z,y) - E\ln e_u(z,y) + \frac{\sigma_{eq}^2-\sigma_u^2}{2}\right)^2 dzdy \\ \iff \tilde{\sigma}_u^2 &= \sigma_{e,q}^2 + \left(\frac{\sigma_{eq}^2-\sigma_u^2}{2}\right)^2 + 0 \quad \Rightarrow \tilde{\sigma}_u^2 = \sigma_{e,q}^2 \quad \text{or} \quad \tilde{\sigma}_u^2 = \sigma_{e,q}^2 + 4 \end{aligned}$$

$\tilde{\sigma}_u = \sigma_{e,q}$ is a solution to the quadratic equation. This verifies our guess.

$\mu_{e,q} = E \ln e_u(z,y) = \ln\left(\frac{p_I \tilde{I}}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}$ and $\sigma_u^2 = \sigma_{eq}^2$ are respectively the mean and standard deviation of within-university log tuitions. Therefore I can now rewrite the problem of the university replacing I with $\tilde{I}$

$$\max_{\tilde{I},\bar{z},\bar{y},r(.)} \tilde{I}^{\omega_1} \bar{z}^{\omega_2} \bar{y}^{-\omega_3} \tag{41}$$

$$\ln \tilde{I} \int_0^1 r_{z,y} dz\, dy = \int r_{z,y} \Big((1-\tau_u) \ln(e_u)^i + \ln(1+a_u) T_u / p_I \Big) dz\, dy$$

$$\ln \bar{z} \int_0^1 r_{z,y} dz\, dy = \int_0^1 r_{z,y} \ln z\, dz\, dy \quad \text{and} \quad \ln \bar{y} \int_0^1 r_{z,y}\, dz\, dy = \int_0^1 r_{z,y} \ln y\, dz\, dy$$

where $r_{z,y}$ denotes the mass of individuals of type (z, y).

The F.O.Cs are $\frac{\omega_1}{\tilde{I}} + \frac{\lambda_1}{\tilde{I}} = 0, \qquad \frac{\omega_2}{\bar{z}} + \frac{\lambda_2}{\bar{z}} = 0$ and $-\frac{\omega_3}{\bar{y}} + \frac{\lambda_3}{\bar{y}} = 0$

$$r_{z,y} = \begin{cases} 0 \quad \text{if} & \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\omega_1}} z^{-\frac{\omega_2}{\omega_1}} (y/\bar{y})^{\frac{\omega_3}{\omega_1}}\right)^{\frac{1}{1-\tau_u}} < e_u(q,z,y) \\ c \in \mathbb{R} \quad \text{if equality} \\ +\infty \quad \text{if strictly larger} \end{cases}$$

where I have solved for the Lagrange multipliers. I guess that in equilibrium, $\bar{y} = \kappa_2 q^{\nu}$. Therefore whenever a college admits a certain student type, the tuition formula is:

$$e_u(q,z,y) = \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1-\nu\tilde{\varepsilon_3}}{\omega_1}} z^{-\frac{\omega_2}{\omega_1}} y^{\frac{\omega_3}{\omega_1}} \kappa_2^{-\frac{\omega_3}{\omega_1}}\right)^{\frac{1}{1-\tau_u}} = \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_1}} z^{-\frac{\varepsilon_2}{\varepsilon_1}} y^{\frac{\varepsilon_3}{\varepsilon_1}} \kappa_2^{-\frac{\varepsilon_3}{\varepsilon_1}}\right)^{\frac{1}{1-\tau_u}}$$

$$\textit{with} \quad \varepsilon_1 = \frac{\omega_1}{1-\nu\tilde{\varepsilon_3}} \qquad \varepsilon_2 = \frac{\tilde{\varepsilon_2}}{1-\nu\tilde{\varepsilon_3}} \qquad \varepsilon_3 = \frac{\tilde{\varepsilon_3}}{1-\nu\tilde{\varepsilon_3}}$$

I can solve for ν and κ_2 using the equilibrium outcome given by the mean income in proposition A.4. I do this later in appendix A.4.1. This confirms the guess for tuition fees (35). Given this guess for tuition, a university is always at the interior solution, therefore always indifferent between all types.

A.3 Other Equilibrium Conditions

A.3.1 Government Budget Constraints

There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any period.

$$\int_0^1 a_y y(i) + a_c c(i) + e(i) di = \int_0^1 e(i)(1+a_u)(1+a_h) di \tag{42}$$

The other three constraints, (44), (43) and (45) pin down T_u, T_y, T_e such that a_y, a_h, a_u parametrize respectively the average rate of income tax, financial aid and transfers to college. Denoting f_q the mass of students in colleges of quality q

$$\int_0^1 y(i)^{1-\tau_y} T_y di = \int_0^1 y(i) di \tag{43}$$

$$(1+a_h)\int_0^1 e(i) di = \int_0^1 e_u(i) di \tag{44}$$

$$\int E_{z,y}[e_u(q,z,y)] f_q dq = \int T_u \left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} f_q dq \tag{45}$$

Lemma 1. *Along the equilibrium path, the government budget constraints* (42),(43),(44) *and* (45) *are given by*

$$\frac{a_{c,t}(1-s_t)}{(1+a_{c,t})} = s_t(1+a_{u,t})(1+a_{h,t}) - \frac{a_{y,t}}{1-a_{y,t}} - s_t \tag{46}$$

$$\ln T_y = \tau_y \ln A + \tau_y \mu \ln \ell + \tau_y \lambda m_h + \frac{\lambda^2}{2}(2-\tau_y)\tau_y \Sigma_{h,t}^2 \tag{47}$$

$$\ln T_e = (-\tau_n \lambda + \alpha_1 \tau_m) m_h + \frac{\alpha_1 \tau_m}{2}(\alpha_1 \tau_m - 1)\sigma_b^2 - \tau_n(\ln A\ell^\mu(1-a_y)) \tag{48}$$

$$+ \left[\lambda^2(1-\tau_y)^2(\tau_n-2)\tau_n + 2\lambda(1-\tau_n)(1-\tau_y)\tau_m\alpha_1 + (\alpha_1\tau_m)^2 - \tau_n\lambda^2(2-\tau_y)\tau_y\right]\frac{\Sigma_h^2}{2} \tag{49}$$

$$\ln T_u = \tau_u \left(\ln A\ell^\mu s(1+a_h)(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2}\right) + \frac{\tau_u}{1-\tau_u}\frac{\sigma_I^2}{2} \tag{50}$$

1. Solving for the aggregate state budget constraint is immediate
2. Solving for T_y. Using (43), and the expression for market income y_m, (21), and using the guess that $\ln h$ is normally distributed one gets:

$$\int_0^1 (A\ell^\mu h^\lambda)^{1-\tau_{y,t}} T_y di = \int_0^1 A\ell^\mu h^\lambda di \iff T_y e^{\lambda(1-\tau_y)m_h + \frac{(\lambda(1-\tau_y))^2}{2}\Sigma_h^2} = A^{\tau_y}\ell^{\tau_y} e^{\lambda m_h + \frac{\lambda^2}{2}\Sigma_h^2}$$

3. Solving for T_e. Using (44), one gets:

$$(1+a_h)\int_0^1 e^i di = \int_0^1 (e_u)^i di \iff (1+a_h)\int_0^1 sy_I di = \int_0^1 \frac{sy(1+a_h)}{T_e z^{-\tau_m} y^{\tau_n}} di$$

$$T_e(1-a_y)^{\tau_n}(A\ell)^{\tau_n(1-\tau_y)}(T_y)^{\tau_n} e^{\lambda(1-\tau_y)m_h+(\lambda(1-\tau_y))^2\frac{\Sigma_h^2}{2}}$$
$$= e^{(\lambda(1-\tau_y)(1-\tau_n)+\tau_m\alpha_1)m_h-\alpha_1\tau_m\frac{\sigma_b^2}{2}+(\lambda(1-\tau_y)(1-\tau_n)+\tau_m\alpha_1)^2\frac{\Sigma_h^2}{2}+\frac{(\alpha_1\tau_m)^2}{2}\sigma_b^2}$$

4. Solving for T_u. Substituting (23) into (45), one gets

$$\int E_{z,y}[e_u(q,z,y)]f_q dq = \int T_u\left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} f_q dq$$
$$\iff \int\left(\frac{p_I I_q}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}} f_q dq = \int \frac{p_I I_q}{(1+a_u)} f_q dq$$
$$\left(\frac{p_I}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}\int I_i^{\frac{1}{1-\tau_u}} di = \frac{p_I}{(1+a_u)}\int I_i di$$
$$\iff \left(\frac{p_I}{1+a_u}\right)^{\frac{\tau_u}{1-\tau_u}}\int I_i^{\frac{1}{1-\tau_u}} di = (T_u)^{\frac{1}{1-\tau_u}}\int I_i di$$

where i indexes households. I then guess that I_i is log-normally distributed with mean μ_I and variance σ_I^2 - I give an expression for these variables in appendix A.6):

$$\left(\frac{p_I}{1+a_u}\right)^{\frac{\tau_u}{1-\tau_u}} e^{\frac{\mu_I}{1-\tau_u}+\frac{\sigma_I^2}{2(1-\tau_u)^2}} = (T_u)^{\frac{1}{1-\tau_u}} e^{\mu_I+\frac{\sigma_I^2}{2}}$$
$$\Rightarrow \ln T_u = \tau_u \ln\left(\frac{p_I}{1+a_u}\right) + \mu_I\tau_u + \frac{\sigma_I^2}{2}\frac{\tau_u(2-\tau_u)}{(1-\tau_u)}$$

Using the guess and from appendix A.3.4, one gets

$$\ln E(I) = \mu_I + \frac{\sigma_I^2}{2} = \ln\frac{A\ell^\mu s(1+a_h)(1+a_u)(1-a_y)}{p_I} + \lambda m_h + \lambda^2\frac{\Sigma_h^2}{2}$$
$$\text{Hence}\quad \mu_I = \ln\frac{A\ell^\mu s(1+a_h)(1+a_u)(1-a_y)}{p_I} + \lambda m_h + \lambda^2\frac{\Sigma_h^2}{2} - \frac{\sigma_I^2}{2}$$

Substituting back into the expression for T_u gives

$$\ln T_u = \tau_u\left(\ln A\ell^\mu s(1+a_h)(1-a_y) + \lambda m_h + \lambda^2\frac{\Sigma_h^2}{2}\right) + \frac{\tau_u}{1-\tau_u}\frac{\sigma_I^2}{2}$$

I derive the expression for σ_I^2 in appendix A.6

A.3.2 Educational Sector: Generalization

In this section I present an extension of the very simple educational sector presented in the main text. I start by assuming that the production function for educational services is given by $y_I = A_I h^{\bar{\lambda}} \ell^{\mu}$ with $\bar{\lambda} \geq \lambda$. The latter assumption says that households with higher human capital have a comparative advantage in working in the educational services sector.

Colleges buy services at price p_I from the educational sector. The latter has two peculiarities. First of all it is made of one non-profit agency that aggregates heterogeneous inputs to produce an homogeneous educational service and whose objective is to minimize costs subject to a non-negative profit condition. Secondly it has full power in the setting of wages. I discuss these two assumptions below. The problem of the educational service agency willing to produce $\bar{I}$ is:

$$\min_{d_h} p_I \quad s.t. \quad \int A_I h^{\bar{\lambda}} \ell^{\mu} d_h dh \geq \bar{I} \quad \text{and} \quad p_I \int A_I h^{\bar{\lambda}} \ell^{\mu} d_h dh \geq \int h^{\lambda} \ell^{\mu} d_h dh \tag{51}$$

where the right hand side of the last line embeds the assumption of bilateral bargaining with full power to the agency and d_h is the indicator function equal to 1 if a household with human capital h is employed in the sector.[44]

A.3.3 Educational Sector: Hiring Rule

From the assumption that $\bar{\lambda}_t > \lambda_t$ (individuals with high human capital have a comparative advantage working for the agency), the hiring rule takes the form of threshold rule where the most educated individuals work in the educational sector.

Proposition A.1. *Provided $a_{u,t}, a_{h,t}, \beta$ are not too high, there exists a unique $\underline{h}_t \in \mathbb{R}$ such that*

$$d_t(h) = 1 \iff h \geq \underline{h}_t \tag{52}$$

[44]The assumption that wages are set through a bilateral negotiation where the agency has full bargaining power implies that wages are determined by the marginal product in the final good sector. These assumptions are not particularly unrealistic and buy us some tractability. The assumption of minimizing the price of educational services will imply that the price of educational services will not reflect the marginal cost of producing these services but rather their average cost. The joint assumption of price-minimizing monopoly allows us to get rid of profit in this sector, that would otherwise need to be given back to either the household sector or the government.

The difference between the product and cost of hiring human capital h.

$$p_I A_I h^{\bar{\lambda}} - A h^{\lambda} \tag{53}$$

Lemma 2. *Equation (53) is increasing in h if*

$$\frac{E\left(h_{\bar{\lambda}}|h_{\bar{\lambda}}\geq 1\right)}{E\left(h_{h_\lambda}|h_\lambda \geq 1\right)} = \frac{m\left(\frac{m_h-\log(\underline{h})}{\Sigma_h} - \bar{\lambda}\Sigma_h\right)}{m\left(\frac{m_h-\log(\underline{h})}{\Sigma_h} - \lambda\Sigma_h\right)} \leq \frac{\bar{\lambda}}{\lambda}$$

$$\text{where} \quad h_{\bar{\lambda}} = \left(\frac{h}{\underline{h}}\right)^{\bar{\lambda}} \quad \text{and} \quad h_\lambda = \left(\frac{h}{\underline{h}}\right)^{\lambda} \quad \text{and } m(.) \text{ denotes the Mills ratio.}$$

Proof. We want to show that this is increasing in h so that the agency—willing to minimize the average cost—wants to hire first the individuals with highest human capital. The condition is: $p_I A_I \bar{\lambda} h^{\bar{\lambda}-1} > A\lambda h^{\lambda-1} \iff \frac{\bar{\lambda}}{\lambda} h^{\bar{\lambda}-\lambda} > \frac{A}{A_I p_I}$. It is sufficient to show that this is true for $h = \underline{h}$ since $\bar{\lambda} > \lambda$, i.e.

$$\frac{\bar{\lambda}}{\lambda}\underline{h}^{\bar{\lambda}-\lambda} > \frac{A}{A_I p_I} \iff \frac{\bar{\lambda}}{\lambda} > \frac{\int_{\underline{h}}\left(\frac{h}{\underline{h}}\right)^{\bar{\lambda}} dh}{\int_{\underline{h}}\left(\frac{h}{\underline{h}}\right)^{\lambda} dh} = \frac{E\left(h_{\bar{\lambda}}|h_{\bar{\lambda}}\geq 1\right)}{E\left(h_{h_\lambda}|h_\lambda \geq 1\right)}$$

where the last line uses the expression for the price p_I given by (55). Let's now show that the RHS of the last line is equal to the ratio of the Mills ratio. Taking the conditional expectation

$$E\left(h_{\bar{\lambda}}|h_{\bar{\lambda}}\geq 1\right) = \frac{e^{\bar{\lambda}(m_h-\log \underline{h})+\frac{\bar{\lambda}^2}{2}\Sigma_h^2}\Phi\left(\frac{\bar{\lambda}(m_h-\log \underline{h})+\bar{\lambda}^2\Sigma_h^2-0}{\bar{\lambda}\Sigma_h}\right)}{1-\Phi\left(\frac{0-\bar{\lambda}(m_h-\log \underline{h})}{\bar{\lambda}\Sigma_h}\right)}$$

$$\Rightarrow \frac{E\left(h_{\bar{\lambda}}|h_{\bar{\lambda}}\geq 1\right)}{E\left(h_{\bar{\lambda}}|h_{\lambda}\geq 1\right)} = e^{(\bar{\lambda}-\lambda)(m_h-\log \underline{h})+\frac{\bar{\lambda}^2-\lambda^2}{2}\Sigma_h^2}\frac{\Phi\left(\frac{m_h-\log \underline{h}}{\Sigma_h}+\bar{\lambda}\Sigma_h\right)}{\Phi\left(\frac{m_h-\log \underline{h}}{\Sigma_h}+\lambda\Sigma_h\right)} = \frac{m\left(\frac{\log \underline{h}-m_h}{\Sigma_h}-\bar{\lambda}\Sigma_h\right)}{m\left(\frac{\log \underline{h}-m_h}{\Sigma_h}-\lambda\Sigma_h\right)}$$

$\square$

Lemma 3. *For $\underline{h}$ high enough,* $\frac{m\left(\frac{\log \underline{h}-m_h}{\Sigma_h}-\bar{\lambda}\Sigma_h\right)}{m\left(\frac{\log \underline{h}-m_h}{\Sigma_h}-\lambda\Sigma_h\right)} \leq \frac{\bar{\lambda}}{\lambda}$

Proof. First denote $m\left(\frac{\log \underline{h}-m_h}{\Sigma_h}-\bar{\lambda}\Sigma_h\right) = m_{\bar{\lambda}}$ and $m\left(\frac{\log \underline{h}-m_h}{\Sigma_h}-\lambda\Sigma_h\right) = m_\lambda$. The

Mills ratio associated with a standard normal is decreasing and convex, which implies

$$\frac{m_{\bar{\lambda}}}{m_{\lambda}} > 1 \iff \bar{\lambda} > \lambda \quad \text{and} \quad \frac{\partial \frac{m_{\bar{\lambda}}}{m_{\lambda}}}{\partial \log \underline{h}} = \frac{m_{\lambda}\left(m'_{\bar{\lambda}} - \frac{m_{\bar{\lambda}}}{m_{\lambda}} m'_{\lambda}\right)}{m_{\lambda}^2} < 0$$

since $m'_{\bar{\lambda}} < m'_{\lambda}$ and $-m'_{\lambda} < -m'_{\lambda}\frac{m_{\bar{\lambda}}}{m_{\lambda}} \Rightarrow m'_{\bar{\lambda}} - \frac{m_{\bar{\lambda}}}{m_{\lambda}} m'_{\lambda} < 0$. I now show that the limit of the ratio of Mills ratio tends to $+\infty$ when $\underline{h}$ tends to 0. Denoting $X_{\lambda}(\underline{h}) = \frac{\log \underline{h} - m_h}{\Sigma_h} - \lambda \Sigma_h$ and similarly for $\bar{\lambda}$, one has

$$\begin{aligned}
\lim_{\log \underline{h} \to -\infty} \frac{m(X_{\bar{\lambda}}(\underline{h}))}{m(X_{\lambda}(\underline{h}))} &= \lim_{\log \underline{h} \to +\infty} \frac{m_{\bar{\lambda}}(-X_{\bar{\lambda}}(\underline{h}))}{m(-X_{\lambda}(\underline{h}))} = \lim_{\log \underline{h} \to +\infty} \frac{\frac{1-\Phi(X_{\bar{\lambda}}(\underline{h}))}{\Phi(X_{\bar{\lambda}}(\underline{h}))}}{\frac{1-\Phi(X_{\lambda}(\underline{h}))}{\Phi(X_{\lambda}(\underline{h}))}} \frac{m(X_{\bar{\lambda}}(\underline{h}))}{m(X_{\lambda}(\underline{h}))} \times 1 \\
&= \lim_{\log \underline{h} \to +\infty} \frac{-\phi(X_{\bar{\lambda}}(\underline{h}))}{-\phi(X_{\lambda}(\underline{h}))} = \lim_{\log \underline{h} \to +\infty} e^{\Sigma_h(\bar{\lambda}-\lambda)\left(\frac{\log \underline{h} - m_h}{\Sigma_h} - (\lambda + \bar{\lambda})\frac{\Sigma_h}{2}\right)} = +\infty
\end{aligned}$$

where the second line uses the symmetry of the standard normal cdf and pdf, the third uses l'Hôpital's rule and the fourth that $\bar{\lambda} > \lambda$.

Hence the ratio of Mills ratios is decreasing, continuous, goes to $+\infty$ when $\underline{h}$ approaches 0 and to 1 when it goes to $+\infty$. Therefore there exists a unique threshold at which it crosses $\frac{\bar{\lambda}}{\lambda}$ and is below it for $\underline{h}$ higher. This finishes the proof. $\square$

Given the market clearing condition (54) pinning down $\log \underline{h}$, it is clear that a_u, a_h, s (and hence β) low enough ensures that $\log \underline{h}$ will be high enough in equilibrium. I have checked that in our calibration this property holds.

A.3.4 Market Clearing

It remains to be checked that the final good and the educational services market clear. By Walras' law, if the latter clears, since college admission markets already clear, the final good market should clear. The market clearing condition in the educational sector requires that the demand for educational services coming from colleges—the left-hand-side of equation (54)—be equal to the supply of human capital supplied by the agency. It implicitly pins down the threshold $\underline{h}_t$. The price of educational services is then given by the no-profit condition in (51) of the agency. The following proposition summarizes these results.

Lemma 4. *1. The threshold to work in the education sector $\underline{h}$ is implicitly given by*

$$(1+a_{u,t})(1+a_{h,t})s_t e^{\lambda_t m_{h,t}+\frac{\lambda_t^2}{2}\Sigma_{h,t}^2}(1-a_{y,t}) = \underline{H}_t = e^{\lambda_t m_{h,t}+\lambda_t^2\frac{\Sigma_{h,t}^2}{2}} \frac{\Phi\left[\frac{m_{h,t}-\log \underline{h}_t}{\Sigma_{h,t}}+\lambda_t\Sigma_{h,t}\right]}{1-\Phi\left[\frac{\log \underline{h}_t - m_{h,t}}{\Sigma_{h,t}}\right]} \tag{54}$$

where $\Phi(.)$ denotes the c.d.f. of a standard normal.

2. Defining $\underline{H}_t^I = \int_{\underline{h}_t}^{+\infty} h^{\bar{\lambda}} dh$, the equilibrium price of education services is

$$p_{I,t} = \frac{A\int_{\underline{h}_t}^1 h^{\lambda_t} dh}{A_I \int_{\underline{h}_t}^1 h^{\bar{\lambda}} dh} = \frac{A}{A_I} e^{(\lambda_t-\bar{\lambda})m_{h,t}+\left(\lambda_t^2-\bar{\lambda}^2\right)\frac{\Sigma_{h,t}^2}{2}} \frac{\Phi\left[\frac{m_{h,t}-\log \underline{h}_t}{\Sigma_{h,t}}+\lambda_t\Sigma_{h,t}\right]}{\Phi\left[\frac{m_{h,t}-\log \underline{h}_t}{\Sigma_{h,t}}+\bar{\lambda}\Sigma_{h,t}\right]} \tag{55}$$

Proof. Demand for educational services is given by $\frac{(1+a_u)(1+a_h)}{p_I} s \int_0^1 y_I di$ and supply is given by $A_I \ell \int_{\underline{h}}^1 (h^i)^{\bar{\lambda}} dh$ hence market clearing is:

$$\begin{aligned}
\frac{(1+a_u)(1+a_h)}{p_I} s \int_0^1 y_{I,t} di &= A_I \ell^\mu \int_{\underline{h}}^1 (h^i)^{\bar{\lambda}} dh \\
\iff (1+a_u)(1+a_h) s A \ell^\mu e^{\lambda m_h + \frac{\lambda^2}{2}\Sigma_h^2}(1-a_y) &= p_I A_I \ell^\mu \underline{H}^I \\
(1+a_u)(1+a_h) s A \ell^\mu e^{\lambda m_h + \frac{\lambda^2}{2}\Sigma_h^2}(1-a_y) &= A \ell^\mu \underline{H} \\
\iff (1+a_u)(1+a_h) s e^{\lambda m_h + \frac{\lambda^2}{2}\Sigma_h^2}(1-a_y) &= \underline{H}
\end{aligned}$$

To derive the expression for the equilibrium price (55), I combine the no-profit condition in (51) with the threshold condition (52) and use the expression for conditional expectation of a log-normal distribution. □

A.4 Quality distribution and within-college distributions

Parent's education and income. Taking the logarithm of (17): $\ln q = (\varepsilon_I + \varepsilon_A)\ln h + \varepsilon_A \ln \xi_b + x$ with
$x = \varepsilon_1 \left(\ln\left(s\frac{(1+a_h)}{T_e}\right)^{1-\tau_u}\left(\frac{(1+a_u)T_u}{p_I}\right)\right) + (\varepsilon_1(1-\tau_u)(1-\tau_n)-\varepsilon_3)\ln(A\ell^\mu)^{1-\tau_y}T_y(1-a_y) + \varepsilon_3 \ln \kappa_2$, where $\varepsilon_I = (\varepsilon_1(1-\tau_u)(1-\tau_{n,t})-\varepsilon_3)(1-\tau_y)\lambda$ and $\varepsilon_A = \alpha_1(\varepsilon_2+\tau_m(1-\tau_u)\varepsilon_1)$. All pairs (h,ξ_y) that verify this condition will go to a university with quality q. The distribution of parents human capital within a given university of quality q can therefore be computed explicitly. The mass of individuals with $\ln h$ and going to $\ln q$

is given by:

$$
\begin{aligned}
&f\left(\frac{1}{\varepsilon_A}(\ln q - x - (\varepsilon_I+\varepsilon_A)\ln h)\cap \ln h\right) = f_{\xi_b}\left(\frac{1}{\varepsilon_A}(\ln q - x - (\varepsilon_I+\varepsilon_A)\ln h)\right) f_h(\ln h)\\
&= \phi\left(\frac{\ln q - x - (\varepsilon_I+\varepsilon_A)\ln h}{\varepsilon_A}, \mu_b, \sigma_b^2\right)\phi\left(\ln h, m_h, \Sigma_h^2\right)\\
&= \phi\left(\ln h, \underbrace{\frac{\ln q - x - \varepsilon_A\mu_b}{\varepsilon_I+\varepsilon_A}}_{\mu_1^q}, \underbrace{\left(\frac{\varepsilon_A\sigma_b}{\varepsilon_A+\varepsilon_I}\right)^2}_{\sigma_1^2}\right)\phi\left(\ln h, m_h, \Sigma_h^2\right)\\
&= \phi\left(\ln h, \mu_1^q, \sigma_1^2\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) = \phi\left(\mu_1^q, m_h, \sigma_1^2+\Sigma_h^2\right)\phi\left(\ln h, \mu_2^q, \sigma_2^2\right)
\end{aligned}
$$

where the RHS is the mass of individuals going to quality q and the LHS is the density of people whose parents have human capital h conditional on college q.

$$
f(\ln h|q) \sim \mathcal{N}\left(\frac{\Sigma_h^{-2}m_h + \left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2}\sigma_b^{-2}\frac{(\ln q - x - \varepsilon_A\mu_b)}{\varepsilon_I+\varepsilon_A}}{\Sigma_h^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2}\sigma_b^{-2}}, \frac{\Sigma_h^2\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2}{\Sigma_h^2 + \left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2}\right) \sim \mathcal{N}\left(\mu_2^q, \sigma_2^2\right)
$$

For future reference I introduce $\mu_2^q = \mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \varepsilon_A\mu_b)$

$$
\text{with}\quad \mu_{2,1} = \frac{\Sigma_h^{-2}}{\Sigma_h^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2}\sigma_b^{-2}} \quad\text{and}\quad \mu_{2,2} = \frac{\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2}\sigma_b^{-2}}{\left[\Sigma_h^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2}\sigma_b^{-2}\right](\varepsilon_I+\varepsilon_A)}
$$

where the second line stems from independence of h and ξ_b. The mass of individuals studying in college of quality q is $\phi\left(\mu_1^q, m_h, \sigma_1^2+\Sigma_h^2\right)$ and the density of $\ln h$ conditional on being in this college is $\phi\left(\ln h, \mu_2^q, \sigma_2^2\right)$. From the distribution of parents' human capital within a college, the distribution of parents' income is

$$
\ln y \sim \mathcal{N}\left(\ln(1-a_y) + (1-\tau_y)\left[\ln A + \lambda\mu_2^q + \mu\ln\ell\right] + \ln T_y, (1-\tau_y)^2\lambda^2\sigma_2^2\right).
$$

Distribution of college quality Since $\phi\left(\mu_1^q, m_h, \sigma_1^2+\Sigma_h^2\right)$—with $\mu_1 = \frac{1}{\varepsilon_I+\varepsilon_A}(\ln q - x - \varepsilon_A\mu_b)$ and $\sigma_1^2 = \left(\frac{\varepsilon_A}{\varepsilon_A+\varepsilon_I}\right)^2\sigma_b^2$—is the mass of students in college of quality q, the distribution of quality is given by : $\ln q \sim \mathcal{N}\left((\varepsilon_I+\varepsilon_A)m_h + x + \varepsilon_A\mu_b, \varepsilon_A^2\sigma_b^2 + (\varepsilon_I+\varepsilon_A)^2\Sigma_h^2\right)$

Students' abilities From the definition of abilities $\ln z = \alpha_1 \ln h + \alpha_1 \ln \xi_b$ and the sorting rule used above $\ln q = (\varepsilon_I + \varepsilon_A) \ln h + \varepsilon_A \ln \xi_b + x$, one gets

$$\ln z = \frac{\alpha_1}{\varepsilon_A} (\ln q - \varepsilon_I \ln h - x) \Rightarrow \ln z | q \sim \mathcal{N} \left(\frac{\alpha_1}{\varepsilon_A} (\ln q - \varepsilon_I \mu_2^q - x), \left(\frac{\alpha_1 \varepsilon_I}{\varepsilon_A} \right)^2 \sigma_2^2 \right)$$

A.4.1 Solving for κ_2 and ν

The initial guess was that $\bar{y} = \kappa_2 q^\nu$. Recall that $\ln \bar{y}$ is the mean log (after tax) income within a college $\ln \bar{y} = \ln(1-a_y) \left(A\ell^\mu\right)^{1-\tau_y} T_y + (1-\tau_y)\lambda \left(\mu_{2,1} m_h + \mu_{2,2}(\ln q - x - \varepsilon_A \mu_b)\right)$

Identifying coefficients with the guess $\ln \bar{y} = \ln \kappa_2 + \nu \ln q$, one gets:

$$\nu = (1-\tau_y)\lambda\mu_{2,2} = (1-\tau_y)\lambda \frac{\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2} \sigma_b^{-2}}{\left[\Sigma_h^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^{-2} \sigma_b^{-2}\right] (\varepsilon_I + \varepsilon_A)}$$

$$\iff \nu = \frac{1}{\left[\Sigma_h^{-2} \left(\frac{\omega_A}{\omega_I+\omega_A}\right)^2 \sigma_b^2 + 1\right] \left[(\omega_1(1-\tau_u)(1-\tau_n) - \omega_3) + \frac{\omega_A}{(1-\tau_y)\lambda}\right] + \omega_3}$$

ν is therefore only a function of Σ_h^2. Identifying the coefficient independent of $\ln q$, and recalling that x_t is a linear function of $\ln \kappa_2$, and defining $\tilde{x} = x - \varepsilon_3 \ln \kappa_2$, one gets:

$$\begin{aligned} \kappa_2 &= (1-a_y) \left(A\ell^\mu\right)^{1-\tau_y} T_y e^{(1-\tau_y)\lambda(\mu_{2,1} m_h - \mu_{2,2}(x+\varepsilon_A \mu_y)} \\ &= \left((1-a_y) \left(A\ell^\mu\right)^{1-\tau_y} T_y e^{(1-\tau_y)\lambda(\mu_{2,1} m_h - \mu_{2,2}(\tilde{x}+\varepsilon_A \mu_b)}\right)^{1-\nu\omega_3} \end{aligned}$$

A.5 Law of motion

Replacing κ_2, T_y, T_e and T_u obtained above in the law of motion for human capital

$$
\begin{aligned}
&\ln h' = \ln \xi_y + \ln \kappa + \alpha_1(1+\alpha_2(\varepsilon_2 + \tau_m(1-\tau_u)\varepsilon_1)) \ln \xi_b + \alpha_h \ln h \\
&+ \alpha_2\omega_1 \left(\ln s + \ln(1+a_h) + \ln \frac{(1+a_u)}{p_I} + \tau_u \left(\ln A\ell^{\mu}(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2} \right) + \frac{\tau_u}{1-\tau_u} \frac{\sigma_I^2}{2} \right) \\
&+ \alpha_2\omega_1(1-\tau_u)(1-\tau_n) \left(\ln A\ell^{\mu} + \ln(1-a_y) + \tau_y \lambda m_h + \frac{\lambda^2}{2}(2-\tau_y)\tau_y \Sigma_h^2 \right) \\
&+ \alpha_2\omega_3 \nu \varepsilon_A \frac{\sigma_b^2}{2} + \alpha_2\omega_3(1-\tau_y)\lambda\mu_{2,1} m_h \\
&+ \alpha_2\omega_1(1-\tau_u) \Bigg[(\tau_n\lambda - \alpha_1\tau_m) m_h + \frac{\alpha_1\tau_m}{2}(1-\alpha_1\tau_m)\sigma_b^2 + \tau_n(\ln A\ell^{\mu}(1-a_y)) \\
&- \left[\lambda^2(1-\tau_y)^2(\tau_n-2)\tau_n + 2\lambda(1-\tau_n)(1-\tau_y)\tau_m\alpha_1 + (\alpha_1\tau_m)^2 - \tau_n\lambda^2(2-\tau_y)\tau_y \right] \frac{\Sigma_h^2}{2} \Bigg]
\end{aligned}
$$

I now take the expectation, I factorize out all the terms in m_h as well as all the terms in σ_b^2. The next steps consist in simplifying the coefficient in front of σ_b^2, of factorizing out all the terms in Σ_h^2 and in using the expression in (A.6) for σ_I^2. I also use the fact that $\mu_{2,1} = 1 - \mu_{2,2}(\varepsilon_I + \varepsilon_A)$. One obtains

$$
\begin{aligned}
&m_h' = \rho m_h - \frac{\sigma_y^2}{2} + \ln \kappa \\
&+ \left[\frac{\tau_u}{1-\tau_u} \left(\frac{\alpha_1(1-\tau_u)}{(1-\nu\omega_3)} (\tau_m + \omega_2(1-\tau_n)\nu) \right)^2 - \alpha_1 \left(\alpha_2 \left(\omega_2 + \omega_1(1-\tau_u)(\tau_m)^2\alpha_1 \right) + 1 \right) \right] \frac{\sigma_b^2}{2} \\
&+ \alpha_2\omega_1 \left(\ln A\ell^{\mu}(1-a_y)s(1+a_u)(1+a_h) - \ln p_I \right) \\
&+ \alpha_2\omega_1 \Bigg[\lambda^2 + \frac{\tau_u}{1-\tau_u} \left(\frac{\alpha_1(1-\tau_u)}{(1-\nu\omega_3)} (\tau_m + \omega_2(1-\tau_n)\nu) \right)^2 \left(\frac{\omega_I}{\omega_A} + 1 \right)^2 \\
&\qquad - (1-\tau_u)\left[\lambda(1-\tau_y)(1-\tau_n) + (\alpha_1\tau_m) \right]^2 \Bigg] \frac{\Sigma_h^2}{2}
\end{aligned}
$$

with $\rho = \alpha_1 + \alpha_3 + \alpha_1\alpha_2\omega_2 + \alpha_2\omega_1\lambda$. Finally taking the variance gives the expression for the law of motion of Σ_h^2: $\Sigma_h^{2'} = \sigma_y^2 + (\alpha_1(1+\alpha_2(\varepsilon_2 + \tau_m(1-\tau_u)\varepsilon_1)))^2 \sigma_b^2 + (\alpha_h)^2 \Sigma_h^2$

Gathering all our results, the law of motion of human capital is given by

$$\ln h_{t+1} \sim \mathcal{N}\left(m_{h,t+1}, \Sigma^2_{h,t+1}\right)$$
$$m_{h,t+1} = \rho_t m_{h,t} + X_1\Big(m_{h,t}, \{\Sigma_h\}_{s=t}^{\infty}\Big)$$
$$\Sigma^2_{h,t+1} = (\alpha_{h,t}(\Sigma_{h,t}))^2\,\Sigma^2_{h,t} + X_{2,t}(\Sigma_{h,t})$$
$$\rho_t = \alpha_1 + \alpha_3 + \alpha_1\alpha_2\omega_2 + \alpha_2\omega_1\lambda_t$$
$$\begin{aligned} X_{1,t}\Big(m_{h,t}, \{\Sigma_h\}_{s=t}^{\infty}\Big) &= -\frac{\sigma_y^2}{2} + \ln\kappa \\ &+ \left[\frac{\tau_{u,t}}{1-\tau_{u,t}}\left(\frac{\alpha_1(1-\tau_{u,t})}{(1-\nu_t(\Sigma_{h,t})\omega_3)}\left(\tau_{m,t} + \omega_2(1-\tau_{n,t})\nu_t(\Sigma_{h,t})\right)\right)^2 \right. \\ &\qquad \left. -\alpha_1\left(\alpha_2\left(\omega_2 + \omega_1(1-\tau_{u,t})(\tau_{m,t})^2\alpha_1\right)+1\right)\right]\frac{\sigma_b^2}{2} \\ &+ \alpha_2\omega_1\left(\ln A\ell_t^{\mu}(\{\Sigma_{h,t}\}_{s=t}^{\infty})(1-a_{y,t})s_t(\{\Sigma_h\}_{s=t}^{\infty})(1+a_{u,t})(1+a_{h,t}) - \ln p_{I,t}(m_{h,t},\Sigma_{h,t})\right) \\ &+ \alpha_2\omega_1\left[\lambda_t^2 + \frac{\tau_{u,t}}{1-\tau_{u,t}}\left(\frac{\alpha_1(1-\tau_{u,t})}{(1-\nu_t(\Sigma_{h,t})\omega_3)}\left(\tau_{m,t}+\omega_2(1-\tau_{n,t})\nu_t(\Sigma_{h,t})\right)\right)^2\left(\frac{\omega_{I,t}}{\omega_{A,t}}+1\right)^2 \right. \\ &\qquad \left. -(1-\tau_{u,t})\left[\lambda(1-\tau_{y,t})(1-\tau_{n,t}) + (\alpha_1\tau_{m,t})\right]^2\right]\frac{\Sigma^2_{h,t}}{2} \end{aligned}$$
$$X_{2,t}(\Sigma_{h,t}) = \sigma_y^2 + (\alpha_1(1+\alpha_2(\varepsilon_2 + \tau_m(1-\tau_u)\varepsilon_1)))^2\,\sigma_b^2$$

A.6 From the distribution of $\ln q$ to the distribution of $\ln I$.

Using the definition of q and the expression for $\bar{z}$ obtained earlier,

$$\ln q = \ln \tilde{I}^{\omega_1}\bar{z}^{\omega_2} = \omega_1 \ln\tilde{I} + \omega_2\left(\frac{\alpha_1}{\varepsilon_A}(\ln q - \varepsilon_I\mu_2^q - x)\right)$$

which implies

$$\ln\tilde{I} = \frac{1}{\omega_1}\left(\ln q\left(1 - \alpha_1\frac{\omega_2}{\varepsilon_A}\right) + \alpha_1\frac{\omega_2}{\varepsilon_A}(\varepsilon_I\mu_2 + x)\right).$$

Given the expression for μ_2^q $\mu_2^q = \mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \varepsilon_A\mu_b)$, one gets

$$\ln\tilde{I} = \frac{1}{\omega_1}\left(\ln q\left(1-\alpha_1\frac{\omega_2}{\varepsilon_A} + \alpha_1\frac{\omega_2}{\varepsilon_A}\varepsilon_I\mu_{2,2}\right) + \alpha_1\frac{\omega_2}{\varepsilon_A}\left(\varepsilon_I\mu_{2,1}m_h + (1-\varepsilon_I\mu_{2,2})x - \varepsilon_I\mu_{2,2}\varepsilon_A\mu_b\right)\right)$$

Hence from the distribution $\ln q$ I can recover the distribution of $\ln\tilde{I} \sim \mathcal{N}\left(\mu_{\tilde{I}}, \sigma^2_{\tilde{I}}\right)$

with

$$\mu_{\tilde{I}} = \frac{1}{\omega_1}\left(\mu_q\left(1-\alpha_1\frac{\omega_2}{\varepsilon_A}+\alpha_1\frac{\omega_2}{\varepsilon_A}\varepsilon_I\mu_{2,2}\right)+\alpha_1\frac{\omega_2}{\varepsilon_A}\left(\varepsilon_I\mu_{2,1}m_h+(1-\varepsilon_I\mu_{2,2})x-\varepsilon_I\mu_{2,2}\varepsilon_A\mu_b\right)\right)$$

and $\sigma^2_{\tilde{I}} = \left(\frac{\alpha_1(1-\tau_u)}{(1-\nu\omega_3)}\left(\tau_m+\omega_2(1-\tau_n)\nu\right)\right)^2\left(\sigma_b^2+\left(\frac{\omega_I}{\omega_A}+1\right)^2\Sigma_h^2\right)$

The last line stems from

$$\begin{aligned}\frac{1}{\omega_1}\left(1-\alpha_1\frac{\omega_2}{\varepsilon_A}+\alpha_1\frac{\omega_2}{\varepsilon_A}\varepsilon_I\mu_{2,2}\right) &= \frac{1}{\omega_1}\left(1-\alpha_1\frac{\omega_2}{\varepsilon_A}+\alpha_1\frac{\omega_2}{\varepsilon_A}\varepsilon_I\mu_{2,2}\right)\\ &= \frac{\alpha_1(1-\tau_u)}{\varepsilon_A(1-\nu\omega_3)}\left(\tau_m+\omega_2(1-\tau_n)\nu\right)\end{aligned}$$

where I used $\nu=(1-\tau_y)\lambda\mu_{2,2}$ and $\varepsilon_l = \frac{\omega_l}{1-\nu\omega_3}$. Finally

$$\begin{aligned}\sigma^2_{\tilde{I}} &= \left(\frac{\alpha_1(1-\tau_u)}{\varepsilon_A(1-\nu\omega_3)}\left(\tau_m+\omega_2(1-\tau_n)\nu\right)\right)^2\sigma_q^2\\ &= \left(\frac{\alpha_1(1-\tau_u)}{(1-\nu\omega_3)}\left(\tau_m+\omega_2(1-\tau_n)\nu\right)\right)^2\left(\sigma_b^2+\left(\frac{\omega_I}{\omega_A}+1\right)^2\Sigma_h^2\right)\end{aligned}$$

Since $\ln\tilde{I} = \ln I-(1-\tau_u)\frac{\sigma_u^2}{2}$ and σ_u^2 is common across all colleges, I have $\ln I\sim\mathcal{N}\left(\mu_I,\sigma_I^2\right)$with $\mu_I=\mu_{\tilde{I}}+(1-\tau_u)\frac{\sigma_u^2}{2}$ and $\sigma_I^2=\sigma_{\tilde{I}}^2$

Expression for σ_u^2 Given that all households save a fraction s of their disposable income and the selection equation into college, one gets

$$\begin{aligned}\ln e_u = \ln\frac{(1+a_h)s}{T_e} &+ \tau_m\frac{\alpha_1}{\varepsilon_A}\left(\ln q-x\right)+\ln h^{(1-\tau_n)(1-\tau_y)\lambda-\tau_m\frac{\alpha_1}{\varepsilon_A}\varepsilon_I)}\\ &+(1-\tau_n)\ln T_y(1-a_y)A^{1-\tau_y}\ell^{(1-\tau_y)\mu}\end{aligned}$$

Hence the within-university variance of tuitions is given by:

$$\sigma_u^2=\left((1-\tau_n)(1-\tau_y)\lambda-\tau_m\frac{\alpha_1}{\varepsilon_A}\varepsilon_I\right)^2\sigma_2^2=\left((1-\tau_y)\lambda\frac{(1-\tau_n)\omega_2+\tau_m\omega_3}{\omega_2+\omega_1(1-\tau_u)\tau_m}\right)^2\sigma_2^2$$

which is indeed constant across universities wince σ_2^2 is an aggregate constant.

A.7 Details on the Positioning Game

In this appendix I give a formal explanation of the positioning game as well as a characterization of the equilibrium. Recall the general environment. There is a continuum of colleges $j \in [0,1]$. At each generation $t \in \mathbb{N}$, they play a positioning game. The games played at any two generations $t > t'$ are independent of each other.

At a given generation $t \in \mathbb{N}$, and before playing the positioning game, each college is given a real number $o \in [0,1]$. The positioning game is sequential and o is the order in which colleges play. Without loss of generality, since all colleges are identical, one can relabel colleges $j = o$ so that their label is also their order.[45] Colleges play sequentially in descending order: j plays before j' if and only if $j > j'$. Each college plays once.

All colleges have the same set of actions: the line of qualities $q \in \mathbb{R}_+$. The history of the sequential game up to college j's turn is a (injective) function $\mathscr{H}_j^+ : (j,1] \to \mathbb{R}_+$ that describes the colleges' actions up to j's turn. A strategy for college j is a choice of quality $q \in \mathbb{R}_+$ whenever it is its turn; abusing notation I denote it $q_j(\mathscr{H}_j^+)$. Denoting $\mathscr{H}_j : [j,1] \to \mathbb{R}_+$ the history including college j's action, one has, for all $k > j$, $\mathscr{H}_j(k) = \mathscr{H}_j^+(k)$ and $\mathscr{H}_j(j) = q_j(\mathscr{H}_j^+)$. $\mathscr{H}_0$ denotes a terminal history.

I now introduce the notion of the *set of available students* at quality q at history $\mathscr{H}_j$. Denote $S(q) \subset \mathscr{I}$ the subset of students who demand quality q and $card(S(q))$ its cardinal, similarly denote $S(q, \mathscr{H}_j)$ the subset of students demanding quality q who are not in a college yet after history $\mathscr{H}_j$ (we call it the set of available students).

The cardinality of the set of available students at each quality to colleges that play later $j' \leq j$ is a function of the positions of colleges that have already played, $j' > j$, because when college j chooses quality q it takes a subset of these students, $S(q, \mathscr{H}_j) \subset S(q, \mathscr{H}_j^+)$. More specifically, I assume that college j picks a subset of students of cardinality $\aleph_1$ (its assumed size). I further assume that at any history $\mathscr{H}_j^+$, if $card(S(q), \mathscr{H}_j^+) \leq \aleph_1$ and j chooses q, then college j takes all the students at quality q and $card(S(q, \mathscr{H}_{j'})) = 0$ for all $j' \leq j$.[46] If $card(S(q, \mathscr{H}_j^+)) > \aleph_1$, I

[45] This assumption of an order across colleges captures in a very direct way the notion that colleges do not start on an equal foot in the competition for prestige. In the real world, there are slow-moving state variables that gives an advantage to some colleges in this race, such as their reputation, their faculty member, their stock of publications, their endowment. Our assumption should be seen as a reduced-form expression of this *ex ante* hierarchy of advantages created by these state variables that this paper abstracts from.

[46] This is indeed a restriction, and not a tautology. It would be possible for a countable number of colleges to offer the same quality q and still respect the size constraint since a countable set of

assume that college j picks a subset of students of cardinality $\aleph_1$ which implies $card(S(q), \mathscr{H}_j) > \aleph_1$.[47]

Recall that the objective of the college is to deliver the highest quality possible. If they faced no constraint, they would all choose to deliver the highest quality. All colleges would like to be Harvard (or Princeton), but there is room for only one. This notion is captured by the size constraint: colleges can't be too small. Specifically, if at history $\mathscr{H}_j^+$ the set of available students at q is lower than the cardinality of the continuum, $card(S(q, \mathscr{H}_j^+)) < \aleph_1$, the payoff of college j if it chooses q is 0, and I say that college j is *not operating*.[48] If $card(S(q, \mathscr{H}_j^+)) \geq \aleph_1$, and college j chooses q, then its payoff is simply q and we say that it is *operating*.

This induces a preference relationship over the set of possible terminal histories. Consider any two terminal histories $\mathscr{H}_0, \mathscr{H}_0'$ in which college j is operating. College j prefers $\mathscr{H}_0$ to $\mathscr{H}_0'$, $\mathscr{H}_0 \succsim \mathscr{H}_0'$ if and only if $\mathscr{H}_0(j) = q_j \geq q_j' = \mathscr{H}_0'(j)$ with strict preference for strictly higher quality. A college always prefers a terminal history in which it is operating over one in which it is not.

Denote $q_{<j}^* = \{q_k^*(\mathscr{H}_k^+)\}_{k \in [0,j)}$ the strategy profile of colleges playing (strictly) after j and $\mathscr{H}_0\left(\mathscr{H}_j^+, q_j(\mathscr{H}_j^+), q_{<j}\right)$ the terminal history that follows history $\mathscr{H}_j^+$ and induced by the strategies of college j, $q_j(\mathscr{H}_j^+)$ and of the colleges playing afterwards $q_{<j}$. A subgame perfect Nash equilibrium of this game is a strategy profile $\{q_j^*(\mathscr{H}_j^+)\}_{j \in [0,1]}$ such that for all j, given the strategies of the colleges playing next $q_{<j}^*$

$$\mathscr{H}_0\left(\mathscr{H}_j^+, q_j^*(\mathscr{H}_j^+), q_{<j}^*\right) \succsim \mathscr{H}_0\left(\mathscr{H}_j^+, q, q_{<j}^*\right)$$

for all $q \in \mathbb{R}_+$.

Detail on the index set of households, $\mathscr{I}$. To be consistent with the notion that there is a continuum of colleges and a continuum of students within each college, it has to be the case that the cardinality of the set of students be strictly higher than the cardinality of the set of colleges, i.e. $card(\mathscr{I}) > card([0,1]) = \aleph_1$. It seems natural to consider the smallest such cardinal. Using the axiom of choice, such a cardinal is

set of cardinal $\aleph_1$ is still of cardinal $\aleph_1$. It is however an inconsequential restriction which allows to associate one college with one quality since in equilibrium it is true that $card(S(q)) = \aleph_1$.

[47] Although this case might arise in some other version of the model, it doesn't happen in any equilibria analyzed in this paper.

[48] The size constraint is what makes the game strategic: the positioning decisions of higher-ranked colleges influence the payoffs of lower-ranked colleges.

$\aleph_2$. To fix ideas, this corresponds for example to the index set $\mathscr{I} = [0,1]^{[0,1]}$.

Assumption 1. *The cardinal of the set of households is the same as the continuum of continua*

$$card(\mathscr{I}) = \aleph_2$$

Equilibrium Characterization. The following lemma says that the quality delivered by each college follows the same order as the order in which colleges play the game.

Lemma 5. *Assume the distribution of students over quality is continuous over* $\mathbb{R}_+$*. Then in equilibrium,*

$$q_j > q_{j'} \iff j > j'.$$

Proof. Since the distribution is continuous over $\mathbb{R}_+$ and there are a cardinal $\aleph_2$ of students, there must be a cardinal $\aleph_1$ of students demanding a given quality q, *i.e.* $card(S(q)) = \aleph_1$ for all $q \in \mathbb{R}_+$. (Otherwise there would be a mass point at some q, contradicting the assumption of a continuous distribution). Hence, by the assumption made earlier, whenever college j chooses a location q that is unoccupied $card(S(q, \mathscr{H}_j^+)) = \aleph_1$, it takes all of its students and no students is left for a college playing later, $card(S(q, \mathscr{H}_{j'})) = 0$ for all $j' \leq j$. This implies that if there exists $\underline{q}$ such that the history up to j is bounded on the left by $\underline{q}$: $\mathscr{H}_j^+((j,1]) = (\underline{q}, +\infty)$, then a college j's optimal location is $\underline{q}$: choosing strictly above $\underline{q}$ would mean not operating by the previous argument, and choosing exactly $\underline{q}$ rather than a strictly lower quality is strictly preferred. This shows that in any equilibrium in which the distribution for quality demanded is continuous over $\mathbb{R}_+$, for any $j > j'$, one has $q_{j'} < q_j$.

□

A.8 Existence and Uniqueness of Equilibrium Path

The set of equations defining an equilibrium path in proposition 4.3 is block-recursive. In particular, the law of motion of Σ_h, is independent and the path of all other variables are pinned-down by the path of Σ_h. It is therefore necessary and sufficient to focus on the existence and uniqueness of the path of Σ_h. I first define new notations:

$$\begin{aligned}
\Sigma_h'^2 &= f(\Sigma_h^2) \\
&= \left[\alpha_1^2 + \left(\frac{A}{1-\nu\omega_3}\right)^2 + \frac{2\alpha_1 A}{1-\nu\omega_3}\right]\Sigma_h^2 + \sigma_y^2 + \left[\alpha_1^2 + \frac{B^2}{(1-\nu\omega_3)^2} + \frac{2B\alpha_1}{1-\nu\omega_3}\right]\sigma_b^2
\end{aligned}$$

with $A = \alpha_1\alpha_2(\omega_2 + \tau_m(1-\tau_u)\omega_1) + \alpha_2\left(\omega_1(1-\tau_u)(1-\tau_n) - \omega_3\right)(1-\tau_y)\lambda$

$$B = \alpha_1\alpha_2\left(\omega_2 + \tau_m(1-\tau_m)\omega_1\right) \qquad \nu = \frac{C}{E\Sigma_h^{-2} + (E+\omega_3)C}$$

$$C = \left(\frac{\omega_A}{\omega_I + \omega_A}\right)^{-2}\sigma_b^{-2} \qquad E = (\omega_1(1-\tau_u)(1-\tau_n) - \omega_3) + \frac{\omega_A}{(1-\tau_y)\lambda}$$

$f(.)$ is differentiable for $\Sigma_h^2 \in (0,\infty)$ and $\lim_{\Sigma_h^2 \to 0} f(\Sigma_h^2) = \sigma_y^2 + \left[\alpha_1^2 + B^2 + 2B\alpha_1\right]\sigma_b^2 > 0$. The derivative $f'(.)$ is:

$$\begin{aligned}
&\left[\alpha_1^2 + \left(\frac{A}{1-\nu\omega_3}\right)^2 + \frac{2\alpha_1 A}{1-\nu\omega_3}\right] \\
&+ \left[\left[\left(\frac{A}{1-\nu\omega_3}\right)^2 + \frac{\alpha_1 A}{1-\nu\omega_3}\right]\Sigma_h^2 + \left[\frac{B^2}{(1-\nu\omega_3)^2} + \frac{B\alpha_1}{1-\nu\omega_3}\right]\sigma_b^2\right]\frac{2\omega_3}{1-\nu\omega_3}\frac{\partial\nu}{\partial\Sigma_h^2}
\end{aligned}$$

with $\dfrac{\partial\nu}{\partial\Sigma_h^2} = \dfrac{CE}{\left(E + \Sigma_h^2(E+\omega_3)C\right)^2}$

Hence $$\begin{aligned}
\lim_{\Sigma_h^2 \to \infty} \frac{\partial f}{\partial \Sigma_h^2} &= \left[\alpha_1^2 + \left(\frac{A}{1 - \frac{\omega_3}{E+\omega_3}}\right)^2 + \frac{2\alpha_1 A}{1 - \frac{\omega_3}{E+\omega_3}}\right] \\
&= \left[\alpha_1 + \alpha_1\alpha_2(\omega_2 + \tau_m(1-\tau_m)\omega_1) + \alpha_2[\omega_1(1-\tau_u)(1-\tau_n) - \omega_3](1-\tau_y)\lambda\right]^2
\end{aligned}$$

Therefore if $\left[\alpha_1 + \alpha_1\alpha_2(\omega_2 + \tau_m(1-\tau_m)\omega_1) + \alpha_2[\omega_1(1-\tau_u)(1-\tau_n) - \omega_3](1-\tau_y)\lambda\right]^2 < 1$, the equation $\Sigma_h^2 = f(\Sigma_h^2)$ has at least one solution since f is continuous and $\lim f(0) > 0$. Moreover, it has to be that an odd number of these solutions are characterized by $f'(\Sigma_h) < 1$, which guarantees local stability of the equilibrium path around these solutions.

Let's now show that the equilibrium path is unique for ω_3 small enough. A first order approximation of f in the neighborhood of $\omega_3 = 0$ is

$$
\begin{aligned}
f(\Sigma_h^2) \simeq & \left[\alpha_1^2 + A^2 + 2\alpha_1 A\right] \Sigma_h^2 \\
& + \sigma_y^2 + \left[\alpha_1^2 + B^2 + 2B\alpha_1\right] \sigma_b^2 + \left[\left[A^2 + \alpha_1 A\right] \Sigma_h^2 + \left[B^2 + \alpha_1 B\right] \sigma_b^2\right] 2\nu\omega_3 \\
f'(\Sigma_h^2) \simeq & \left[\alpha_1^2 + A^2 + 2\alpha_1 A\right] \\
& + \underbrace{\left(\left[\left[A^2 + \alpha_1 A\right] \Sigma_h^2 + \left[B^2 + \alpha_1 B\right] \sigma_b^2\right] \frac{E}{E + EC\Sigma_h^2} + \left[A^2 + \alpha_1 A\right] \Sigma_h^2\right) \frac{C}{E + EC\Sigma_h^2} 2\,\omega_3}_{F(\Sigma_h^2)}
\end{aligned}
$$

with$\nu = \frac{C}{E\Sigma_h^{-2} + EC}$. Since I have assumed that $[\alpha_1^2 + A^2 + 2\alpha_1 A] < 1$, and $F(\Sigma_h^2)$ is bounded for $\Sigma_h^2 \in (0, \infty)$, there exists an ω_3 small enough such that for all Σ_h^2, $\frac{\partial f(\Sigma_h^2)}{\partial \Sigma_h^2} < 1$. This is sufficient for the existence and uniqueness of a globally stable steady-state.

A.9 Rise in returns to human capital

The total derivative of the IGE with respect to λ is given by

$$
\begin{aligned}
& \left[\frac{\partial \nu}{\partial \lambda} + \frac{\partial \nu}{\partial \Sigma_h^2}\frac{\partial \Sigma_h^2}{\partial \lambda}\right] \left[\alpha_1\alpha_2\left(\frac{\partial \varepsilon_2}{\partial \nu} + \tau_m \frac{\partial \varepsilon_1}{\partial \nu}\right) + \alpha_2\left(\frac{\partial \varepsilon_1}{\partial \nu}(1 - \tau_n) - \frac{\partial \varepsilon_3}{\partial \nu}\right)(1 - \tau_y)\lambda\right] \\
& + \alpha_2(\varepsilon_1(1 - \tau_n) - \varepsilon_3)(1 - \tau_y)
\end{aligned}
$$

I then compute the derivatives:

$$
\frac{\partial \varepsilon_l}{\partial \nu} = \varepsilon_l \varepsilon_3 > 0 \qquad \frac{\partial \nu}{\partial \Sigma_h^2} = \frac{CE}{\left(E + (E + \omega_3)C\Sigma_h^2\right)^2} > 0
$$

with C and E have been defined in the proof of existence and uniqueness.

$$
\frac{\partial \nu}{\partial \lambda} = \frac{2C\left(\frac{\omega_A}{\omega_A + \omega_I}\right)\frac{1}{\omega_I}\left[E\Sigma_h^{-2} + \omega_3 C\right] + C\frac{\omega_A}{(1 - \tau_y)\lambda^2}}{\left(E\Sigma_h^{-2} + (E + \omega_3)C\right)^2} > 0
$$

$$
\frac{\partial X_2}{\partial \lambda} = \sigma_b^2 \alpha_1 \left(1 + \alpha_2(\varepsilon_2 + \tau_m \varepsilon_1)\right) \alpha_1\alpha_2\varepsilon_3(\varepsilon_2 + \tau_m\varepsilon_1)\varepsilon_1 \frac{\partial \nu}{\partial \lambda} > 0
$$

$$
\frac{\partial \Sigma_h^2}{\partial \lambda} = \frac{\frac{\partial X_2}{\partial \lambda} + \Sigma_h^2 2 \frac{\partial \alpha_h}{\partial \lambda}\alpha_h}{1 - (\alpha_h)^2 - \Sigma_h^2 2\frac{\partial \alpha_h}{\partial \Sigma_h^2}\alpha_h - \frac{\partial X_2}{\partial \Sigma_h^2}} > 0
$$

where $\frac{\partial \alpha_h}{\partial \lambda}$ has to be understood as the partial derivative of α_h w.r.t. λ keeping Σ_h^2 constant. The last line stems from the fact that the steady-state is locally stable - which requires that $1-(\alpha_h)^2-\Sigma_h^2 2\frac{\partial \alpha_h}{\partial \Sigma_h^2}\alpha_h-\frac{\partial X_2}{\partial \Sigma_h^2}=\frac{\partial (\Sigma_h')^2}{\partial (\Sigma_h)^2}>0$. Hence, putting everything together yields

$$\frac{\partial \alpha_h}{\partial \lambda}=\left[\underbrace{\frac{\partial \nu}{\partial \lambda}+\frac{\partial \nu}{\partial \Sigma_h^2}\frac{\partial \Sigma_h^2}{\partial \lambda}}_{>0}\right]\varepsilon_3[\alpha_1\alpha_2\left(\varepsilon_2+\tau_m\varepsilon_1\right)$$
$$+\alpha_2\left(\varepsilon_1(1-\tau_n)-\varepsilon_3\right)(1-\tau_y)\lambda]+\alpha_2(\varepsilon_1(1-\tau_n)-\varepsilon_3)(1-\tau_y)>0.$$

This proves not only that the steady-state IGE is increasing in λ but that the variance of human capital in the economy is as well. Given that the variance of market income is given by $\lambda^2\Sigma_h^2$ it is immediate that it increases too. Turning to the private spending on higher education, given by s, it is immediate to see from the expressions (14) and (16) that it is increasing in the future path of λ. Let's now turn to the ratio of within college variance of (log) parental income over economy-wide variance of (log) income:

$$\frac{V(\ln y|q)}{V(\ln y)}=\frac{1}{\lambda^2\Sigma_h^2}\lambda^2\frac{\Sigma_h^2\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2}{\Sigma_h^2+\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2}=\frac{\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2}{\Sigma_h^2+\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2}$$
$$\Rightarrow \frac{\partial \frac{V(\ln y|q)}{V(\ln y)}}{\partial \lambda}=\frac{\sigma_b^2\frac{\partial B}{\partial \lambda}\left[\Sigma_h^2+\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2\right]-B\sigma_b^2\left[\frac{\partial \Sigma_h^2}{\partial \lambda}+\sigma_b^2\frac{\partial B}{\partial \lambda}\right]}{\left[\Sigma_h^2+\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2\right]^2}=\frac{\sigma_b^2\frac{\partial B}{\partial \lambda}\Sigma_h^2-B\sigma_b^2\frac{\partial \Sigma_h^2}{\partial \lambda}}{\left[\Sigma_h^2+\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2\sigma_b^2\right]^2}<0$$

with $B=\left(\frac{\varepsilon_A}{\varepsilon_I+\varepsilon_A}\right)^2$ and since $\frac{\partial \Sigma_h^2}{\partial \lambda}>0$ and $\frac{\partial B}{\partial \lambda}<0$.

Finally the variance of (log) college quality is given by $\varepsilon_A^2\sigma_b^2+(\varepsilon_I+\varepsilon_A)^2\Sigma_h^2$. It is immediate that it increases with λ since $\varepsilon_I,\varepsilon_A,\Sigma_h^2$ increase with λ.

Monotonic transition path. From the law of motion of Σ_h^2, in the first period the initial increase in λ raises α_h and triggers the initial increase in the dispersion of human capital. Since $X_2(\Sigma_h)$ and $\alpha_h(\Sigma_h)$ are both increasing in Σ_h it further increases Σ_h^2 at the following period and so on... This establishes that Σ_h^2 is strictly increasing over the transition path. This also establishes the monotonic increase in α_h and all ω's.

Turning to the private spending on higher education, given by s, it is easy to see that it is increasing in the future path of λ, α_h and ε_1. Since these three variables are increasing over the transition path, s also increases. The variance of log college quality is also increasing because $\varepsilon_I, \varepsilon_A, \Sigma_h^2$ are increasing over the transition path. The ratio of within college variance of (log) parental income over economy-wide variance of (log) income will decrease monotonically over the transition path because of the initial increase in λ, this is the first term in the derivative $\sigma_b^2 \frac{\partial B}{\partial \lambda} \Sigma_h^2$, and then decreases further as Σ_h increases, this is the second term $B\sigma_b^2 \frac{\partial \Sigma_h^2}{\partial \lambda}$.

B Estimation - Details

B.1 The College Problem in the Quantitative Version

For future reference, I call M1 the benchmark model explored in section 2 and 3 and M2 the model with intergenerational financial transfers and enrollment decision.

In order to keep the college problem tractable despite the loss of closed-form expressions for the distribution of students within the college and equilibrium tuition, I assume that the problem of the college is still given by (41). While it is not possible to derive (41) from the primitive problem (8), the following re-interpretation gives support to the reduced-form formulation: there is a loss in the efficiency with which resources are used when the inequality of tuition fees among students rises. One can interpret it as a rise in human resources and administrative costs or as an increase in the sentiment of unfairness among students. The first order conditions for this problem are the same as in M1, see appendix A.2 for more details.

B.2 External Calibration

Income Tax Schedule a_y, τ_y. In order to calibrate τ_y, I take an average between the value estimated by Heathcote, Storesletten, and Violante (2017) and the ones needed to match the ratio between the market income and after tax and transfers Gini in the U.S in 2000, $\tau_y = .23$. The latter estimate τ_y directly using two different datasets over the period 2000-2005: using the data provided by the CBO—itself based on the SOI and the CPS— they obtain $\tau_y = .2$ and using the PSID, they find $\tau_y = .18$. The value needed to rationalize the .12 difference in the Gini coefficient of households market income (.56) and after tax and transfers income (.44) in 2000-2005, within the log-normal framework of M1 is .26.[49] The discrepancy is due on the one hand to the log-normal assumption and on the other to the slightly different different measure of income used—for example, Heathcote, Storesletten, and Violante (2017) exclude Medicare benefits from their measure of transfers. I calibrate a_y using average income tax rate provided by CBO: $a_y = .2$.

College Subsidies, a_h, a_u, τ_u. From the IPEDS, I compute a_u by dividing the total amount of public subsidies by the total revenues before public aid. From the NPSAS I obtain a_h by dividing the total amount of public financial aid by the sum of out of pocket payments. According to the specification for subsidies to university, τ_u can be estimated in a weighted least-square regression of (log) total revenues per student on (log) revenues before public transfers in the cross-section of colleges, where the weights are given by students enrollment. I run this regression in a companion paper Capelle (2019) and find $\tau_u = .35$ at the beginning of the 2000s.

Frish Elasticity, η. η is set to match the Frisch elasticity of labor supply, $\varepsilon_{\ell,w} = \frac{1}{\frac{\eta}{\mu}-1}$. Empirical estimates of $\varepsilon_{\ell,w}$ range from .2 to .7 (Chetty, Guren, Manoli, and Weber, 2011). I explain in the next paragraph why my preferred estimate in the conservative value $\varepsilon_{\ell,w} = .2$. This implies $\eta = \mu\left(1 + \frac{1}{.2}\right) = 2$. Ideally we would have elasticities of lifetime household income to wages in order to be consistent with the model. Estimates however are at the individual and yearly level. I argue that they are likely upper bounds for two reasons. First they do not capture intra-household substitution. Second they do take into account the intertemporal substitution stemming from temporary fluctuations in wages.

Calibrating $\bar{\lambda}$*, and* μ. The returns to human capital in the educational services sector is set to $\bar{\lambda} = \lambda$ so that the price of educational services is given by $p_I = \frac{A_I}{A}$. I calibrate $\mu = 1$.

B.3 Internal Calibration

General Strategy. The algorithm used to estimate the parameters is akin to a Simulated Method of Moments. If $\bar{z}$ denotes the vector of the twelve parameters to be estimated, $M(\bar{z})$ the vector of model-generated moments, $\hat{M}(\bar{z})$ one such realization and $\hat{m}$ the vector of empirical moments, one seeks to find $\hat{\bar{z}}$ such that $\hat{\bar{z}} = \arg\min_{\bar{z}} \left[\hat{m} - \hat{M}(\bar{z})\right]' W \left[\hat{m} - \hat{M}(\bar{z})\right]$ with $E\left[\hat{M}(\bar{z})\right] = M(\bar{z})$ and W a weighting

[49] CBO (The Distribution of Household Income and Federal Taxes, 2013, CBO and own calculations

matrix. I depart from this standard expression in two ways. First the empirical moments themselves $\hat{m}$ are function of a subset of parameters, $\bar{z}_1 = \{\alpha_1, \omega_2\}$. Many of the targeted moments are coefficients in a regression involving one of two variables that need to be constructed using $\bar{z}_1$: students ability z and college quality q. Secondly, as I have explained earlier, the procedure to generate the student ability variable, based on test scores, isn't a deterministic function of the observables and the parameters but involves some randomness. Our estimation procedure actually takes the following form:

$$\hat{\bar{z}} = \arg\min_{\bar{z}} \left[\hat{m}(\bar{z}_1, \varepsilon) - \hat{M}(\bar{z})\right]' W \left[\hat{m}(\bar{z}_1, \varepsilon) - \hat{M}(\bar{z})\right]$$

where ε denotes the noise introduced in the process of constructing the student ability variable.

Despite the large dimension of the parameter space, the algorithm is quick and the global minimum to the loss function can be easily and with certainty found. I proceed in two steps. First I estimate M1, the version of the model without outside option and financial asset. The closed-form solutions to the model enables me to compute the exact value for the moments—$\hat{M}(\bar{z}) = M(\bar{z})$—and to run the estimation on a fine and large grid. This in turn allows me to check numerically that the parameters are well identified, in the sense that the loss function is steep at the global minimum, which I do in B.4. The estimates are reported in the third column of table 1. I then estimate the augmented version M2, the version of the model with an outside option and a financial asset. When estimating the richer quantitative version, I use the estimates from the first step as initial values. The key assumption here is that the parameters estimated with M1 are not too far from the true parameters in M2. It turns out that almost all estimates but one, ω_2, are identical to their counterpart in M1.

Assumption of Steady-State. I assume that the economy is at steady-state at the beginning of the 2000s. The assumption of steady-state really matters only for the identification of σ_y^2. Moreover, the endogenous convergence speed is quite fast—the half life of the AR(1) for the variance of income is given by $-\ln(2)/\ln(\alpha_h^2) \simeq= .5$, which corresponds to half a generation. But one could still be concerned that the movement of exogenous variables—especially persistent increase in λ—have not reached a steady-state. However, the future increase in λ feedbacks into the current allocation only through the aggregate investment rate into higher education—at least in the version of the model for which I have closed-forms—and through labor supply, so would not change the cross-sectional and inequality implications, on which most of the identification is based.

Identification of σ_b^2 and Construction of Model-Consistent Abilities. At this point of the procedure, I construct a grid on α_1. Assume the relationship between children' high school abilities and parental human capital given by equation (56) is true. In NLSY97, I observe gross parental income $y_{m,i}$ and the ranking in test scores $\text{rank}(z_i)$. For a given (α_1, λ), there exists a unique σ_b^2 that matches the correlation between parental income and the rank of the child at the test, $\rho(y_{m,i}, \text{rank}(z_i))$. This identifies $\sigma_b^2(\alpha_1)$.

I can then construct a model-consistent measure of abilities $\{\ln z_i\}(\alpha_1, \varepsilon)$ that is consistent with $(\alpha_1, \lambda, \sigma_b^2(\alpha_1))$ and $\text{rank}(z_i)$ to use in subsequent steps. This step implies a random draw that introduces some noise in the measure of ability, i.e. $\{\ln z_i\}$ are not deterministic function of $\alpha_1, \lambda, \sigma_b^2(\alpha_1)$ and $\text{rank}(z_i)$.

The Progressivity of Financial Aid, τ_n, τ_m and ω_3. In M1, the government financial aid and the equilibrium tuition schedule are given by (57) and (58). In the NCES-NPSAS dataset, one observes parental income $y_{m,i}$, test score, institutional aid, government aid as well as out of pocket payment. Regressing the (log) ratio of after-government aid payment on before-government aid payment over parental income and student ability gives τ_n and τ_m.

The second equation relates between before-government aid payment and college fixed effect, parental income and student ability. It tells us that the elasticity of before-government aid tuition to parental income $\varepsilon_{e,y_m}(\alpha_1)$ identifies the progressivity of institutional financial aid ω_3, what I called the social objective parameter. In the data I find that the fit of the second equation to the data is very high, $R^2 = 80\%$. In the model M2, I run the second regression on a simulated population.

Constructing a Measure of Quality, q_i. At this point of the procedure, I need to define a grid on $\frac{\omega_2}{\omega_1}$. At each point of the grid $(\alpha_1, \frac{\omega_2}{\omega_1})$, I construct a measure of annual quality delivered by all colleges, indexed by j. I collect average real spending per student by college using IPEDS and median test score within a college. The median test score is defined as the mean between the bottom and top quartile—the only available data in the IPEDS—which is exactly the median if the distribution is symmetric. When test scores data are not available for 2000, I either use years up to 2004, or impute them based on a regression of test scores on spending per student and other characteristics. This measure based on ACT or SAT is converted into a model-consistent measure of abilities using a scale based on publicly available quantiles of the distribution of these scores and using the guess on $\frac{\omega_2}{\omega_1}$. I construct the annual quality delivered by a college consistent with the functional form for the production function given by 9 and the guess for $\frac{\omega_2}{\omega_1}$.[50]

I then construct a measure of quality received by an individual i. q_i is a weighted average of the q_j where the weights depend on the time spent in each college and whether they have graduated at any point. I have checked the robustness of the results to alternative ways of aggregating annual college qualities.

Estimating the Elasticity of Quality to Ability, $\varepsilon_{q,z}\left(\alpha_1, \frac{\omega_2}{\omega_1}\right)$ *and Identifying* $\frac{\omega_2}{\omega_1}$*.*

According to the equilibrium sorting rule in M1, the elasticity of quality to ability identifies $\frac{\omega_2}{\omega_1}$ as can be seen from (59). Since the measure of quality is based on a guess on $\frac{\omega_2}{\omega_1}$, finding $\frac{\omega_2}{\omega_1}$ is a fixed point problem. Empirically, the elasticity of quality to ability doesn't change a lot with the guess on $\frac{\omega_2}{\omega_1}$, the latter is therefore tightly identified. Although this relationship should hold perfectly in the model, the R^2 in the data associated with this regression is 23%. Finally notice that even in M1, it is not possible to recover $\frac{\omega_2}{\omega_1}$ directly from the elasticity of quality to abilities without numerically solving the model, since it depends on the endogenous variable, $h(\Sigma_h)$.

The Human Capital Accumulation Function $\alpha_2, \alpha_3, \lambda$*.* According to the law of motion for human capital $\ln y'_{m,i} = c_y + \lambda \ln z_i + \alpha_2 \omega_1 \lambda \ln q_i + \alpha_3 \ln y_{m,i} + \ln \xi_{y,i}$ running a regression of children' income on parental income, abilities and quality identifies α_2 and α_3 and λ. I run the regression using the NLSY97, where I observe parental income, children ability, children college quality and children earnings. At this point, I gather all the moments that depend on $\left(\alpha_1, \frac{\omega_2}{\omega_1}\right)$, $\left(\sigma_b^2 \quad \tau_n \quad \tau_m \quad \varepsilon_{e,y_m} \quad \varepsilon_{q,z} \quad \alpha_2 \quad \alpha_3 \quad \lambda\right)$

The remaining targeted moments are independent of $\left(\alpha_1, \frac{\omega_2}{\omega_1}\right)$.

Using the Gini Coefficient for Income to Identify σ_y^2*.* I target the Gini coefficient for income. In M1, the Gini is given by equation (62) and (63). The best estimate for the Gini coefficient of lifetime labor earnings is from Kopczuk, Saez, and Song (2010) who have access to administrative data. There would be two issues with the NLSY97: first children labor earnings are observed only up to 2015, *i.e.* in their first years of labor market experience and a lot of them are not in a households yet. Secondly,

[50] I do not need to take into account the within college heterogeneity, corresponding to σ_u^2 because, at least in the model, it is common to all colleges and will therefore factor out and leave our regression coefficients unchanged.

top income are censored. Kopczuk, Saez, and Song (2010) finds that the eleven-year Gini coefficient is between .45 and .50. This is slightly lower than the annual Gini coefficient, which is between .49 and .57—depending on the exact measure of gross income used—in 2000 according to the CBO, probably because of transitory income shock. I keep a Gini of lifetime labor earnings of .45 as a target.

The Intergenerational Elasticity identifies α_1. From the steady-state equilibrium law of motion of human capital in M1, the children income elasticity to their parents income is the IGE: $\ln y'_{m,i} = c + \alpha_h \ln y_{m,i} + \varepsilon_i$ The IGE directly informs α_1. There is disagreement in the literature regarding the magnitude of the IGE, with estimates ranging from .3 to .6. Even the recent literature that uses administrative data is not immune to the short-panel and lifecycle biases.[51] I take the intergenerational elasticity from Mazumder (2015) who provides the most robust estimates, $\alpha_h = .5$.

β *and the Share of Private Spending for Higher Education in GDP.* I calibrate the intergenerational discount factor, β, to match the average private spending on higher education in GDP. In M1, the latter is given by equation (64). The OECD reports that share of private spending for higher education in GDP in the U.S. over the period 2000-2004 is 1.3%.[52]

Calibrating $\underline{q}$ *and the enrollment rate.* To calibrate $\underline{q}$—the outside option to college—it is natural to target the enrollment rate: the lower $\underline{q}$, the stronger the incentives to go to college. The immediate enrollment rate, provided by the NCES, in the U.S. in the 2000s is about 70%.

Calibrating limits to intergenerational financial transfers, $\underline{a}, \bar{a}$. There is no limit to how much individuals can bequeath, $\bar{a} = +\infty$. For $\underline{a}$, I target the official limit on student loans, as a percentage of lifetime GDP per capita, which amounts to 3%.

r *and the income-sorting channel.* First I consider a small open economy, and do not try to find the interest rate that ensures market clearing of the financial asset market. Instead, I target the elasticity of quality to parental income. It turns out that in M1, the elasticity resulting from the estimation is too high compared to what is in the data—.4 instead of .2. By increasing r, one gives incentives to individuals to avoid debt, which relaxes the borrowing constraint and decreases the dependence

[51] Given that only the first years of the children earnings are observed while the parents are observed when they are already older, the estimate of the intergenerational elasticity from the NLSY97 is biased downward—I indeed find $\hat{\alpha}^h = .3$.

[52] For reference, they also report that the share of public spending is 1%, making spending in higher education 2.3% of GDP.

of college quality to parental income. I find a generational net interest rate of 180% which corresponds to an annual interest rate of 3.5% for a generation length $H = 30$ years.

B.4 Analytical Evidence of Identification in M1

Lemma 6. *In M1, given our set of targets (detailed in column 4 of table (1)), if $\omega_3 = 0$ and $\{z_i\}$ are observables, there exists a unique set of parameters $\bar{z}_1$ consistent with the model-implied restrictions.*

From equations (57) and (58) one gets $\tau_n, \tau_m, \omega_3/\omega_1$. From the coefficient in front of z in (59), one identifies ω_3/ω_1. From (60), one identifies $\alpha_2\omega_1$, and α_3. The IGE is obtained with (61) (although I use another, more reliable source, as explained in the main text), which together with the previously obtained $\alpha_2\omega_1, \alpha_3, \omega_2/\omega_1, \omega_3/\omega_1, \tau_u, \tau_n\, tau^m, \tau_y, \lambda$ gives α_1. From equation (56) one gets σ_b^2. The computation of the steady-state Gini identifies σ_y^2 (equation (62) and (63)). Finally, targeting the LHS of (64) and computing the steady-state value of V gives us β.

The assumption $\omega_3 = 0$ implies $h(\Sigma_h) = h$ in equation (59). When ω_3, the set of restrictions for the estimation is given by

$$\ln z_i = \ln(\xi_{b,i} h_i)^{\alpha_1} = \frac{\alpha_1}{\lambda} \ln y_{m,i} + \alpha_1 \ln \xi_{b,i} \quad \text{with } V(\ln \xi_{b,i}) = \sigma_b^2 \tag{56}$$

$$\ln \frac{e_i^h}{e_i^u} = \tau_n (1-\tau_y) \ln y_{m,i} - \tau_m \ln z_i + c_0 \tag{57}$$

$$\ln e_{i,j}^h = \gamma_j + \left(\frac{\omega_3}{\omega_1}(1-\tau_u) + \tau_n\right)(1-\tau_y) \ln y_{m,i} - \left(\frac{\omega_2}{\omega_1(1-\tau_u)} + \tau_m\right) \ln z_i + c_1 \tag{58}$$

$$\ln q_i = c + h\left[\left((1-\tau_u)(1-\tau_n) - \frac{\omega_3}{\omega_1}\right)(1-\tau_y) \ln y_{m,i} + \left(\frac{\omega_2}{\omega_1} + \tau_m (1-\tau_u)\right) \ln z_i\right] \tag{59}$$

$$\ln y'_{m,i} = c_y + \lambda \ln z_i + \alpha_2 \omega_1 \lambda \ln q_i + \alpha_3 \ln y_{m,i} + \ln \xi_{y,i} \tag{60}$$

$$\ln y'_{m,i} = c + \alpha_h \ln y_{m,i} + \varepsilon_i \tag{61}$$

$$\text{Gini}(y_m) = 2\Phi\left(\lambda \sqrt{\frac{(\Sigma_h^2)^{SS}}{2}}\right) - 1 \tag{62}$$

$$\left(\Sigma_h^2\right)^{SS} = \frac{\sigma_y^2 + \left(\alpha_1 [1 + \alpha_2 (\varepsilon_2(\Sigma_h^{SS}) + \tau_m (1-\tau_u) \varepsilon_1 (\Sigma_h^{SS}))]\right)^2 \sigma_b^2}{1 - (\alpha_h)^2} \tag{63}$$

$$s(1-a_y) = \frac{\beta \alpha_2 \omega_1 (1-\tau_u) V (1-a_y)}{1 - \beta + \beta \alpha_2 \omega_1 (1-\tau_u) V} \tag{64}$$

C Counterfactuals: Details

Policy Counterfactual: Random Allocation In this appendix, I provide details regarding the computation of the counterfactual with random allocation of students. All individuals receive the same amount of college quality $\bar{q}$, so that the law of accumulation of human capital is: $h' = z\bar{q}^{\alpha_2} h^{\alpha_3}$ where $\bar{q}$ is consistent with the average child ability in the economy and the level of spending per student: $\bar{q} = (\bar{s} \times \text{Disp Income})^{\omega_1} \bar{z}^{\omega_2}$with $\ln \bar{z} = E[\ln z]$ where $\bar{s}$ denotes the aggregate spending rate for higher education and "Disp Income" is the average disposable income in the economy. Given that private agents stop spending on higher education in this counterfactual, all resources have to be financed through taxes and transfers to colleges. I set the proportion of disposable income going to higher education, $\bar{s}$, equal to what it was in the *status quo* equilibrium, *i.e.* $\bar{s} = s(1+a_u)(1+a_h)$. Although inconsequential for mobility and inequality, the choice of $\bar{s}$ has a first order effect on the steady-state level of GDP. One can easily show the laws of motion for the mean and the variance are given by $m' = \rho m + \ln \kappa - \frac{\sigma_y^2}{2} - \alpha_1 \left(1 + \alpha_2 \omega_2\right) \frac{\sigma_b^2}{2} + \alpha_2 \omega_1 \left[\ln(\bar{s} A (1 - a_y) \ell^{\mu} + \frac{\lambda^2}{2} \Sigma_h^2\right]$ and $\Sigma_h^{2'} = (\alpha_1 + \alpha_3)^2 \Sigma_h^2 + \sigma_y^2 + \alpha_1^2 \sigma_b^2$. These two moments are sufficient to compute the income Gini and GDP.

Counterfactuals with respect to the returns to education Changing λ in the first counterfactual changes not only the returns to education but also the level of output for a given distribution of human capital. Although it has no impact on inequality and mobility—our two measures of interest—in M1, this change in the level of GDP matters in M2 through the enrollment choice. A lower λ decreases GDP which results in a decrease in the enrollment rate for a given $\underline{q}$. To address this issue, I consider three possible assumptions: i) Keep $\underline{q}$ unchanged, ii) Adjusting $\underline{q}$ to target an enrollment rate of 50% which corresponds to its level in 1980, iii) Adjusting $\underline{q}$ to target an enrollment rate of 70% which corresponds to its level in the original calibration. The results for the main variables of interest are reported in table (A1):

Table A1: Change in s, Gini coefficient and IGE in counterfactuals

	Assumption	**Enrollment**	**s**	**Gini $\mathbf{y_m}$**	**IGE**	**Endog. Amplif. of Gini $\mathbf{y_m}$**
$\lambda = .\mathbf{67}$		70%	1.3%	.45	.5	-
	1	5%	-92%	-23%	-14%	+ 35%
$\lambda = .\mathbf{55}$	2	50%	-36%	-17%	-2.8%	+ 6.3%
	3	70%	-18%	-16%	-.6%	+ 2.4%

Legend: The three assumptions are as follows: 1- Keep $\underline{q}$ unchanged. 2- Adjusting $\underline{q}$ to target an enrollment rate of 50% which corresponds to its level in 1980. 3- Adjusting $\underline{q}$ to target an enrollment rate of 70% which corresponds to its level in the original calibration.

Fixing Tuition and Spending within Counterfactual 1. Start from the price schedule in equilibrium faced by HH in 1980: $e(q, z, y) = z^{-\tau_m} y^{\tau_n} \frac{T_e}{(1+a_h)} \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_1}} z^{-\frac{\varepsilon_2}{\varepsilon_1}} \left(\frac{y}{\kappa_2}\right)^{\frac{\varepsilon_3}{\varepsilon_1}} \right)^{\frac{1}{1-\tau_u}}$. Consider the marginal distributions of high school ability and income, denoted $F_{z,1980}(z)$,$F_{y,1980}(y)$. As well as the distribution of colleges quality $F_{q,1980}(q)$. Denote $F^{-1}_{hs_s,1980}(.)$, $F^{-1}_{y,1980}(.)$, $F^{-1}_{q,1980}(.)$ the respective quantile function. The object I fix is the following function

$$e(\mathrm{rk}_q, \mathrm{rk}_z, \mathrm{rk}_y) = C \left(F^{-1}_{q,1980}(\mathrm{rk}_q)\right)^{\frac{1}{\varepsilon_1(1-\tau_u)}} \left(F^{-1}_{z,1980}(\mathrm{rk}_z)\right)^{-\frac{\varepsilon_2}{\varepsilon_1(1-\tau_u)} - \tau_m} \left(F^{-1}_{y,1980}(\mathrm{rk}_y)\right)^{\frac{\varepsilon_3}{\varepsilon_1(1-\tau_u)} + \tau_n}$$

where rk (rank) denotes the quantile in their respective distribution and $C = \frac{T_e}{(1+a_h)}\left(\frac{p_I}{(1+a_u)T_u}\left(\frac{1}{\kappa_2}\right)^{\frac{\varepsilon_3}{\varepsilon_1}}\right)^{\frac{1}{1-\tau_u}}$ as well as $\varepsilon_1, \varepsilon_2, \varepsilon_3, \tau_u$ are fixed at their 1980 value. In other words, I fix the rank of a college a household gets into.

From the constant spending rate across all households at a given time t, I get the following relationship:

$$sy = C\left(F^{-1}_{q,1980}(\mathrm{rk}_q)\right)^{\frac{1}{\varepsilon_1(1-\tau_u)}}\left(F^{-1}_{z,1980}(\mathrm{rk}_z)\right)^{-\frac{\varepsilon_2}{\varepsilon_1(1-\tau_u)}-\tau_m}\left(F^{-1}_{y,1980}(\mathrm{rk}_y)\right)^{\frac{\varepsilon_3}{\varepsilon_1(1-\tau_u)}+\tau_n}$$

$$\mathrm{rk}_q = F_{q,1980}\left[\left(\frac{sy}{C}\right)^{\varepsilon_1(1-\tau_u)}\left(F^{-1}_{z,1980}(\mathrm{rk}(z))\right)^{\varepsilon_2+\tau_m\varepsilon_1(1-\tau_u)}\left(F^{-1}_{y,1980}(\mathrm{rk}(y))\right)^{-\varepsilon_3-(1-\tau_u)\varepsilon_1\tau_n}\right]$$

In the counter-factual I fix the spending rate to the one in the final steady-state of the benchmark economy. This identifies the partial effect of colleges' reactions to market forces keeping the policy rules of families constant. The next step consists in mapping rk_q to an actual quality. Here I do two experiments. In the first, the level of quality remains constant at what it was in 1980, and in the second I allow for change in peer effects but not in spending. For this, I create a grid on $\mathrm{rk}_q = [0,1]$, put people in bins according to their choice of rank, and take the geometric average. This gives us a mapping $\hat{\hat{z}}(\mathrm{rk}_q) : [0,1] \to \mathbb{R}^+$. I then combine it with our fixed mapping of investment per student to get quality q.