Monetary Policy and Wage Inequality: the Labour Mobility Channel*

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Abstract

In times of rising inequality and widespread resignation we study the distributional consequences of monetary policy through labour mobility. We estimate the impact of monetary policy shocks with CPS data and quantile regressions on local projection specifications. We find that contractionary policy reduce wage inequality by disproportionally increasing the separation for bottom earners, by increasing the wages of bottom earners that remain in the market and by lowering those of top earners. This speaks of a *selection effect* through reallocation. Next, we build a monetary model with uninsurable risk, agents heterogeneous in income risk, talents and wealth and in which participation and occupational allocation decisions take place through a period-by-period discrete choice optimization on value functions across occupations. The key novel transmission runs through the dependence of the transition probabilities on wealth and income. Their decline, following a tightening, increases separation and reduces re-employment probabilities, more so for bottom earners. Their exit results in a rise of wages for the bottom earners that remain in the market. This, coupled with the fact that model-based regressions replicate the empirical counterparts, confirm the selection through reallocation through the lens of the model.

Keywords: Occupational Mobility, Labour Participation, Heterogeneous Agents, Uninsurable Risk, Heterogenous Skills, Wage and Wealth Inequality, Discrete Choice Optimization, Separation, Bottom Earners. JEL: E5, J22, J23, J31, J62, E21, D31.

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1. Introduction

The rise in wage inequality has sparked interest in the distributional consequences of monetary policy. An expanding empirical literature is documenting the role of monetary policy for wealth inequality (see Doepke and Schneider (2006) and Coibion, Gorodnichenko, Kueng and Silvia (2017)). Influential theoretical work (see Auclert (2019) among others referenced further below) has examined transmission of monetary shocks to the distribution of wealth through the saving elasticity using Bewley (1980)-Aiyagari (1994)-style models with nominal rigidities. Most of this work, however, considers settings with exogenous income earnings, thereby studying the distributional consequences of monetary policy for wealth inequality rather than for wage inequality. Wealth and wage inequality have however increased in tandem in recent years. Beyond that, an essential part of the monetary transmission is through the labour market and especially occupational mobility, as has long been recognized (see Keynes (1936)).[1] This is even more so in times of rising polarization. Changes in monetary policy can affect marginal incentives of workers to participate in the labour market and to shift across occupations, both through direct effects on the labour supply elasticities and through indirect effects via changes in wealth. Reallocation of workers in turn affects wage inequality. Those mechanisms have become even more remarkable after the recent recessions. All indeed have induced large reallocation and have marked the sharp rise of the Great Resignation, a *en masse* quit from the labour market. The question on how monetary policy can tame adverse effects on this front or foster positive reallocation is therefore very topical. Our work, developed through both an empirical analysis and a theoretical quantitative model, shows indeed that a significant impact of monetary policy on wage inequality runs through occupational mobility, particularly so through separation and the reallocation incentives of bottom earners.

We start our investigation with an empirical analysis that employs CPS data to study the impact of high-frequency identified monetary policy shocks, also interacted with income percentiles, on wage inequality and transition probabilities. In there we find that contrac-

1 For work on the role of labour reallocation in the data see Davis, Faberman and Haltiwanger (2006). For earlier work on the role of worker turnover see Murphy and Topel (1987) or Blanchard, Diamond, Hall and Murphy (1990).

tionary monetary policy reduces inequality by disproportionally rising the separation rates of bottom earners, who exit the labour market making the bottom tail of wage distribution thinner, and by consequently rising the wages of stayers due to a selection through reallocation.[2] On the theory side, the study of the monetary transmission through labour market reallocation requires a major departure even from the most modern version of the Bewley (1980)-Aiyagari (1994)-style model with nominal rigidities, which consists in having earnings being endogenously determined by the equilibrium in various occupational islands. This in turn requires modeling the choice of heterogenous workers of participating in the labour markets and shifting across occupations, conditional on their wealth. We embark into this task by augmenting a model with uninsurable income risk and heterogenous wealth with a period-by-period discrete choice dynamic optimization across occupations.

Our study starts with an empirical analysis with data from the Current Population Survey (CPS) for the United States over the period 1989-2019[3] to estimate the impact of high frequency identified monetary policy shocks (see Gürkaynak, Sack and Swanson (2004) or Gorodnichenko and Weber (2016) among others) through local projection methods on both wage inequality[4]and transition probabilities. We find that a monetary tightening reduces the dispersion of changes in both, real and nominal, wages[5] across quintiles, hence wage inequality. To inspect the mechanism we interact the monetary policy shock with income percentiles and find that wages of bottom earners rise, while those of top earners decline. To assess the role of labour market reallocation we employ the same specification, but with transition probabilities as dependent variables. We find little and insignificant effects on finding rates, while strong and significant effects on separations, which decline disproportionately more for bottom earners. Those results taken together speak of a *selection through reallocation* channel. Bottom earners exit the labour market by more and have lower re-employment probabilities: this results, first and foremost, in a thinning of the bottom tail of the wage

2 Recent studies on the role of selection through reallocation are Barrero, Bloom and Davis (2020) documenting it during the recent Covid-19 induced recession and Dustmann, Lindner, Schönberg, Umkehrer and Vom Berge (2022) who document it in responses to changes in the minimum wage.

3 This long sample period retains enough time-series variation in the monetary policy stance, which on reverse is mostly flat at the zero lower bound in most recent sample period.

4 We measure wage inequality through regressions on monthly changes in wages split across quintiles. Dispersion measured this way is particularly affected by selection and composition.

5 The distinction between real and nominal wages allows us to discuss and purge for the distributional impact of inflation discussed in Doepke and Schneider (2006).

distribution, hence in a statistical decline of its dispersion. Furthermore, the exit of bottom earners induce a rise in their wages, due both to a decline in their labour supply and since the stayers tend to move to occupations with higher wages.

To elucidate on the underlying mechanism and to quantitatively match our evidence we build a model that augments an heterogenous agent set-up with uninsurable risk in the Bewley (1980)-Aiyagari (1994)-style with an occupational choice, in the Roy (1951) tradition. A key feature of our model is that transition probabilities are endogenous, and part of a period-by-period dynamic discrete choice optimization,[6]. Essential to this optimization is that an occupational switch is risky, and hence part of the self-insurance problem agents solve. There is a two-way link between wage and wealth inequality, hence saving decisions, and labour market decisions. Further, this set of choices is affected by monetary policy changes and because of this, the model includes nominal rigidities.

More specifically in our model agents, heterogenous in their idiosyncratic income shocks, wealth and talents, make in every period participation and occupational decisions through a random discrete choice based on the comparison, through Gumbel distributions,[7] of maximized value functions (see Rust (1987)). The resulting optimal switching probabilities depend on value functions, hence on wealth and income distributions. Once in an occupation households make consumption-saving decisions in liquid and non-liquid assets.[8] The equilibrium results from the solution of a fixed point problem between the occupational and the consumption saving decisions, which in itself provides a methodological advance. The model is completed by a monopolistically competitive production sector that faces adjustment costs on prices and determines labour demand by hiring a menu of occupations with partial substitutability. Overall a monetary policy tightening reduces average wages, and their dispersion. The decline in wages, hence in value functions, induces both top and bottom earners to exit the labour force, but separations are more pronounced for bottom earners. This in turn induces a mean preserving spread of the wage distribution, hence a reduction in dispersion. Wages of bottom earners decline on impact by more due to the decline in demand, but rise relatively to top

6 See also (Rust 1987) for first modeling dynamic discrete choices.

7 Gumbel taste shocks represent compensating differentials and other non-pecuniary aspects such as job amenities. See Taber and Vejlin (2020).

8 The inclusion of two assets provide a rich characterization of wealth dynamic and it also allows us to compare results to most recent versions of general equilibrium models with uninsurable risk.

earners in subsequent periods due to a combination of a decline in their labour supply and a reallocation of stayers toward higher wage occupations.

We develop our arguments through several steps, which involve analytical derivations and numerical simulations. First, in a simple stopping time model we show that an increase in nominal wages, induced by a monetary expansion, increases participation. Second, within our expanded structural model we derive the impact of interest rate changes on the Gini coefficient. This changes because different occupations have different elasticities of labour supply: participation and reallocation are affected directly by the change in wages and indirectly through general equilibrium changes in the value of wealth and its distribution. A decline in income and wealth reduces participation and transition probabilities, the more so for bottom earners. The decline in mobility reduces the dispersion of wages.

Numerically we characterize the monetary transmission mechanism first through impulse response functions. Furthermore, using simulated data we estimate model based regressions that map the empirical counterparts. The model is solved through the sequence-space Jacobian method (see Auclert, Bardóczy, Rognlie and Straub (2019)), which we augment with the fixed point between the occupational and consumption-saving choice. Noteworthy, the parametrization of key occupational characteristics is done through a k-means machine learning algorithm on O-NET data[9], something which enhances the credibility of our empirical matching. The resulting steady state distributional statistics for wealth and occupational shares are in line with their equivalent from Survey of Consumer Finance. In our model for instance income-poor households are more likely to transition out of the labour force, a trend which is well in line with the Great Resignation being more marked for low-paid jobs. The lower re-employment probabilities of those same bottom earners is also compatible with classical duration dependence.

The dynamic simulations of our model show that in response to a monetary tightening consumption declines due to a precautionary savings motive, investment declines due to the paradox of thrift, and assets dynamics exhibit Baumol-Tobin flight to liquidity.[10] The classical distributional channels operating in the general class of Bewley (1980) models,

9 See also Bonhomme, Lamadon and Manresa (2017) and Grigsby (2020)).

10 Precautionary saving which characterize models with uninsurable risk induces households to save more and to move toward more liquid assets.

namely earnings-heterogeneity and the interest rate exposure persist here.[11] Our model however features also novel channels stemming from changes in labour demand and supply. A key novel transmission runs from the heterogenous declines in wealth and wages onto the transition probabilities through the value functions. Likewise in the data monetary policy affects first and foremost the separations. Following the monetary tightening labour demand and wages fall, and by more for bottom earners. Hence, like in the data monetary policy hits dis-proportionately more bottom earners. The decline in income and wealth reduces the participation and transition probabilities for workers on both tails. Likewise in the data, this results in a mean preserving spread of the wage distribution. Wages of bottom earners decline on impact by more due to the decline in demand, but rise relatively to top earners in subsequent periods due to a combination of a decline in their labour supply and a reallocation of stayers toward higher wage occupations.

To confirm the mechanism quantitatively and to corroborate the empirical fit of the model we estimate model-based regressions, which replicate the empirical counterparts linking wage inequality and separation rates to the monetary policy shock, alone and interacted with income percentiles. The coefficient estimates match the empirical ones: monetary policy reduces wage inequality, rise wages of bottom earners (while decreasing those of top earners), albeit with a lag, and increases separation, more so for bottom earners.

We inspect the mechanism further by tilting key labour market primitives. We focus in particular on the role of task-specialization, which affects labour demand elasticity, and the distribution of workers' skills, which may depend on educational systems. Higher elasticity of labour demand across occupations has little impact on the transmission channels: absent nominal wage rigidity macro shocks have little impact on relative labour demand across occupations. On the contrary, under a less skewed talent distribution (more similar talents) the fraction of workers that leaves the labour market in response to the monetary contraction is smaller. The wages of bottom earners decline by more and stay lower for longer, resulting in an increase of wage inequality.

Related Literature. Our study relates to the expanding empirical literature that studies

11 Monetary expansions by changing wages and asset prices affect households differentially along the income and wealth distribution.

the impact of monetary policy on inequality. The pioneering work by Coibion et al. (2017) focuses on wealth inequality and finds that it increases in response to expansionary policy. The transmission goes through the increase in the value of wealth at the top triggered by the rise in asset prices. Our results parallel and complement those by focusing on wage inequality and on the labour, rather than the saving channel. Earlier work by Doepke and Schneider (2006) studies the distributional consequences of inflation, which by reducing the value of nominal bonds, hits mostly the rich. Subsequent work by Doepke, Schneider and Selezneva (2015) and Aladangady (2017) finds further evidence of distributional consequences through housing wealth. Evidence on the impact of expected inflation on the cash holdings of different households is found also in Erosa and Ventura (2002). The study by Amberg, Jansson, Klein and Rogantini Picco (2021) documents and summarizes many of the wealth channels using Swedish data.

While models with exogenous uninsurable risk can account well for the impact of monetary policy on wealth inequality, they remain silent on labour adjustment and its impact on wage inequality. Some empirical work on that front includes Engbom and Moser (2017), who by employing Brasilian data find evidence of workers' reallocation across firms in response to a monetary softening. Recently Broer, Kramer and Mitman (2021), using administrative data for Germany, find effects on job finding and separation rates. Our evidence complements those works using U.S. data: likewise Broer et al. (2021) we employ local projection methods (see Jordà (2005)) and high frequency identified monetary shocks a' la Gürkaynak et al. (2004) or Gorodnichenko and Weber (2016). An advantage of our evidence lies in the use of monthly data, which are well suited for the high frequency nature of the monetary transmission channel.

On the theoretical side there is an expanding literature that builds heterogenous agent models with uninsurable risk and nominal rigidities to study the transmission channel of monetary policy on wealth inequality through saving elasticities.[12] Our study provides a theoretical and methdological contribution to this literature by augmenting the Bewley (1980)-

12 A non-exhaustive list includes McKay, Nakamura and Steinsson (2016), McKay and Reis (2016), Guerrieri and Lorenzoni (2017), Gornemann, Kuester and Nakajima (2016),Kaplan, Moll and Violante (2018), Bayer, Lütticke, Pham-Dao and Tjaden (2019), Ravn and Sterk (2017), Auclert et al. (2019), Auclert and Rognlie (2018), Auclert, Rognlie and Straub (2018), Auclert, Rognlie and Straub (2020) or Dupor, Karabarbounis, Kudlyak and Mehkari (2018), Debortoli and Galí (2017).

Aiyagari (1994) framework with a discrete choice dynamic optimization for occupational decisions, a link which makes the distribution of earnings fully endogenous to the labour market equilibrium across occupational islands. Technically this is done also by extending the sequence-space Jacobian method of Auclert et al. (2019) with a fixed point algorithm iterating between the consumption-saving and occupational decisions.

Our occupational decision follows the Roy (1951) tradition, whose theoretical underpinnings are laid down in Heckman and Honore (1990) and Buera (2006), while theoretical foundations through discrete choice models are provided in Boskin (1974), Rust (1987), Keane and Wolpin (1997), Yamaguchi (2012), Card, Cardoso, Heining and Kline (2018). Recently Grigsby (2020) studied the role of reallocation during the financial crisis through static Roy model and with a rich occupational dataset. None of the above studies embeds occupational choice in a general equilibrium model with uninsurable risk. The work by Eeckhout and Sepahsalari (2020) studies the link between uninsurable risk and labour market choices, through a directed search model. The discrete choice setting allows us to derive the shift probability endogenously and to link model predictions to the data (see also (Lamadon, Mogstad and Setzler 2022) who study labour market power through the lens of a static discrete choice model). Beyond that our study explicitly focuses on the role of monetary policy also along the dynamic and on the impact of monetary policy. Finally, an multi-sector version of this model is employed in Faia, Kudlyak and Shabalina (2021) to study reallocation during the recent Covid-19 pandemic.

2. Empirical Evidence

We start our investigation with an empirical analysis aimed at estimating the impact of exogenous monetary policy shocks, through local projection methods, on changes in wage inequality and in job flows. The combined evidence on those two allows us to establish the role of job mobility in the transmission of monetary policy onto wage inequality.

Data Constructions and Econometric Specification. For our empirical analysis we employ data from the U.S. Current Population Survey (CPS in short) on wages and occupational probabilities for the period 1989-2019. CPS data are particularly well suited to study the monetary transmission mechanism due to their monthly frequency. The long time span that we employ has the advantage of providing enough time series variation in the monetary policy stance, which on reverse has been mostly at the zero lower bound in the more recent period. We exclude the post pandemic period, which features large volatility. Earnings are measured weekly to reflect wage levels rather than extensive margin fluctuations in the working time. For workers paid on hourly basis, earnings are given by the product of the hourly wage and the hours worked, while for salaried workers earnings are given by their per-week salary. All earnings are not netted out by taxes and other deductions.

The CPS data structure[13] lends itself well to our high frequency analysis. For our purposes, the most important months in that panel are the 4th and final months, which is when the respondents also report their weekly earnings and wages. This allows us to construct a year-over-year change in earnings for each individual. We define $\Delta w_{i,t} = \frac{w_{i,t+12} - w_{i,t}}{\frac{1}{2}(w_{i,t+12} + w_{i,t})}$ as the percent change in earnings from one period to the next.[14] We employ both nominal and real wages (the latter obtained by deflating with CPI) to account for the distributional consequences of inflation, an aspect which we discuss further below. Finally, note that to precisely track changes in the wage distribution we work with a pseudo panel, which means that we bin workers into 20 groups sorted based on similar characteristics. This allows us to track changes in the wage distribution for similar workers.

Exogenous monetary policy shocks are obtained through high frequency identification in the same way as Gürkaynak et al. (2004). The underlying rationale is as follows: Since market prices only react to new information, it is possible to extract the *unexpected* component of monetary policy decisions by measuring high-frequency movements of market prices in a narrow window around the policy announcement. Then from these high frequency shocks, we convert them into a monthly measure using weights akin to those employed in Gorodnichenko and Weber (2016) or Ottonello and Winberry (2020). More formally the monetary policy shock is given, in each month t, by the variable $\Delta m_t = \tau(t)\Delta_w i$, where $\Delta_w i$ is the change in

13 The data structure involves a 4-months of panel data, an 8-month interregnum followed by another 4-months of responses.

14 This formulation implies that earnings are 0 for one of the periods.

the federal funds rate within a small time window around the announcement. The variable $\tau(t)$ controls for the time of the announcement during the month.[15]

We measure wage inequality, our primary dependent variable, by constructing monthly wage changes for different percentiles and estimate the impact of monetary policy shocks. This way of measuring wage inequality is superior to the use of the Gini coefficient, whose response can be highly non-linear and hard to interpret.[16] We measure labour mobility through transition probabilities both from employment to non-employment, $\mathbb{I}_{EN}$ or its reciprocal, $\mathbb{I}_{NE}$, and across occupations. To construct the transitions across occupations we note the following. Workers can change occupations either directly, without an intervening spell of non-employment or through non-employment. Upon considering all possibilities we end up with five different combinations for the transitions probabilities.

The econometric specification uses the local projections by Jordà (2005) and applies it to the channels of our study. The regressors includes lags of several labour market variables on top and above the externally identified monetary policy shock. The specification reads as follows:

$$\mathbb{I}_{i,t+j} = \beta_j \Delta m_t + \Gamma_j x'_{i,t} \Delta m_t + \Theta Z_{i,t} + \epsilon_{i,t+j} \tag{1}$$

where $\mathbb{I}_{i,t+1}$ is either the monthly change in (nominal or real) wages or the transition probabilities, each of them measured for each income percentile. $x_{i,t}$ is a vector of worker characteristics that may predict transitions, such as prior income percentile, education, job tenure and age. $Z_{i,t}$ are controls. In the baseline specification the coefficients of interest for our analysis are the β_j, namely the reaction of the dependent variable to the exogenous monetary policy shock. We estimate equation 1 also in a variant in which the monetary policy shock is interacted with the income percentiles. This variant is meant to capture the heterogenous impact of the shock, on both wages and transition probabilities, along the income distribution.

15 Specifically $\tau(t) = \frac{\text{days in the month}}{\text{days remaining in the month after the announcement}}$.

16 See Amberg et al. (2021) for evidence on this.

2.1. Impact of Monetary Policy on Wage Inequality.

Table Table 1 shows regression coefficients for the local projection estimates of monthly changes in wages on monetary policy shocks and controls, as per specification in 1. The upper part of the table shows the results for the full sample, the bottom part of the table shows results when splitting the sample with a dummy for earnings above and below the median. The latter provides a further inspection of the quantile results, which we complement further below with regressions across earning ventiles.

Let us start with discussing the results for the full sample (upper part of Table Table 1). Rows show the estimates of the regression coefficients for either nominal or real wages, both in levels and in logs. We distinguish between real and nominal since, as noted in previous literature (see Doepke and Schneider (2006)), inflation may also have independent and heterogenous effects along the income distribution. While our time sample mainly focuses on a low inflation environment, still the general lack of indexation in the U.S. labour market landscape may induce stagnation for wages at the bottom, thereby steepening inequality. The estimates are done for each quintile of the income distribution. The coefficient, all significant, are negative, implying that a monetary tightening reduces the dispersion of changes in wages across quintiles, hence wage inequality. As expected the impact on nominal wages is larger due to the additional distributional impact induced by inflation. Note that monetary policy is unlikely to affect wages of incumbent workers, which cannot be renegotiated on a monthly basis. Its impact is therefore more likely passed through changes in the labour force or through changes of movers' wages. A reduction in wage dispersion may indeed arise for two reasons. First, reallocation following a contractionary shock may induce a selection channel by which only the best among the bottom earners remain in the labour market or even shift to a higher paying job. The rise in the bottom earners wages would reduce wage dispersion. Second, some of the bottom earners exit the labour market and have low probability of re-entering due to duration dependence. This instance results in a mean preserving spread of the wage distribution, as the bottom tail becomes thinner, and hence in a reduction of dispersion. Both channels are related to labour mobility and reallocation and both maybe present in the data.

To test the first channel we now repeat our previous regression by distinguishing the coefficient upon first impact for earners across the income distribution, splitting above and below the median. Results are shown in the bottom part of Table 1. Results for the regression that includes a dummy for each income ventile are plotted in Figure 1. In the regression, the coefficients are both significant, but negative for top earners and positive for bottom earners (or else wage of bottom earners rise and those of top earners decline), hence providing evidence for the selection channel. The figure, splitting by ventiles reinforces that this is driven particularly by the very bottom of the distribution. Beyond helping to disentangle the exact channels, those regressions are later used to judge the empirical fit of the model we present below. Within the model, we repeat the same specification using simulated data and show that the model-based regressions successfully replicate the empirical counterpart.

Table 1: Regression coefficients for changes in monthly earnings (either nominal of real) from CPS data for the period 1989-2019, on high frequency identified monetary policy shocks. Econometric specification for local projections is: $\mathbb{I}_{i,t+j} = \beta_j \Delta m_t + \Gamma_j x'_{i,t} \Delta m_t + \Theta Z_{i,t} + \epsilon_{i,t+j}$, where $\mathbb{I}_{i,t+j}$ is the change in monthly change in wages and the latter are measured on weekly basis, $x_{i,t}$ is a vector of worker characteristics that would often predict transition, such as prior income quintile, education, job tenure and age. $Z_{i,t}$ are controls including $x_{i,t}$. Time sample is 1989-2019. Data on nominal and real earnings are from Current Population Survey. Percent year-on-year change in earnings is defined as $\Delta w_{i,t} = \frac{w_{i,t+12} - w_{i,t}}{\frac{1}{2}(w_{i,t+12} + w_{i,t})}$.

Dependent variable	Coefficient	T-stat
Nominal earnings	-0.050	-4.80
Real earnings	-0.0879	-2.87
Earning Dummy Bottom 50%	0.185	121.53
Earning Dummy Top 50%	-0.0879	-55.39
Monetary policy shock × Bottom 50%	0.005	0.68
Monetary policy shock × Top 50%	-0.006	-0.88

2.2. Impact on Finding and Separation Rates.

We evaluate the role of job market exits on the wage distribution by re-estimating the specification in Equation (1) and replacing the dependent variable with finding and separation rates. Once again quintiles regressions help us to assess the heterogenous impact of the shock along the income distribution. Table 2 shows results. While we find no significant or sizable effects on the finding rates, we see significant effects on the separation rates. Most importantly

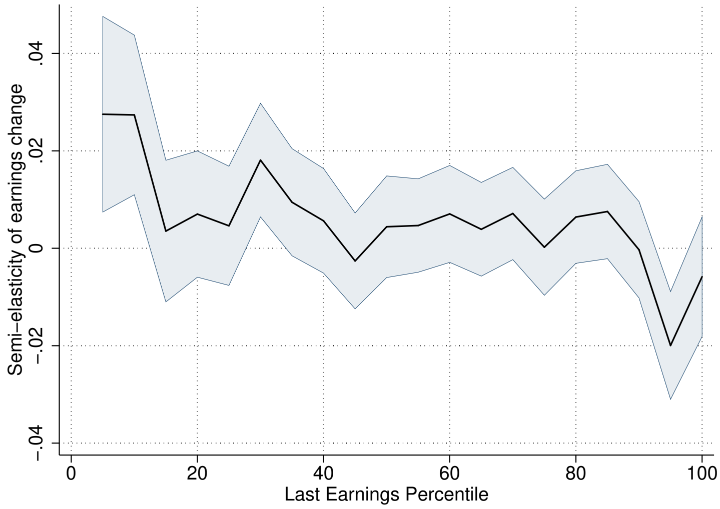

Figure 1: Semi-elasticity of earnings changes for each ventile of past earnings. Empirical estimates of impact coefficient of the exogenous monetary policy shock interacted with income ventiles. Estimation time period 1989-2019. Econometric specifications is: $\mathbb{I}_{i,t+j} = \beta_j \Delta m_t + \Gamma_j x'_{i,t} \Delta m_t + \Theta Z_{i,t} + \epsilon_{i,t+j}$, where the controls x include the income ventiles. The figure plots the coefficients Γ_j for each income ventile and its 95% confidence intervals.

a monetary tightening tends to increase separation rates and more so for bottom earners. It is therefore the lower tail that plays a larger role in transmitting the monetary shock. The fact that separation rates are larger for bottom earners also confirms our conjecture on the mean preserving change of the wage distribution: as bottom earners exit the labour market very low wages simply cease to be part of the wage distribution, resulting in a decline of its dispersion.

To further dissect the heterogenous effects of monetary policy we re-estimate the specification for separation rates by interacting the shock with income ventiles. Results are reported in Figure 2, which plots the estimated coefficient on the interaction term, against the income ventiles, reported on the x-axis. The plot unequivocally confirms that monetary policy disproportionally affects the separation rates of bottom earners. Coefficients are significant, more so for bottom earners. Once again the exit of those workers from the labour market would result in a decline in wage dispersion, even if equilibrium wages were unchanged, hence in absence of any selection. Even the estimates for the separation and finding rates will be compared later on to their model-counterparts to confirm the mechanisms envisaged so far, albeit through the lens of the model.

Table 2: Regression coefficients for changes in finding (top part of the table) and separation (bottom part of the tables rates on high frequency identified monetary policy shocks. Econometric specification for local projections is: $\mathbb{I}_{i,t+j} = \beta_j \Delta m_t + \Gamma_j x'_{i,t} \Delta m_t + \Theta Z_{i,t} + \epsilon_{i,t+j}$, where $\mathbb{I}_{i,t+j}$ is the change in separation rates, $x_{i,t}$ is a vector of worker characteristics that would often predict transition, such as prior income quintile, education, job tenure and age. $Z_{i,t}$ are controls including $x_{i,t}$. Time sample is 1989-2019.

Regression Finding Rates		
Dependent variable	Coefficient	T-stat
Unemployment to Employment Transition (full sample)	0.0028563	0.32
Constant,	0.451545	0.4467076
Unemployment to Employment Transition (below median)	-0.007706	-0.13
Constant,	0.2320822	146.10
Unemployment to Employment Transition (above median)	0.0014917	0.841
Constant,	0.2102231	105.98
Regression Separation Rates		
Dependent Variable	Coefficient	T-stat
Overall	-0.035	-3.58
Bottom decile	-.21264454	-1.83
Top decile	.008837071	0.91

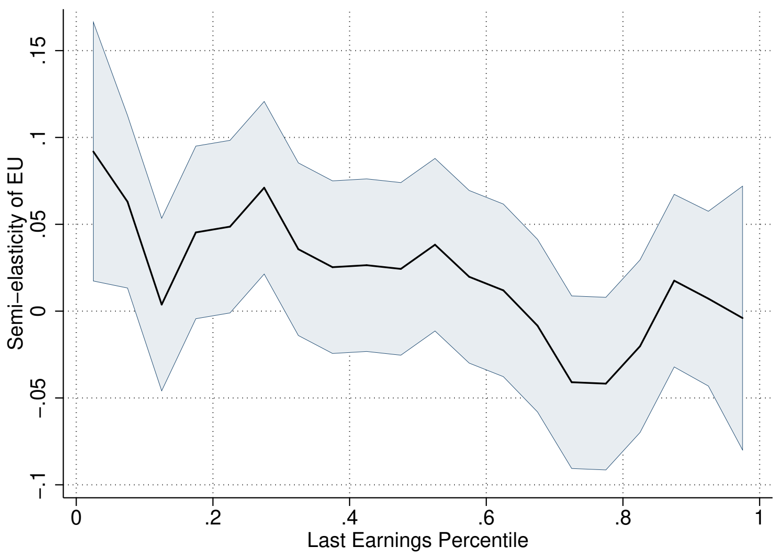

Figure 2: Semi-elasticity of separation rates (transitions from employment to unemployment) for each ventile of the earning distribution. Empirical estimates of impact coefficient of the exogenous monetary policy shock interacted with income ventiles. Estimation time period 1989-2019. Econometric specifications is: $\mathbb{I}_{i,t+j} = \beta_j \Delta m_t + \Gamma_j x'_{i,t} \Delta m_t + \Theta Z_{i,t} + \epsilon_{i,t+j}$, where the controls x include the income ventiles. The figure plots the coefficients Γ_j for each income ventile and its 95% confidence intervals..

3. Model with Dynamic Occupational Choice and Uninsurable Risk

A key novelty of our work consists in embedding endogenous occupational mobility in a multi-occupation Bewley (1980)-Aiyagari (1994) model with nominal rigidities. In the model, risk-averse infinitely lived households, heterogeneous in their idiosyncratic income shocks, wealth and skills, make consumption-saving decisions, choosing between liquid and non-liquid assets, and participation and occupational decisions, including on the intensive margin. Households' optimization is obtained as fixed point problem between a two-stage period by period dynamic optimization. In the first stage, households choose whether to work and in which occupation by solving a discrete choice in which they compare value functions across all possible choices. Smoothing maximal value functions through extreme value type I taste shocks allows us to obtain closed form solution for the shift probabilities. Those in turn are determined by wages and wealth distributions through their dependence on the value functions. Given the occupational choice, in the second stage, households solve a consumption-saving problem, also choosing between liquid and non-liquid assets, the latter featuring a liquidation cost. Portfolio decisions among different assets allow us to obtain a rich characterization of the wealth dynamic and also to compare the dynamic implied by our model to most recent general equilibrium models with uninsurable risk.

The production sector is composed of monopolistically competitive producers, who employ workers through a menu of partially substitutable occupations, employ capital and set prices facing quadratic menu costs. Nominal rigidities are introduced to study monetary policy.

3.1. Consumption-Saving and Occupational Choice

Households can be employed in one of many different occupations indexed by $o \in \{1, ..., O\}$ or non-employed. When we discuss occupational choice, we refer to the choice out of $O + 1$ alternatives, where option $O + 1$ is non-employment. These options differ by the associated flow values and the taste shocks.

Skill heterogeneity. Households differ in their skills and, hence, efficiency units of labor that they can provide in each occupation. The household's skill-type (hereafter, type) is indexed by $j \in \{1, ..., J\}$. The mass of each type j is denoted by m_j. Each type j is characterized by vector $\boldsymbol{\gamma_j}$, which describes the efficiency units that the type j can provide in each occupation $o \in \{1, ..., O\}$. $\boldsymbol{\Gamma}$ is the $J \times O$ matrix that stacks the (transposed) vectors for all J types. The matrix captures the way in which household's skill heterogeneity relates to occupations and plays a crucial role in determining the degree of reallocation across occupations. Each matrix element γ_j^o characterizes the comparative advantage of worker of type j in performing the tasks in occupation o.

Non-employment state. The menu of possible household's activity choices includes non-employment. In the non-employment state, which encompasses out of the labor force and non-employment, individuals receive the flow value of non-work expressed in real income units, h, which encompasses home-production and self-employment. The flow value of non-work, h, is the same for all household types.

Idiosyncratic productivity risk. At the end of each period t, employed and non-employed households experience idiosyncratic productivity shock e_t. The shock follows an n_e-state Markov process with transition matrix $P(e_{t+1}|e_t)$ and stationary distribution $\pi(e)$. The shock is independent of the household's skill type or current occupation, or the non-employment state.

Labor income. Real after-tax labor income $\xi_{j,t}^o$ of individual of type j at time t in occupation o is a function of efficiency units of type j in occupation o, γ_j^o, idiosyncratic productivity state e_t, real wage in occupation o w_t^o, labor hours n_t^o, and tax rate τ_t:

$$\xi_{j,t}^o = (1 - \tau_t) e_t w_t^o \gamma_j^o n_t^o \tag{2}$$

Heterogeneity enters labor income through three components. The first component is the time-invariant efficient labor units supplied by household of type j in occupation o. The second component is the occupational wage, which is endogenously determined in the labor market. The third component is the exogenous idiosyncratic productivity shock. Occupation specific tasks and workers' skills are the primitives that affect wages. This is in line with large empirical evidence (see Gathmann and Schönberg (2010) among others) which finds that human capital is occupation-specific.

Let $\boldsymbol{\xi_{j,t}}$ denote the $(O+1)$-dimensional vector of the real after-tax labor income of type j individuals in time t in all possible occupations $o \in O$, which consists of elements $\xi^o_{j,t}$, and the real after-tax income-equivalent in the non-employed state, $\xi^{O+1}_{j,t} = h$ for all $j \in J$.

Taste shocks. Each period, households draw idiosyncratic occupational-preference shock ϕ^o_t for all $o \in O+1$, in terms of utility. The taste shock is assumed to be i.i.d. across workers and occupations and follows a type 1 extreme value distribution, with density $f(\phi)$ and cumulative distribution function $F(\phi)$. The distributional assumptions follow the tradition of discrete choice models and generate a tractable form for the choice probabilities (see McFadden and Zarembka (1974)). The occupation-specific taste shock serves as a smoothing device in discrete choice models (see Rust (1987); Iskhakov, Jørgensen, Rust and Schjerning (2015)).

Sequence of decisions. Each period a household of type j decides on the occupational choice or non-employment $o \in \{1, ..., O, O+1\}$, their consumption and savings. Hours are equal across occupations $n^o = n$ for all o and are chosen by representative heads. The state in period t is $(e_t, a_{t-1}, b_{t-1}, \boldsymbol{\phi_t})$ where e_t is the idiosyncratic productivity shock, a_t are the illiquid assets, b_t are the liquid assets, and $\boldsymbol{\phi_t}$ is the $(O+1)$-vector of the taste shocks across all occupations and the non-employment state. Household of type j maximizes the following value function, given the state:

$$\begin{aligned} V_j(e_t, a_{t-1}, b_{t-1}, \boldsymbol{\phi_t}) = & \max_{o_t, c_t, a_t, b_t} u(c_t, n^o_t) + \phi^o_t + \beta E_{\boldsymbol{\phi}} E_e V_j(e_{t+1}, a_t, b_t, \boldsymbol{\phi_{t+1}}) \\ \text{s.t.} \quad & c_t + a_t + b_t = \xi^o_{j,t} + (1+r^a_t)a_{t-1} + (1+r^b_t)b_{t-1} - \Phi(a_t, a_{t-1}) \\ & a_t \geq 0, \quad b_t \geq \underline{b} \end{aligned} \tag{3}$$

where $\xi^o_{j,t}$ is the income in occupation o, r^a_t and r^b_t are the interest rates on illiquid and liquid assets, respectively, $\Phi(.)$ is the convex portfolio adjustment costs defined below, and β is the time discount factor. The instantaneous utility takes the standard separable form $u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\rho}}{1+\rho}$. For liquid assets, equation $b_t \geq \underline{b}$ is a borrowing constraint. The value function depends on the idiosyncratic income shock, e_t, through $\xi^o_{j,t}$.

The household's problem can be broken into two stages and solved backwards as follows. First, households make their consumption-saving decision, conditional on the occupation choice at time t and taking into account all possible occupations that they can work at in the future. Second, after substituting the consumption-saving policy functions for each occupation into the value functions, the household determines the probabilities of working in each occupation by comparing value functions across occupations. The resulting equilibrium is obtained through a fixed point problem between the policy functions that characterize the two stages.

Consumption-saving decision. Consider consumption-saving decision of a household that has optimally chosen occupation o. Conditional on occupational choice o, each household of type j in period t chooses consumption, c_t, savings in liquid, b_t, and illiquid, a_t, assets to maximize the following value function:

$$\begin{aligned} V_j^o(e_t, a_{t-1}, b_{t-1}) &= \max_{c_t, a_t, b_t} u(c_t, n_t) + \beta E_{\boldsymbol{\phi}} E_e V_j(e_{t+1}, a_t, b_t, \boldsymbol{\phi_{t+1}}) \\ \text{s.t.} \quad c_t + a_t + b_t &= \xi_{j,t}^o + (1 + r_t^a) a_{t-1} + (1 + r_t^b) b_{t-1} - \Phi(a_t, a_{t-1}) \\ a_t \geq 0, &\quad b_t \geq \underline{b} \end{aligned} \tag{4}$$

where $V_j^o(e_t, a_{t-1}, b_{t-1})$ is the value function in time t of type j conditional on occupation o and ϕ_t^o evaluated at 0.

The portfolio adjustment costs take the following functional form:

$$\Phi(a_t, a_{t-1}) = \frac{\chi_1}{\chi_2} \left| \frac{a_t - (1 - r_t^a) a_{t-1}}{(1 + r_t^a) a_{t-1} + \chi_0} \right|^{\chi_2} [(1 + r_t^a) a_{t-1} + \chi_0] \tag{5}$$

with $\chi_0 > 0$, $\chi_1 > 0$ and $\chi_2 > 1$. Note that $\Phi(a_t, a_{t-1})$ is bounded, differentiable, and convex in both arguments. Consumption-savings and portfolio decisions follow the standard Euler conditions, shown in Appendix B.

Labor hours are chosen by the family as a whole and are determined through the intra-temporal first order condition:

$$\varphi n_t^\rho = \sum_{o=1}^{O} \sum_{j=1}^{J} m_j \int u_c(e_t, a_{t-1}, b_{t-1}, o_t) \theta_j(o|e_t, a_{t-1}, b_{t-1}) \frac{\partial \xi_{j,t}^o}{\partial n_t^o} dD_j(e_t, a_{t-1}, b_{t-1}) \tag{6}$$

where $\theta_j(o|e_t, a_{t-1}, b_{t-1})$ are optimal probabilities of each occupation (see derivation below).

The solution to the optimization problem is a collection of type- and occupation-specific policy functions $c_j^o(e_t, a_{t-1}, b_{t-1})$, $a_j^o(e_t, a_{t-1}, b_{t-1})$ and $b_j^o(e_t, a_{t-1}, b_{t-1})$ that depend on the path $\left\{r_s^a, r_s^b, \tau_s\right\}_{s>t}$, taken as given by households.

Occupational decision. Following the tradition of random discrete choice models, we model the period by period occupational choice as a comparison of the value functions, which is an inherently dynamic operator. A household of type j chooses occupation o by comparing value functions from each choice, evaluated at the optimal consumption-saving policies:

$$o = \text{argmax}_{[1,....,O,O+1]} [\tilde{V}_j^o + \phi_t^o] \tag{7}$$

where $\tilde{V}_j^o$ is the value function $V_j^o(e_t, a_{t-1}, b_{t-1})$ evaluated at the optimal consumption-saving policy $c_j^o(e_t, a_{t-1}, b_{t-1})$, $a_j^o(e_t, a_{t-1}, b_{t-1})$ and $b_j^o(e_t, a_{t-1}, b_{t-1})$. We skip the dependence of the value function on the state for notational convenience. Note that the taste shock, ϕ_t^o, only affects preferences for occupation and does not influence the consumption-saving decision. The solution o to the occupational choice problem (7) satisfies:

$$F(\tilde{V}_j^o + \phi_t^o \geq \tilde{V}_j^{o'} + \phi_t^{o'}) \forall o' \neq o \in O+1 \tag{8}$$

where $F(.)$ is the cumulative density of the taste shock. Taking expectation with respect to the current period taste shocks of the household value function in (3) and evaluating that at the optimal consumption-saving choice yields:

$$E_{\boldsymbol{\phi}} \tilde{V}_j(e_{t+1}, a_t, b_t, \boldsymbol{\phi_{t+1}}) = E_{\boldsymbol{\phi}} \max_{o_t} \left[\tilde{V}_j^o(e_t, a_{t-1}, b_{t-1}) + \phi_t^o \right] = \ln \Big(\sum_{o_t} \exp(\tilde{V}_j^o(e_t, a_{t-1}, b_{t-1})) \Big) \tag{9}$$

where the last equality follows from the properties of the Gumbel distribution. Combining (7) and (9) yields the following closed-form solution for the occupational choice probabilities:

$$\theta_j(o|e_t, a_{t-1}, b_{t-1}) = \frac{\exp(\tilde{V}_j^o(e_t, a_{t-1}, b_{t-1}))}{\sum_o \exp(\tilde{V}_j^o(e_t, a_{t-1}, b_{t-1}))}. \tag{10}$$

Equation 10 shows that the probabilities of moving across occupations and the non-work state depends on the households' value functions. This implies that the wage distribution, the wealth accumulation and the idiosyncratic income shocks have an impact on occupational mobility. Such dependence leads to a precautionary search channel: households at the bottom of the income and wealth distributions have higher incentives to shift to higher wage occupations, a mobility channel on which we elaborate analytically further below. This implies that in face of a shock that reduces wages workers are less likely to move to jobs offering lower wages, and hence are more likely to stay out of the labour force. We solve the two-stage optimization problem numerically through a guess-and-verify procedure described in Appendix B.

Household aggregates. We obtain the aggregate policy functions for consumption and asset holdings by integrating occupation-specific policy functions weighted by occupation choice probabilities over the measure of households in state e_t that own assets in sets A_- and B_- at the start of date t, which is given by $D(e_t, A_-, B_-,) = Pr(e = e_t, a_{t-1} \in A_-, b_{t-1} \in B_-)$, and aggregating across all occupations $o \in O + 1$ and across all household types $j \in J$. The aggregate policy functions for households for every period $i \geq t$ are:

$$\mathcal{A}_t(r_i^a, r_i^b, \tau_i, N_i) = \sum_{j=1}^{J} m_j \sum_{o=1}^{O+1} \Big(\int a_j^o(e_t, a_{t-1}, b_{t-1}) \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) \Big) \tag{11}$$

$$\mathcal{B}_t(r_i^a, r_i^b, \tau_i, N_i) = \sum_{j=1}^{J} m_j \sum_{o=1}^{O+1} \Big(\int b_j^o(e_t, a_{t-1}, b_{t-1}) \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) \Big) \tag{12}$$

$$\mathcal{C}_t(r_i^a, r_i^b, \tau_i, N_i) = \sum_{j=1}^{J} m_j \sum_{o=1}^{O+1} \Big(\int c_j^o(e_t, a_{t-1}, b_{t-1}) \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) \Big) \tag{13}$$

where N_i is total hours, in employment and non-employment. Total labor hours supplied in each occupation o are given by:

$$N_t^o = n_t^o \sum_{j=1}^{J} m_j \int \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) \tag{14}$$

where n_t^o is the labor hours per occupation. The total effective labor supply in occupation o is given by the efficiency-units-weighted and idiosyncratic-shock-weighted employment in each occupation:

$$L_t^{o,Supply} = n_t^o \sum_{j=1}^{J} m_j \gamma_j^o \int e_t \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) \tag{15}$$

3.2. Monopolistic Competitive Production Sectors: Input and Pricing Decisions

Monopolistically competitive firms produce output by combining total labor input and capital using the production function:

$$y_t = z_t k_{t-1}^{\nu} L_t^{1-\nu} \tag{16}$$

where y_t is the variety, z_t is the total factor productivity and ν is the capital share, k is capital and L is labour input. The optimal demand for each variety is given by $p_t = \left(\frac{Y_t}{y_t}\right)^{\frac{1}{\eta}} P_t$, where P_t is the aggregate price index and is normalized to 1 in the steady state. Each firm employs a menu of occupations, in terms of occupation-specific effective labor hours $l_{o,t}$, with different shares α_o and elasticity of substitution σ. Labor aggregator is specified as follows:

$$L_t = \left(\sum_{o=1}^{O} \alpha_o l_{o,t}^{\sigma}\right)^{\frac{1}{\sigma}} \tag{17}$$

The elasticity of substitution across occupations, $\frac{1}{\sigma}$, plays a role in determining the extent of reallocation across occupations as we show in the analytical results in Proposition 4 in Section 4. This is so since it determines the elasticity of substitution of labor demand with respect to wages. Through this it also affects the degree of occupational specialization.

Firms choose labor demand, $l_{o,t}$, for each occupation $o \in O$, capital demand, k_t, and investment, I_t, to maximize the sum of future discounted real profits, which recursively reads as follows:

$$J_t\left(k_{t-1}\right) = \max_{p_t, k_t, I_t, l_{o,t}} \left\{ \frac{p_t}{P_t} y_t - \sum_{o=1}^{O} w_t^o l_{o,t} - I_t - \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 k_{t-1} - \frac{\eta}{2\kappa}\ln(1+\pi_t)^2 Y_t + \frac{J_{t+1}(k_t)}{1+r_{t+1}} \right\} \tag{18}$$

$$\text{s.t.} \qquad k_t = (1-\delta)k_{t-1} + I_t - k_t^{adj} \tag{19}$$

$$p_t = \left(\frac{Y_t}{y_t}\right)^{\frac{1}{\eta}} p_t \tag{20}$$

$$y_t = z_t k_{t-1}^{\nu} L_t^{1-\nu} \tag{21}$$

where equation (19) is the capital accumulation equation, δ is the depreciation rate of capital, and $\frac{\eta}{2\kappa}\ln(1+\pi_t)^2 Y_t$ is the quadratic price adjustment cost. Firms face quadratic adjustment costs on physical capital, with a standard functional form of $k_t^{adj} = \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 k_{t-1}$, which leads to a variable price of capital. Adjustment costs on capital have several appealing empirical implications, but most importantly for us they are commonly employed in structural models with nominal rigidities, to which we are bound to compare the dynamic of our own model. The first order condition for labor demand is:

$$l_{o,t} = \left(\frac{mc_t(1-\nu)\alpha_o}{w_t^o}\right)^{\frac{1}{1-\sigma}} \left(z_t k_{t-1}^{\nu}\right)^{\frac{\sigma}{(1-\nu)(1-\sigma)}} y_t^{\frac{\sigma-1+\nu}{(1-\nu)(\sigma-1)}}, \tag{22}$$

Defining q_t as the Lagrange multiplier on the capital accumulation and, hence, as the shadow price of capital, the firm's first order condition for the capital stock:

$$(1+r_{t+1})q_t = \nu z_{t+1}\left(\frac{L_{t+1}}{k_t}\right)^{1-\nu} mc_{t+1} - - \left[\frac{k_{t+1}}{k_t} - (1-\delta) + \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_{t+1}-k_t}{k_t}\right)^2\right] + \frac{k_{t+1}}{k_t} q_{t+1} \tag{23}$$

where mc_{t+1} is the Lagrange multiplier on the production constraint and represents real marginal cost. The first order condition with respect to investment is:

$$q_t = 1 + \frac{1}{\varkappa \varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right) \tag{24}$$

The first order condition with respect to prices leads to the Phillips curve:

$$\log(1+\pi_t) = \kappa \left(\frac{p_t}{P_t}\right)^{-\eta}(mc_t - \frac{1}{\mu_p}\frac{p_t}{P_t}) + \frac{Y_{t+1}}{Y_t}\log(1+\pi_{t+1})\frac{1}{1+r_{t+1}} \tag{25}$$

where $\mu_p = \frac{\eta}{\eta-1}$.

3.3. Equilibrium Conditions and Policy

The general equilibrium model is completed with a set of equilibrium conditions for good and financial markets and with the specification of the policy stance.

Asset Returns. Let r_t denote the ex-post return on government bonds. Let v_t be the ex-dividend price of equity and d_{t+1} be the firm dividend. The real return on equity is $\frac{d_{t+1}+v_{t+1}}{v_t}$. The no-arbitrage condition is: $v_t = \frac{d_{t+1}+v_{t+1}}{1+r_{t+1}}$. The combined return on illiquid assets is:

$$(1+r_t^a) = \left(\frac{v_t}{\mathcal{A}_t}\right)\frac{d_t+v_t}{v_{t-1}} + \frac{B^g - \mathcal{B}_t}{\mathcal{A}_t}(1+r_t) \tag{26}$$

The government does not issue new debt and pays interest on the constant level B^g. The return on liquid assets is set by a representative competitive financial intermediary which transforms illiquid into liquid assets through a technology that operates at a proportional cost ψ. Arbitrage between the two assets, coupled with a zero profit condition on the intermediary, leads to the following expression for the return on liquid assets: $r_t^b = r_t - \psi$.

Monetary and Fiscal Policy. Fiscal policy G_t follows a balanced-budget policy:

$$\tau_t \sum_{o=1}^{O} w_t^o l_{o,t} = r_t B^g + G_t, \tag{27}$$

Monetary policy follows a classical Taylor-type rule, which endogenously responds to macroeconomic conditions as follows:

$$i_t = r_t^* + \phi_\pi \pi_t + \phi_y (Y_t - Y_{ss}) \tag{28}$$

where i_t is the monetary policy interest rate, ϕ_π is the weight on inflation π_t, ϕ_y is the weight on output gap, $(Y_t - Y_{ss})$, r_t is the real interest rate, r_t^* is the natural interest rate, which is equal to the real interest rate in the steady state, and $1 + r_t = \frac{1+i_{t-1}}{1+\pi_t}$.

Market Clearing: Labor, Goods and Asset Markets. Wages are determined in equilibrium by equating labor supply and demand for each occupation, hence:

$$\underbrace{L_t^{o,Demand} = l_{o,t}}_{Demand} = \underbrace{n_t^o \sum_{j=1}^{J} m_j \gamma_j^o \int e_t \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) = L_t^{o,Supply}}_{Supply} \tag{29}$$

Aggregate supply of goods is equal to aggregate demand of goods, hence:

$$Y_t + hL_t^{O+1} = \mathcal{C}_t + G_t + I_t + \psi \mathcal{B}_t + \Phi_t, \tag{30}$$

where $\psi \mathcal{B}_t$ is the resource cost from liquidating assets, L_t^{O+1} is the number of individuals in the non-employment state, and where all variables above are aggregated through the joint distribution, D_t, defined above. Finally, asset markets clearing implies:

$$\mathcal{A}_t + \mathcal{B}_t = v_t + B^g, \tag{31}$$

where again aggregation is obtained through the joint distribution D_t.

Equilibrium.

Definition 1: Competitive Equilibrium

A Competitive Equilibrium of the economy satisfies the following definition:

- The sequence $[c_t, a_t, b_t]_{t=0}^{\infty}$ solves households' consumption-saving decision in eq. (4), given the distribution of idiosyncratic shocks, $P(e_{t+1}|e_t)$ and the sequence of prices $r_t^a, r_t^b, r_t, \boldsymbol{w_t}$.

- The sequence of probabilities $[\theta_t]_{t=0}^{\infty}$ solves households' occupational choice problem in (7).
- The policy functions resulting from the consumption-saving and the occupational problem are a in fixed point equilibrium.
- Aggregate assets holdings and consumption of the households are equal to the product of the individual optimal functions and the distribution of households across occupations and assets.
- Firms choose labor demand for each occupation, $l_{o,t}$, and capital inputs, k_t to solve discounted profit optimization, given in section 3.2.
- Market clearing and the aggregate resource constraints are satisfied.
- Monetary policy determines the short term interest rate according to eq. (28) and fiscal policy follows a balanced budget rule as in eq. (27).

4. Analytical Results

In this section we discuss through means of analytical derivations the impact of changes in the interest rate through labour market channels. First, to isolate the role of monetary policy for participation decisions, we lay down a toy model in which mobility decisions are framed in terms of optimal stopping time. In there we show that a decline in the interest rate fosters participation, hence a rise restrains it. Second, we derive the direct and indirect impact of interest rate changes on wage inequality in our structural model. Specifically we derive a model-based Gini index and show that the impact of interest rate changes is channelled on it through the relative labour elasticities across occupations, which in turn affect mobility incentives. Most importantly we decompose the monetary transmission into a direct effect, that goes through the impact of wages onto labour supply elasticities, and an indirect one, that goes through general equilibrium changes in the value of wealth. The latter by affecting precautionary savings also affect the incentives to participate in the labour market and to shift across occupations.

4.1. Simple Stopping Time Framework of Reallocation

We start by laying down a simple toy model in which the mobility decision is framed as an optimal stopping time problem. The worker faces the choice between two states, $O = [0, 1]$, which may capture the mobility decision between two jobs or between employment and non-employment. In each state the worker receives earnings which are subject to random fluctuations, acting as Bewley (1980) shocks. For analytical tractability earnings, e, are modelled through a continuous distribution, with cumulative $F(e' \mid e)$, over the support $[0, E]$, where the prime indicates future shocks.[17] Different states are characterized by different earning distributions. Occupation or state $o = 1$ offers earnings, whose distribution is a mean preserving spread of the one in occupation $o = 0$. For instance one contract may offer larger average earnings, but feature more risk. The optimal stopping time problem consists in finding the optimal threshold of earning realization today that makes the worker willing to participate in the labour market or shift state.[18] To simplify the model we neglect the consumption-saving decision, by assuming a risk-neutral worker. Workers discount the future at a rate β. At last, changes in the monetary policy rate affect the time discount, as on a first order approximation it holds true that: $\beta = \frac{1}{1+r} + \psi(\sigma_e^2)$, where $\psi(\sigma_e^2)$ is a premium for cross-sectional variation in income risk.

At time t the worker is employed in occupation $o = 0$ and has to decide upon which earning realization she/he would shift to another state. The discrete choice problem can be written as follows:

$$V(e) = max_{[stay,move]} \left[e + \beta \int_0^E V(e) dF^0(e' \mid e), -c + \beta \int_0^E V(e') dF^1(e' \mid e) \right] \tag{32}$$

where the present value of staying in the current state is $e + \beta \int_0^E V(e) dF^0(e' \mid e)$, while $-c + \beta \int_0^E V(e') dF^1(e' \mid e)$ is the value of shifting and where $\int_0^x [F^0(e' \mid e) - F^1(e' \mid e)] de' \geq 0$ $\forall x$, which is to say that $F^0(e' \mid e)$ is a mean preserving spread of $F^1(e' \mid e)$. In this set-up we are assuming that the choice is once and for all, but in the larger structural model the

17 The conditionality in the density function captures the Markov nature of the shock.
18 This provides the equivalent of the occupational shifting probabilities in the structural model.

worker can reconsider her/his occupational decision in every period. The solution consists in finding the threshold for the income shock, $\bar{e}$, such that at $e = \bar{e}$ we have that:

$$V(e) = e + \beta \int_0^E V(e')dF^0(e' \mid e) = -c + \beta \int_0^E V(e')dF^1(e' \mid e) \tag{33}$$

The solution to the optimal stopping time or the shifting threshold implies the following lemma.

Lemma 1. *A decline in the monetary policy rate reduces the shifting threshold and increases the shift probability.*

Proof. Using integration by parts:

$$0 \leq \int_0^E [F^0(e' \mid e) - F^1(e' \mid e)]de' = -\int_0^E e'[dF^0(e' \mid e) - dF^1(e' \mid e)] \tag{34}$$

the optimal threshold can be written as:

$$\tilde{e} = -c - \beta \int_0^E V(e')[dF^0(e' \mid \bar{e}) - dF^1(e' \mid \bar{e})] \tag{35}$$

If β increases due to a decline in the interest rate, the shifting threshold declines and the shifting probability increases.

4.2. Decomposing the Impact of Monetary Policy on Occupational Wage Inequality: Direct and Indirect Effects

We now analyze the impact of changes in the interest rate on income inequality. Any measure of labor income inequality could be written as a function of labor income levels and the shares of households that receive each level of income. In our structural model both depend on the occupational and consumption-saving decisions. The impact of changes in interest rates on those two passes through *direct* and *indirect* effects. Broadly speaking the first captures the impact of interest rate on wage changes and in turn on labour supply and occupational decisions. Formally, and in our framework, this is channelled through the impact of interest rates on savings and labour hours, both captured by the Euler conditions. The indirect effects capture the general equilibrium component of the interest rate pass-through. The latter affects asset values and the wealth distribution, which in turn affects saving and labour decisions as well as transition probabilities.

Let G be a measure of labor income inequality, for example the Gini index, $\{\xi^o_{j,t}\}$ is a vector of labor income levels in the economy at time t ($o \in \{1, ...O+1\}$, $j \in \{1, ...J\}$) and $\{\theta^o_{j,t}\}$ is a vector of shares of households that receive the corresponding levels of income. The Gini index reads as follows:

$$G_t = f(\{\xi^o_{j,t}\}, \{\theta^o_{j,t}\}) \tag{36}$$

$$\frac{\partial G_t}{\partial r_t} = \frac{G_t}{r_t}\left(\sum_{o,j} \varepsilon_{\xi^o_{j,t},r_t} \cdot \varepsilon_{f,\xi^o_{j,t}} + \sum_{o} \varepsilon_{\theta^o_{j,t},r_t} \cdot \varepsilon_{f,\theta^o_{j,t}}\right) \tag{37}$$

where $\varepsilon_{i,j}$ is the elasticity of variable i to changes in variable j. The function f is the chosen measure of inequality, which depends on the weights assigned to changes in income or to the population shares. We define with $\varepsilon_{f,\cdot}$ the elasticities for each function f. Independently from the inequality measure chosen the ultimate objects of interest are the elasticities of income levels and population shares in each income percentile to a monetary policy shock. As an example consider an economy with two levels of income, such that $\xi^o \geq \xi^{o'}$, the change in Gini index could be specified as follows:

$$\frac{\partial G_t}{\partial r_t} = \frac{G_t}{r_t}\left(\varepsilon_{\xi^o_t,r_t}\frac{\xi^o_t}{\xi^o_t - \xi^{o'}_t} - \varepsilon_{\xi^{o'}_t,r_t}\frac{\xi^{o'}_t}{\xi^o_t - \xi^{o'}_t} + \varepsilon_{\theta^o_t,r_t} + \varepsilon_{\theta^{o'}_t,r_t}\right) \tag{38}$$

The objects of interest in this example are $\varepsilon_{\xi^o_t,r_t}$, $\varepsilon_{\xi^{o'}_t,r_t}$ and $\varepsilon_{\theta^o_t,r_t}$, $\varepsilon_{\theta^{o'}_t,r_t}$, namely the elasticities with respect to changes in the interest rate, of the two income levels and of the corresponding population shares holding them. The weights are $\frac{\xi^o_t}{\xi^o_t-\xi^{o'}_t}$, $-\frac{\xi^{o'}_t}{\xi^o_t-\xi^{o'}_t}$, 1, 1. Thus, changes in the Gini with respect to changes in the interest rate are ultimately given by each of the elasticities. In what follows we will focus on those and we will compute them within the structural model outlined in section 3.

Lemma 1. *The elasticity of labor income to monetary policy shocks reads as follows*:

$$\varepsilon_{\xi^o_t, r_t} = \underbrace{-\frac{\tau_t}{1-\tau_t}\varepsilon_{\tau_t, r_t} + \sigma \varepsilon_{n_t, r_t} + (1-\sigma)\frac{r_t}{r_t+\delta} + \sigma \left(\sum_{o''=1}^{O} \alpha_{o''}^{\frac{1}{1-\sigma}} w_{o'',t}^{\frac{-\sigma}{1-\sigma}} \cdot \varepsilon_{w_{o'',t}, r_t}\right)\left(\sum_{o''=1}^{O} \alpha_{o''}^{\frac{1}{1-\sigma}} w_{o'',t}^{\frac{-\sigma}{1-\sigma}}\right)^{-1}}_{\text{aggregate GE effects}} -$$

$$\underbrace{-(1-\sigma)\frac{n_t}{l_{o,t}}\sum_{j=1}^{J} m_j \gamma_j^o \int e_t \theta_j^o(e_t, a_{t-1}, b_{t-1}) \cdot \varepsilon_{\theta^o_{j,t}, r_t} \cdot dD_j(e_t, a_{t-1}, b_{t-1})}_{\text{occupation-specific effect}} \tag{39}$$

Proof. See Appendix A.

The elasticity is equal across types j as skill endowments are constant. The first term on the right-hand side of eq. (39), stems from the endogenous response of the fiscal policy to changes in the interest rate. As the government has to maintain a balanced budget changes in the interest rate, by affecting debt services, require a tax adjustment. The second term stands for changes in the supply of hours worked or the intensive margin. Recall that in our model those are set equal for each family j member. The next two terms are changes in labor demand induced by changes in factor prices. The last term represents changes in labor income induced by reallocation. This term depends itself on the elasticity of the shift probabilities with respect to changes in the interest rate and on the changes of the population shares in each income-wealth state. The latter are in turn induced by changes in general equilibrium asset returns and wages. In sum, reallocation affects inequality by changing the shares of households in each occupation and by inducing changes in labor income (hence in the population shares in each income state) for each occupation. The latter arise in general equilibrium as a result of reallocation. Given this an alternative decomposition is presented in the next corollary.

Proposition 1. *Changes in inequality induced by monetary policy shocks can be decomposed into changes in income levels induced by changes in prices and changes due to reallocation.*

$$
\begin{aligned}
\frac{\partial G_t}{\partial r_t} = & \underbrace{\frac{G_t}{r_t}\sum_o \varepsilon_{P_t,r_t}\cdot\varepsilon_{f,\xi_t^o}}_{\text{price effects}} + \\
& + \underbrace{\frac{G_t}{r_t}\sum_o \left(\varepsilon_{\theta_t^o,r_t} - (1-\sigma)\frac{n_t}{l_{o,t}}\sum_{j=1}^{J} m_j\gamma_j^o \int e_t\theta_j^o(e_t,a_{t-1},b_{t-1})\cdot\varepsilon_{\theta_t^o,r_t}\cdot dD_j(e_t,a_{t-1},b_{t-1})\right)\cdot\varepsilon_{f,\theta_t^o}}_{\text{reallocation effects}}
\end{aligned}
\tag{40}
$$

Proof. See Appendix A. Note that there is a mapping between the decomposition in eq. (39) and the decomposition in eq. (40). Indeed the term ε_{P_t,r_t} in eq. (40) corresponds to the general equilibrium effect present in eq. (39).

Sufficient Statistics for Earning Elasticities. At this stage it is important to highlight the empirical relevance of our decompositions. They can indeed lead to some testable implications. Note that the number of occupations in the model is presumably smaller than the large dis-aggregated varieties that one can gather in the data. Hence an exact parallel is not feasible. However, we can map the model-based changes in the occupational distributions to the empirical one by matching the mean elasticity and the dispersion of the earnings elasticities. This in turn implies that we can assess the empirical relevance of changes in the model-based wage distribution, by targeting the response to monetary policy shocks of average wage and the wage dispersion in the data. This testable implication provides the basis for the comparison between the estimates of the empirical specification presented in section 2 and the model equivalent, which will be discussed later on. Both the empirical specification as well as the corresponding model-based, presented later on in Section 6.3 indeed compute precisely elasticities of wage dispersion to interest rates. Dissecting the earnings elasticities even further and relating them to the model primitives allow us to provide a structural interpretation to their estimates. The next corollary does a step forward into that direction.

Corollary 1. *Average elasticity and the dispersion of earnings elasticities to interest rates are given by the following relations.*

$$\mu_\varepsilon = \sum_{o=1}^{O+1} \varepsilon_{\xi_t^o, r_t} \sum_{j=1}^{J} m_j \int \theta_j^o(e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1})$$

$$= \varepsilon_{P_t, r_t} \lambda_t^E - (1-\sigma) \sum_{o=1}^{O} \frac{Cov_I(\gamma^o e_t, \frac{\partial \theta^o}{\partial r_t}) + \Lambda E_I \frac{\partial \theta^o}{\partial r_t}}{Cov_I(\gamma^o e_t, \theta^o) + \Lambda E_I \theta^o} \lambda_{o,t} \tag{41}$$

$$\sigma_\varepsilon^2 = (1-\sigma)^2 Var \left(\frac{Cov_I(\gamma^o e_t, \frac{\partial \theta^o}{\partial r_t}) + \Lambda E_I \frac{\partial \theta^o}{\partial r_t}}{Cov_I(\gamma^o e_t, \theta^o) + \Lambda E_I \theta^o} \right) \tag{42}$$

where I stands for the cross-sectional moments,
$\lambda_t^E = \sum_{o=1}^{O} \sum_{j=1}^{J} m_j \int \theta_j^o(e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1})$ and
$\lambda_{o,t} = \sum_{j=1}^{J} m_j \int \theta_j^o(e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1})$ stand for the total employment and employment shares in each occupation, ε_{P_t, r_t} is again the aggregate general equilibrium effect, $\Lambda = \sum_{j=1}^{J} m_j \gamma_j^o$ is a normalization constant, $\frac{\partial \theta_j^{o'}}{\partial r_t}$ are calculated using job flows, $\gamma_j^{o'} e_t$ stands for the total productivity of the household and is measured as a ratio between the worker's wage and the average wage in her/his occupation. Finally Var stands for the variance across occupations, excluding non-employment.

Proof. See Appendix A.

The main takeaway from the above corollary is that earning elasticities in our model depend on the transition probabilities across occupations and the share of workers in each occupation. For instance equation (42) shows that the higher the variance of the changes in transition probabilities to changes in the interest rate, the larger is the earning elasticity, or else in environments with more mobility the dispersion of changes in earnings is larger. This result is compatible with our empirical evidence showing that contractionary monetary policy reduces earnings inequality primarily by reducing mobility.

Note that in our empirical evidence we also estimated the elasticities of transition probabilities to the monetary shock. Once more, dissecting analytically the elements and primitives that compose the elasticities of transition probabilities to interest rates helps in guiding the interpretation of their empirical estimates.

Proposition 2. *The elasticity of the shift probability across occupations for each household j with respect to monetary policy shocks is as follows:*

$$\varepsilon_{\theta^o_{j,t},r_t} = \underbrace{r_t u_c \left(\frac{\partial r^a_t}{\partial r_t} a_{t-1} + \frac{\partial r^b_t}{\partial r_t} b_{t-1} - (\Phi'_1 + 1) \frac{\partial a^{o,*}_t}{\partial r_t} - \frac{\partial b^{o,*}_t}{\partial r_t} + \frac{\partial \xi^o_{j,t}}{\partial r_t} \right)}_{\text{income effect}} +$$
$$+ \underbrace{r_t u_n \frac{\partial n_t}{\partial r_t}}_{\text{labor hours effect}} + \underbrace{\beta r_t E_\phi E_e \frac{\partial V^o_j(e_{t+1}, a_t, b_t)}{\partial r_t}}_{\text{continuation value effect}} \underbrace{- r_t \frac{\sum_{o''=1}^{O+1} exp(V^{o''}_j(e_t, a_{t-1}, b_{t-1})) \frac{\partial V^{o''}_j}{\partial r_t}}{\sum_{o''=1}^{O+1} exp(V^{o''}_j(e_t, a_{t-1}, b_{t-1}))}}_{\text{granularity effect}} \tag{43}$$

Proof. See Appendix A.

Changes in the shift probabilities depend upon changes in the value functions across occupations. The latter in turn depend upon changes in occupational income, in labour hours in each occupation and in the future value function (continuation value). Continuation values depend upon future asset values and upon labour market concentration. The first term on the right-hand side of eq. (43) are changes in the shift probabilities induced by changes in both capital and labour income for each occupation. The second term takes into account that changes in income and wealth induce workers to modify the number of hours worked, or else the intensive margin. The third term captures the changes in the continuation value: monetary policy affects future asset values, which in turn affect savings, portfolio and occupational decisions. The last term accounts for the granularity of occupations. If shift probabilities change, occupations shares change too, leading to different labour market concentration. The latter in our framework is akin to the concept of labour market tightness.

Labour Demand Substitability. Some of the terms in the previous decompositions, specifically those related to changes in occupational income, are due to changes in labour demand. We therefore complete this section by dissecting the model primitive that determine the elasticity of labour demand to wages. The economics behind this will be useful also in interpreting the dynamic of our simulated model under different labour demand elasticities.

Proposition 3. Occupational Specialization and Rents. *Consider the economy outlined in section 3. The elasticity of labor demand with respect to wages in occupation* o *is* $\varepsilon^o_{l,w} = \frac{1}{\sigma-1}\left(1 - \frac{l_o w^o}{\sum_{o'=1}^{O} l_{o'} w^{o'}}\right)$ *and declines with* σ *(*$\varepsilon^o_{l,w} = \frac{1}{\sigma-1}$ *with infinite number of occupations) and the mark-up or rent extracted by each occupation are given by the inverse of* $\mu_w = \frac{1-\frac{l_o w_o}{\sum_{o'=1}^{O} l_{o'} w'_o}}{\sigma-\frac{l_o w_o}{\sum_{o'=1}^{O} l_{o'} w'_o}}$ *(with infinite number of occupations* $\mu_w = \frac{\varepsilon_{l,w}}{\varepsilon_{l,w}+1} = \frac{1}{\sigma}$*).*

***Proof*.** See A.

The above proposition highlights the role of job substitutability, $\frac{1}{\sigma}$, on reallocation as driven by cross-sectional changes in labor demand. Higher σ increases substitutability of occupations and the elasticity of labor demand to wages.[19] This in turn provides a link between compression in wage dispersion and compression in labour demand dispersion. A monetary policy shock that changes labour demand across occupations will affect the equilibrium wage distribution conditional on the degree of substitability.

5. Model Parametrization and Solution Method

As baseline numerical algorithm we employ the sequence-space Jacobian (see Auclert et al. (2019)). The latter however has to be modified to accommodate the fixed point iteration between the consumption-saving and the occupational decision stage. We discuss this in detail below. In terms of calibration, beyond the baseline parameters, we devote particular attention to obtain a precise calibration of the occupational parameters. This is crucial to guarantee the soundness of our model empirical fit. We will use k-means on ONET data to cluster occupations and calibrate the main parameters related to them. Furthermore, we verify that our calibration delivers long run distributional statistics which are in line with data from the Survey of Consumer Finance. At last, a key measure that allows us to map

19 This is the sense in which lower substitability represents a mobility cost (see Kennan and Walker (2011)).

our model into the data is the construction of the transition probabilities, which in a model with a high dimension state, are complex objects and require extensive analytical derivations.

5.1. Measuring Occupations in the Data: k-Means Algorithm.

A key element of the reallocation channel is the matrix Γ of skills distribution. Each vector of this matrix provides the set of talents that each household type J has for each occupation. Hence, each element of the matrix, γ_j^o, provides the comparative advantage, in terms of efficiency units of labor, of each household j for occupation o. We employ O-NET data to calibrate this matrix. Those are indeed the best suited to quantify the concept of task-specific human capital (Gathmann and Schönberg (2010)). The O-NET dataset contains up to thousands occupational categories. To map those into the model and reduce the curse of dimensionality the 3-digit or 968 occupations in the dataset are grouped into 8 categories with similar job requirements using k-means clustering. The entries of the matrix Γ are then filled using the skill-requirements vectors provided in O-NET.[20] A k-means clustering algorithm is used to identify clusters of 3-digit occupations with similar requirements. Mathematically, let M be distinct elements in the vector of job requirements reported by O-Net and let G denote a set of detailed occupations in O-Net, each characterized by the M-dimensional vector of requirements $\mathbf{h}_g$. Given the set of observations ($\mathbf{h}_1$,$\mathbf{h}_2$, ..., $\mathbf{h}_G$), the k-means clustering partitions the G observations into k ($\leq G$) sets $\mathbf{O} = (O_1, O_2, ..., O_k)$ by minimizing the within-cluster sum of squares (i.e. variance):

$$\arg\min_{\mathbf{O}} \sum_{i=1}^{k} \sum_{\mathbf{h} \in O_i} \|\mathbf{h} - \mathbf{m}_i\|^2 \tag{44}$$

where $\mathbf{m}_i$ is the mean of points in S_i. k is set to 8. The resulting 8 occupational clusters are summarised based on the following labels: (1) Manual trade occupations, (2) Management and supervisory occupations, (3) Machine operators, (4) Engineering occupations, (5) Healthcare and community occupations, (6) Personal services, (7) Technical-support occupations, and

20 Each detailed requirement is ranked by its importance on a scale from 0 to 7. For example, O-Net contains information on 52 different elements within abilities - such as "Problem Sensitivity", "Mathematical Reasoning" or "Manual Dexterity". Skill requirements are "Management of Financial Resources", "Social Perceptiveness" or "Complex Problem Solving" etc.

(8) Office and administrative support. The entries of the Γ matrix are then filled with a comparative advantage index constructed from O-NET skill-requirements. We report both the procedure and the value of the matrix in Appendix B.

Calibrating Other Parameters. The values of the other parameters are summarized in table 3. Time is in quarters. Most of them are in line with those used in models with heterogeneous agents and uninsurable risk or are consistent with long run targets on asset distributions. The steady state interest rate on government bonds is calibrated to 0.0125, close to the 5-year average of the Effective Federal Funds Rate (see Auclert et al. (2019)). The time discount factor, β, is then obtained from the asset market clearing condition. The portfolio adjustment cost parameter χ_1 is set to 6.19 to match the quarterly interest rate of 0.0125 and the debt to output ratio of 1.04. Following Auclert et al. (2019) the portfolio adjustment costs parameters are set as follows $\chi_0 = 0.25$ and $\chi_2 = 2$ and the steady state mark-up is set to 1.015 in order to match the steady state sum of liquid and non-liquid assets, which in our model is 16.26. The steady state level of government spending is set to 0.2 of GDP (a widely used "great ratio" from the literature). The value of taxes is then obtained from the fiscal policy rule and is equal to 0.4. Monetary policy follows Taylor-type rule with 1.5 for the weight on the inflation gap and 0 for the weight on the output gap. The slope of the Phillips curve, the capital depreciation rate and the capital adjustment costs are calibrated to 0.1, 0.02, and 4, respectively, following Auclert et al. (2019). The flow value of non-employment, h_t, is calibrated using data from Chodorow-Reich and Karabarbounis (2016) who estimate the opportunity cost of employment using a wide range of data sources, namely Consumer Expenditure Survey (CES), the Panel Study of Income Dynamics (PSID) and the Current Population Survey (CPS). Under separable preferences, which is the case considered in our model, they report a ratio of opportunity cost of employment to the marginal product of employment of 47%. This value is then multiplied by our average steady state income for all employed households and delivers a value of 0.16. For comparison, households' steady state income (see eq. (2)) varies in our model between 0.06 and 1.15. This opportunity cost includes unemployment insurance and other benefits like Medicaid as well as foregone value of non-working time expressed in units of consumption.

For the production sector the parameter ν is calibrated so as to deliver a capital share of 0.40. This results from the average of capital and labour shares across sectors in the EU-KLEMS data. The baseline σ is set to 0.2. The parameter is then varied in the simulations to assess the impact of job specialization on reallocation. The α shares are calibrated as follows. Production, $y_t = K_t^{\nu} L_t^{1-\nu}$, and aggregate labor demand, $L_t = \left(\sum_{o=1}^{O} \alpha_o l_{o,t}^{\sigma}\right)^{\frac{1}{\sigma}}$, are substituted in the firms' first order conditions for labor: $(1-\nu) K_t^{\nu} L_t^{-\nu} \alpha_o \frac{1}{\sigma} \left(\sum_{o=1}^{O} \alpha_o l_{o,t}^{\sigma}\right)^{\frac{1}{\sigma}-1} \sigma l_{o,t}^{\sigma-1} = w_o$. Then taking the ratios, $\frac{\alpha_o}{\alpha_{o'}} = \left(\frac{l_{o,t}}{l_{o',t}}\right)^{1-\sigma_s} \frac{w_o}{w_o'}$ and summing up delivers: $\frac{1}{\alpha_{o'}} = \sum_{o=1}^{O} \left(\frac{l_{o,t}}{l_{o',t}}\right)^{1-\sigma} \frac{w_o}{w_o'}$. From the BLS Occupational Employment Statistics (OES) for the 2014-2019, data on wages for each of our occupational clusters are extracted and, once weighted by employment in each occupations, they are used to calibrate the steady state real wages in occupational clusters 1-8. The same employment shares and wages are then used in equation **??** to calibrate the α_o. Specifically occupation shares, that is the $\alpha_{s,o}$, are: 0.078, 0.254, 0.092, 0.280, 0.035, 0.085, 0.070, 0.106. The wage distribution, that is w_o, is as follows: 14.1, 40.8, 16.7, 46.3, 6.2, 14.9, 12.8, 18.9.

5.2. Measuring Job Flows in the Data and the Model

A key aspect of our analysis is the comparison of the elasticities of transition probabilities, summarized by separation and findings, in the data and in the model. To this purpose we need to obtain analytical expressions for the job flows, both gross and net. The analytical derivations of the formula are contained in appendix A. The final formulas are as follows for gross flow between t and $t+1$:

$$\sum_{j=1}^{J} m_j \sum_{o} \int_{e_t, a_{t-1}, b_{t-1}} \theta_j(o|e_t, a_{t-1}, b_{t-1}) * \sum_{e_{t+1}} (1 - \theta_j(o|e_{t+1}, a_t, b_t)) P(e_{t+1}|e_t) dD_j(e_t, a_{t-1}, b_{t-1}) \tag{45}$$

Table 3: Parameter Values, Description and Source

Parameter		Description	Value and source
Skills and Occupations			
	O	Number of occupations	8, clustered by k-means
	J	Number of skill types	8, clustered from O*NET
	m_j	Distribution of skill types	$1/J$ (uniform across J)
	Γ	Skill transferability matrix	See Section 5.1
Final Composite Good			
	P_t	Aggregate Price	Normalized to 1 in the steady state.
Production Function			
	σ	Elasticity of substitution between occupations	0.2 (baseline)
	ν	Capital share	0.4, KLEMS, Section 5.1
	w^o	steady state wage per efficiency unit in occupation o	OES-BLS, see Section 5.1
	δ	Capital depreciation	0.02, Auclert et al. (2019)
	$\imath$	Capital adj. parameter	4, Auclert et al. (2019)
Households			
	β	Time discount factor	0.979, see Section 5.1
	χ_0	Portfolio adj. cost pivot	0.25, Auclert et al. (2019)
	χ_1	Portfolio adj. cost scale	6.19 (target $\mathcal{B}_h = 1.04Y$, Auclert et al. (2019))
	χ_2	Portfolio adj. cost curvature	2, Auclert et al. (2019)
	σ	EIS	0.5 Auclert et al. (2019)
	ρ	Inverse Frisch elasticity	1 Auclert et al. (2019)
	ρ_z	Autocorrelation of earnings	0.966, Auclert et al. (2019)
	σ_z	Cross-sectional std of log earnings	0.92, Auclert et al. (2019)
	h_t	Flow value of non-employment	47% of average income, see Section 5.1 Auclert et al. (2019)
	φ	Dis-utility parameter	1.71 (target $n = 1$)
Asset Markets			
	r	Real interest rate	0.0125, Auclert et al. (2019)
	ψ	Liquidity premium	0.005, Auclert et al. (2019)
	μ_p	steady state markup	1.015, Auclert et al. (2019)
Monetary and Fiscal Policy			
	ϕ	Coefficient on inflation in Taylor rule	1.5, Auclert et al. (2019)
	ϕ_y	Coefficient on output gap in Taylor rule	0, Auclert et al. (2019)
	τ	Tax rate	0.401, Auclert et al. (2019)
	$\mathcal{B}_g$	Bond supply	2.8, Auclert et al. (2019)
	κ	Slope of the Phillips curve	0.1, Auclert et al. (2019)

where $a_t = a(e_t, a_{t-1}, b_{t-1})$, $b_t = b(e_t, a_{t-1}, b_{t-1})$ and $P(e_{t+1}|e_t)$ is an element of the transition matrix. The matrix has a property $\sum_{e_{t+1}} P(e_{t+1}|e_t) = 1$. The net flows across occupations are as follows:

$$\frac{1}{2}\sum_{o}\left|\sum_{j=1}^{J} m_j \int_{e_{t+1},a_t,b_t} \theta_j(o|e_{t+1}, a_t, b_t) dD_j(e_{t+1}, a_t, b_t) - \sum_{j=1}^{J} m_j \int_{e_t,a_{t-1},b_{t-1}} \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1})\right| \tag{46}$$

where $|\cdot|$ is the absolute value.[21]

5.3. Solution Method: Extending the Sequence-Space Jacobian with Fixed Point.

We solve the model using the Sequence-Space Jacobian Solution Method built by Auclert et al. (2019). The method obtains policy functions through a first order approximation of the model equilibrium conditions. As the approximation involves the Jacobians, the latter are evaluated numerically at each point of the wealth distribution, hence the sequence concept. Among the many benefits, the method also accounts for the impact of shocks on expectations. This is important for our study, whose goal is to study the role of policy, as it avoids the risk of naïve conclusions. We extend the solution method to include a fixed point iteration between the consumption-saving and the occupational decision. Specifically, using a guessed value function, we obtain the policy functions for the consumption-saving problem in each occupation. The latter are then substituted into the value functions to solve for the optimal discrete occupational choice. The guess is updated until convergence. Appendix B contains a detailed description of our algorithm.[22]

21 One half in the equation is coming from the fact that inflow in one occupation is an outflow from another occupation and this flow should be calculated only once.

22 Note that the equilibrium is not locally indeterminate based on the winding number check introduced by Auclert et al. (2019), which is analogous to the Blanchard-Kahn condition.

6. Quantitative Results

In this section our model is simulated to assess its quantitative properties and its matching to the data. While the primary goal is the study of the short run impact of monetary policy, we start by assessing the empirical-fit of the model in relation to long-run distributional statistics, with a focus on occupational sorting. This is generally an important validation step for this class of models. Next, we simulate the dynamic response of the model under a standard 25 basis point contractionary monetary policy shock and assess the monetary transmission mechanism of the model using impulse response functions. To highlight the role of the novel elements we compare the responses of the model with endogenous mobility to the ones of the model in which occupational shares are fixed. This is the closest proximate to the more recent class of heterogenous agents models with nominal rigidities. We close our numerical analysis with the model-based regressions, whose goal is to replicate the ones presented in the empirical analysis.

6.1. Steady State Results

Occupational Sorting. We start by characterizing the forces that determine the long run allocation of labour shares, which have been calibrated on the data as described in section 5.1. Table 4 shows effective labor hours in the steady state supplied in each occupation by each household type. The rows correspond to different household types, and the columns correspond to different occupations. Each entry equals the effective labor hours supplied in each occupation aggregated across idiosyncratic income shocks and asset holdings, i.e. $\int \gamma_j^o e_t \theta_j(o|e_t, a_{t-1}, b_{t-1}) n_{j,t}^o dD_{j,t}(e_t, a_{t-1}, b_{t-1})$. Households supply most hours in the occupation in which they have the highest comparative advantage (i.e., in each column, the entry on the diagonal of the matrix is the highest), a fact well in line with the Roy tradition. Across occupations, effective labor supply is higher for all households in occupations with higher wages, namely occupation 2 (Management) and occupation 4 (Engineering). This too is a classical pattern motivating a large class of models studying occupational mobility or directed search (see for instance Burdett and Mortensen (1998)).

Table 4: Effective Labor Supply by Occupation and Household Type, in the Steady State. Each matrix entry shows the effective labor hours supplied in each occupation in the steady state aggregated across idiosyncratic income shocks and asset holdings, i.e. $\int \gamma_j^o e_t \theta_j(o|e_t, a_{t-1}, b_{t-1}) n_{j,t}^o dD_{j,t}(e_t, a_{t-1}, b_{t-1})$. Each row represents a household's skill type. Each column represents an occupation. The diagonal entries represent households' types with highest level of occupational skills. See text for the definition of households' types and how they relate to the occupations. The last row presents the sum across household types.

Household Type	Occ. 1	Occ. 2	Occ. 3	Occ. 4	Occ. 5	Occ. 6	Occ. 7	Occ. 8
Type 1	0.029	0.250	0.019	0.649	0.008	0.019	0.012	0.022
Type 2	0.017	0.871	0.020	0.239	0.012	0.018	0.015	0.024
Type 3	0.015	0.254	0.041	0.632	0.007	0.018	0.013	0.022
Type 4	0.017	0.168	0.020	0.996	0.012	0.018	0.016	0.023
Type 5	0.016	0.392	0.020	0.525	0.013	0.018	0.014	0.024
Type 6	0.015	0.302	0.019	0.639	0.010	0.035	0.014	0.024
Type 7	0.015	0.298	0.019	0.607	0.008	0.018	0.026	0.025
Type 8	0.017	0.405	0.020	0.526	0.010	0.020	0.015	0.056
Sum across types	0.141	2.940	0.178	4.813	0.079	0.164	0.126	0.221

The probability of choosing an occupation depends on skills, income and wealth. Table 5 shows the steady state probabilities of choosing an occupation for households in the lower ten percent of the wealth distribution for three different realizations of the idiosyncratic income shock. First, regardless of the realization of the income shock, the households choose occupations with the highest wages (occupations 2 and 4).[23] Second, households with the low income realization are more likely to choose non-employment than the households with high or medium realization of income shock. This is well in line with the trend that the Great Resignation is actually stronger for low-paid jobs, something which can be documented with both BLS and CPS data.[24] Third, for the households with the high income realization skills play a less important role in their occupational choice as compared to the households with lower income realization. We conclude by reporting the long run labour shares in table 5. Income-rich households have larger employment shares than income-poor households. Finally, the link between low income and non-employment shares is broadly compatible with classical duration dependence.

Long Run Income and Wealth Distributions. Although the focus of our study is on wage inequality, we are bound to verify that our model preserves the properties of wealth

23 This is in line for instance with recent evidence by Böhm, Gaudecker and Schran (2019) who document the empirical link between high wages and expanding occupations.

24 See for instance https://www.bls.gov/opub/mlr/2022/article/the-great-resignation-in-perspective.htm.

Table 5: Occupational Choice of Asset-Poor Individuals, in the Steady State. The table shows the probabilities of choosing occupations and the non-employment state for the asset-poor households, aggregated across all skill types. The asset-poor are those at the bottom 10% of both the liquid assets distribution and the illiquid assets distribution.

Income shock	Occ. 1	Occ. 2	Occ. 3	Occ. 4	Occ. 5	Occ. 6	Occ. 7	Occ. 8	Non-employment
Low	0.056	0.119	0.056	0.139	0.055	0.056	0.056	0.057	0.405
Medium	0.053	0.162	0.053	0.464	0.055	0.053	0.054	0.053	0.053
High	0.052	0.230	0.049	0.388	0.063	0.050	0.053	0.047	0.069

inequality. Table 6 shows the income and asset distribution statistics in the model and in the data. We compute the empirical statistics from the Survey of Consumer Finance (SCF). Liquid assets consist of the following categories: transactions accounts, directly held bonds, directly held stocks, and credit card balances. Non-liquid assets consist of certificate of deposits, savings bonds, cash value of insurance, other managed assets, retirement accounts, stock holdings, and primary residence net of mortgage home loans. To facilitate the comparison with the steady state numbers in the model, we average the SCF statistics across all available years, 1989-2019. We compute the average asset holdings by different wealth and income percentiles.

The model captures well the average holdings of liquid and illiquid assets (the first two rows in Table 6). The model's Gini coefficients compare well to the empirical Gini coefficients reported in the World Inequality Database and Kaplan et al. (2018). Next, the model matches fairly well the mean assets holdings by different percentiles of the income distribution, except for illiquid asset holdings of the top 10%, which is larger in the data. This tail is generally hard to replicate without including in the model investment in housing. Overall the matching is good relatively to the class of Bewley (1980)-Aiyagari (1994) models and also considering that the labor income distribution is endogenous in our model. The SCF data also contain information on income and assets by broad occupational groups. We compare the SCF group "Managers" to our occupations 2 and 4, and the SCF group "Technical, Sales, Services" to our occupations 5, 6 and 7. The ratios of mean income to liquid assets in the model compare well to the data equivalent.

Finally, Appendix C provides further classical validation checks. In there we plot the wealth and wage distributions. The first is skewed and exhibits a peak at zero. This is due

Table 6: Income and Asset Distributions, Model's Steady State Values and Data. The model statistics are calculated at the steady state. Income statistics from the model includes pre-tax labor income and capital gains but do not include inactivity benefits. Data on income, liquid and non-liquid assets are from the Survey of Consumer Finance, averages over 1989-2019. Liquid assets consist of transactions accounts, directly held bonds, directly held stocks, and credit card balances. Non-liquid assets consist of certificate of deposits, savings bonds, cash value of insurance, other managed assets, retirement accounts, stock holdings, and primary residence net of mortgage home loans. The deciles for illiquid assets holdings are based only on the value of primary residence. The data on Gini coefficients are from the World Inequality Database and Kaplan et al. (2018).

Statistics	**Data**	**Model**
Wealth distribution		
Mean Liquid Assets/GDP	0.26	0.17
Median Illiquid/GDP	2.92	3.89
Gini coefficients		
Income	0.52	0.41
Liquid assets	0.98	0.82
Illiquid assets	0.81	0.47
Shares of liquid assets per income percentile		
less than 20th percent.	0.05	0.03
20th-40th percent.	0.10	0.19
40th-60th percent.	0.08	0.04
60th-80th percent.	0.13	0.40
80th-100th percent.	0.63	0.31
Shares of illiquid assets per income percentile		
less than 20th percent.	0.07	0.07
20th-40th percent.	0.09	0.09
40th-60th percent.	0.11	0.14
60th-80th percent.	0.15	0.28
80th-100th percent.	0.57	0.28
Income/Liquid Assets, by Occupation		
Managers and Professionals	1.80	2.04
Technical, Sales and Services	2.74	2.81

to the share of the borrowed constrained households in the model. In our simulations at the recession's trough, the upper tail becomes thinner and the lower tail fatter. This is even more so for the non-liquid assets distribution.

6.2. Dynamic Responses to Monetary Policy Shock

The main focus on our paper is on the impact of monetary policy on participation, labour reallocation and wage inequality. We measure this quantitatively in two ways. First, through means of impulse response functions we discuss the monetary transmission channel in our model, also linking to the baseline intuition proposed through the analytical results. Next, we simulate our model in response to monetary policy shocks and compute the model-based counterpart of our empirical regression specification linking wage inequality and separations, defined as employment to unemployment shifts, to monetary policy shocks. The goal of this comparison is to evaluate the empirical fit of the model.

Impulse Response Analysis. We start by simulating the model's response to a standard 25 basis point contractionary monetary policy shock. To appreciate the role of occupational reallocation and endogenous participation decisions impulse responses are compared in the models with and without mobility, that is when labour shares remain fixed at the steady-state values.[25] The latter provides a good experimental counterfactual to assess the role of mobility.

Figure 3 displays the impulse responses to a contractionary 25 basis points monetary shock for selected variables, namely GDP, investment, consumption, policy rule, average wage (weighted by shares in each occupation), total effective labor hours, non-employment share, the labour aggregator, the wage Gini, liquid and non-liquid assets dynamic and mobility, measured as weighted average of the transition probabilities. Blue lines provide the responses of our model, while orange lines provide the responses of the model in absence of mobility. The baseline channels noted elsewhere in the class of models with uninsurable risk and nominal rigidities are operative here as well. Aggregate consumption falls for two reasons. The first is the rise in precautionary savings, which follows the contractionary shock and

25 Note that this scenario is not exactly equivalent to the most proximate models with uninsurable risk and nominal rigidities.

is stronger in models with uninsurable risk.[26] The second is the loss of wealth or interest rate exposure.[27] The decline in asset prices, induced by the monetary tightening, reduces the value of assets, so that both aggregate liquid and non-liquid assets decline. This in turn induces a fall in consumption for households, who are not borrowed constrained. The increase in the interest rate equally reduces consumption for households who are borrowed constrained: as their income falls, their consumption falls too. Note that non-liquid assets decline by much more: this is still due to precautionary savings motives which induce a flight to liquidity.[28]

Our framework features a number of additional channels, both on the labour supply and demand side. A key novel transmission runs from the heterogenous declines in wealth and wages to the transition probabilities. In the model with mobility, the stronger decline in wages for bottom earners induces them to leave the labour force and discourages their mobility across occupations. The mobility patterns in turn affect the changes in the wage distribution. More specifically the novel mechanism runs as follows.

First, the rise in interest rates and the contraction in output induce a decline in labour demand, which in turn reduces wages in all occupations (see first panel of figure 4). At the aggregate level this effect alone would heighten precautionary saving and steepen the contraction. Across occupations the decline in wages is stronger for low-wage occupations (see again first panel of figure 4), confirming the disproportionate impact of monetary policy on low-wage workers. The decline in income and wealth affects the value-functions, which in turn reduce transition probabilities, more so for all workers on the tails of the wage distribution. Likewise in the data the monetary transmission mechanisms runs primarily through separation and re-employment rates. Workers in the highest wage occupation mostly stay in the labor force (see the third panel of the figure 4) whereas others much more likely transition to non-employment.[29] The reduction of labour supply at the tail of the wage

26 The rise in saving is stronger for households with higher marginal propensity to consume. See Auclert (2019) among others.

27 See Auclert (2019).

28 See Bayer, Born, Luetticke and Müller (2020) for a model with uninsurable risk highlighting this mechanism.

29 The highest wage occupation is occupation 4. Households of type 4, whose skills are best-suited for the occupation 4, mostly choose this occupation (80% of them are employed in occupation 4 in the steady state). The increase in the non-employment probability for households of type 4 following a monetary policy shock (in brown, panel 3 of the figure 4) is much less than the responses of other types of households.

distribution produces a mean preserving spread of the latter, hence a decline in its dispersion, as shown by the wage Gini.

The dynamic of wages results from a combination of channels. On impact labour demand falls by more for low-wage workers. As they exit the labour force the decline in their labour force puts an upward pressure on wages. Furthermore, the low-wage workers that remain in the labour force tend to reallocate toward higher paying jobs, something which rises their wages even further. The rise in wages of low-paid jobs is compatible with the evidence provided in Section 2. This selection through reallocation channel, coupled with the mean preserving change in the workers distribution, explains the decline in wage inequality.

Two further considerations are noteworthy to characterize the transmission mechanism of our model. First, in the model with mobility the recession is stronger. As more workers exit the labour force and have low probability of reallocating, the decline in production and employment is stronger. Second, note that in the model with fixed probabilities, changes in labour shares are due only to changes in the intensive margin and are much lower than in the model with workers' mobility. The lower intensity of the reallocation implies that wages vary much more in the model with fixed shares. In particular, and following the initial decline, wages of low earners increase by more in this model. This results in a further squeeze of the wage Gini. This is the sense in which labour reallocation tames the impact of macro and policy shocks onto the macroeconomy.

To confirm the empirical fit of the model also in quantitative terms our next step is to employ the model-simulated data to reproduce the regressions linking wage inequality and separation rates to the monetary policy shock that we estimated in the data.

6.3. Model Based Regressions: Wage Inequality and Separations

To assess the empirical fit of the model we now estimate the econometric specifications presented in Section 2 by using simulated data. The comparison between the empirical estimates and the ones obtained in the model will confirm the empirical fit of the latter. To tighten up the model-data comparison we replicate both the wage and the finding-separation specifications. This will allow us also to confirm that the mechanics of the shock transmission

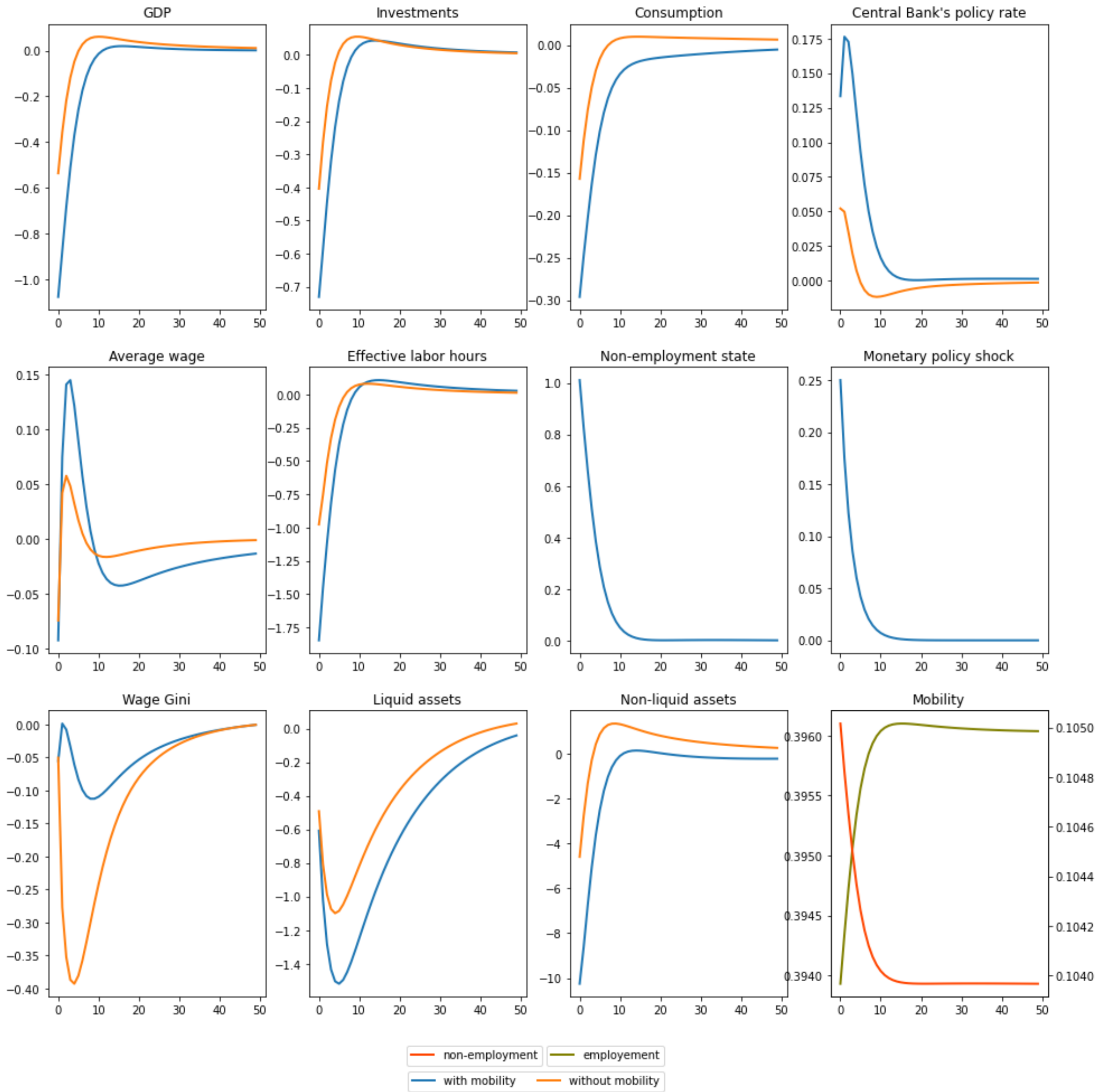

Figure 3: Impulse Responses to a Monetary Policy Shock. The figure shows the percentage deviations of the specified series from the steady state in response to a monetary policy shock. The shock is a 25 b.p. increase in the Central Bank's interest rate; the shock follows an AR(1) process with the persistence parameter 0.7, such that the shock almost dissipates by the 13^{th} quarter. The figure shows the impulse responses from the model with the occupational mobility (in blue) and from the model without it (in orange). Central Bank follows the Taylor rule. The average wage is the weighted average wage across occupations where the weights are occupational employment shares in terms of efficiency units. The effective labor hours is the sum of the labor hours across occupations. The non-employment share shows the share of households that choose non-employment. Mobility panel shows probability (aggregated across households and occupations) of being employed and probability of being in the non-employed state (aggregated across households). Probability of being in the non-employed state is shown on the right axis. X-axis shows the time in quarters.

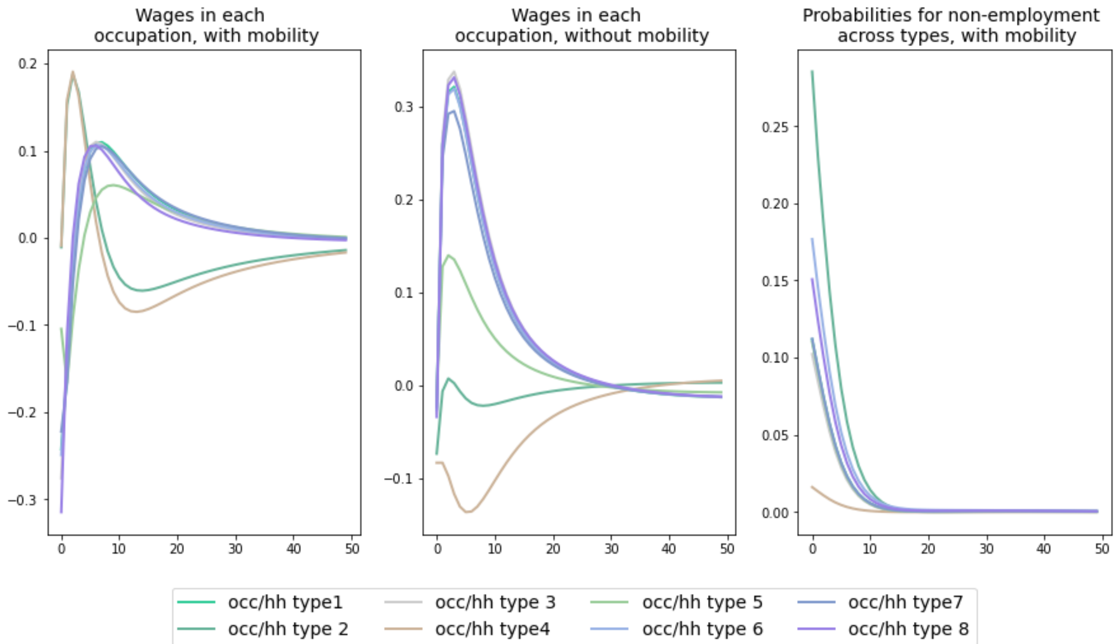

Figure 4: Impulse Responses to the Monetary Policy Shock. The figure shows the responses of the selected variables in response to a monetary policy shock. The shock is a 25 b.p. tightening and the shock follows an AR(1) process with the persistence parameter 0.7, such that the shock almost dissipates by the 13^{th} quarter. Wages are shown for each occupation and in percentage deviations from the steady state. Probabilities of non-employment are shown for different types of households and are presented in deviations from the steady state. X-axis shows the time in quarters.

is equivalent between the model and the data. Noteworthy is that the dynamic of the wage distribution in our model is endogenous and determined by the reallocation channels: hence any replication of the empirical results can be driven only by the equivalence of the transmission mechanism and cannot be induced by convenient parameter choices. All results for the model based regression are shown in Table 7 and Table 8.

The upper part of Table 7 is a general exploration of the link between contractionary shocks and inequality, measured through the model-based Gini. Monetary policy affects inequality with a lag. On impact the coefficient is positive and insignificant, while the one on the lagged shock is negative and significant at the 1% confidence interval. This confirms that in the model, likewise in the data, a contractionary monetary policy shock reduces inequality. The next step is to dissect the channels as we did in our empirical analysis. The second block of Table 7 starts by examining the impact of the contractionary shock on separation rates of earners above and below the median. Here too the model confirms the empirical estimates closely. The impact of the contractionary shock is negative and stronger for the bottom earners. The latter exit the market, hence their wages cease to be part of the wage distribution, causing a decline in its dispersion.[30]

Next, and likewise in our empirical analysis, we estimate the impact of the contractionary shock on year-on-year wage changes with quantile regressions. Table 8 shows the coefficient for workers earning above and below the median. The model replicates once again the empirical estimates. The coefficients are positive for bottom earners and negative for top earners. Through the lens of the model this can only be due to a selection effect. Workers that remain in the labour market only move toward higher paying jobs, if they change jobs. The size of the coefficient for the bottom earners is somewhat larger than the one in the data. This s likely due to the fact that the model features no nominal wage rigidities or further hiring frictions that may impede adjustment along the labour demand, an avenue that we plan to pursue in future research.

Table 7: Regression coefficients for real quarterly earnings on monetary policy shocks in the top part of the table; regression of Wage Gini on monetary policy shock in the middle part of the table; regression for separation rates, defined as employment to unemplopyment transitions, for different income percentiles (and for given wealth percentile at 20%) over the monetary policy shock in the bottom part of the table. The errors are calculated using heteroscedasticity and autocorrelation robust covariance matrix. Estimation is performed using WLS regression with weights assigned according to the households' distribution.

Regression Wage Gini		
	Coefficient	T-stat
Monetary policy shock	0.1861	1.42
Lag of monetary policy shock	-2.3596	-19.73
Lag of dlog(Gini)	0.9871	21.39
Second lag of dlog(Gini)	-0.1980	-4.23
Regression Separation Rates		
	Coefficient	T-stat
Bottom decile	0.0074	3.39
Top decile	-0.0078	-2.77
Bottom half	0.0010	3.62
Top half	-0.0010	-2.05

30 Note that since the model tracks both the wealth and the income distribution, to make the regression comparable to the empirical counterpart we run the quantile estimates for given level of wealth, which in this case is given by the top 20%. Results are robust also across other deciles of the wealth distribution.

Table 8: Regression coefficients for log differences of real quarterly earnings on monetary policy shocks. The differences are for a one-year change in wages in line with the data regressions. Estimation is performed using WLS regression with weights assigned according to the households' distribution. The errors are calculated using heteroscedasticity and autocorrelation robust covariance matrix.

Regression without top/bottom dummies	Coefficient	T-stat
Intercept	-0.007	-237.95
Monetary policy shock	-0.001	-3.59
Regression with top/bottom dummies	Coefficient	T-stat
Dummy Bottom 50%	-0.010	-10.54
Dummy Top 50%	-0.003	6.23
Monetary policy shock * dummy Bottom 50%	0.012	4.23
Monetary policy shock * dummy Top 50%	-0.014	-8.10

6.4. The Role of Job Specialization and Skill-Transferability

The transmission of monetary policy through the labour market may be affected by structural and institutional characteristics. On the structural side there is increasing evidence that recent technological trends, such as offshoring and automation, have increased skill-tasks specialization[31]. As workers' skills become less substitutable, the elasticity of labour demand declines and so does the extent of reallocation. Our proposition 4 had indeed highlighted the role of occupational substitutatibility $\frac{1}{\sigma_s}$ for the extent of reallocation. We now examine the role of labour elasticity numerically. Specifically we compare the responses under the baseline calibration to those with higher σ. The latter represent a counterfactual of a more fluid labour market.

Figure 5 shows the dynamics of the usual selected variables in response to the monetary policy shock and figure 6 displays the pattern of wages and transition probabilities to non-employment. Overall differences in the elasticity of labour demand seem to have little effect both, on the cross-sectional variation of wages and transition probabilities and, as a result, on the macro dynamic. We conjecture that, absent nominal wage rigidity, the impact of the macro shocks on labour demand is more muted.

Countries and regions may also differ in the extent to which their educational system

31 See Autor, Levy and Murnane (2003) for early evidence on task-biased technological change. See Cortes, Lerche, Schönberg and Tschopp (2020) for similar evidence for Germany. See Faia, Ottaviano and Spinella (CEPR, w.p. 2022) for evidence on core-task biased technological change.

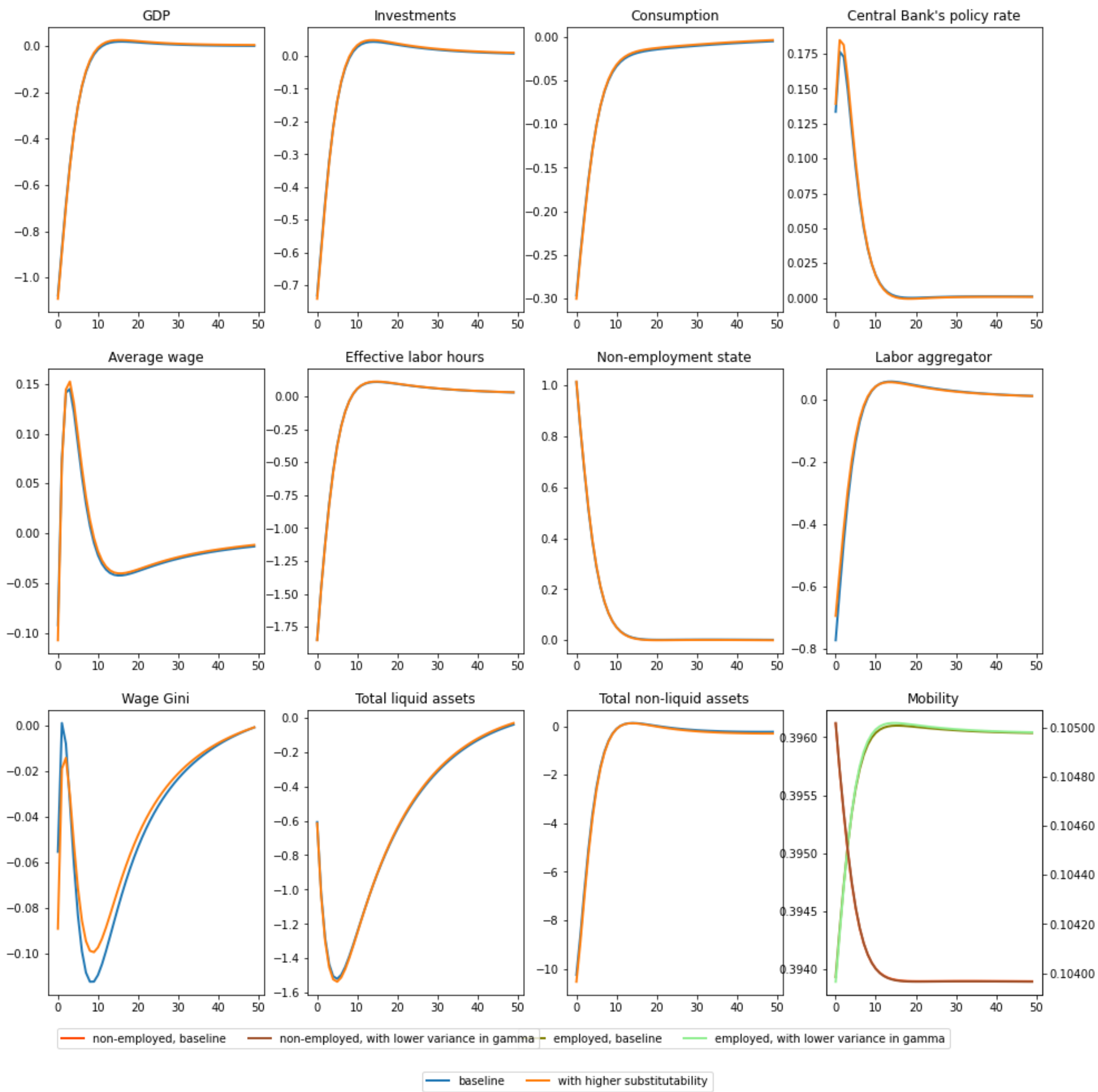

Figure 5: Impulse Responses to the Monetary Policy Shock under Different Substitutability of Labor. The figure shows the percentage deviations of the selected variables from the steady state in response to a monetary policy shock. The shock is a 25 b.p. tightening and the shock follows an AR(1) process with the persistence parameter 0.7, such that the shock almost dissipates by the 13^{th} quarter. The figure shows the impulse responses from the model with the baseline substitutability of labor (with $\sigma_s = 0.2$ in blue) and from the model with a higher substitutability of labor (with $\sigma_s = 0.5$ in orange). The average wage is the weighted average wage across occupations where the weights are occupational employment shares in terms of efficiency units. The effective labor hours is the sum of the labor hours across occupations. The non-employment share shows the share of households that choose non-employment. Labor aggregator is L_s and shows the aggregated labor used in the production. Mobility panel shows probability (aggregated across households and occupations) of being employed and probability of being in the non-employed state (aggregated across households). Probability of being in the non-employed state is shown on the right axis. X-axis shows the time in quarters.

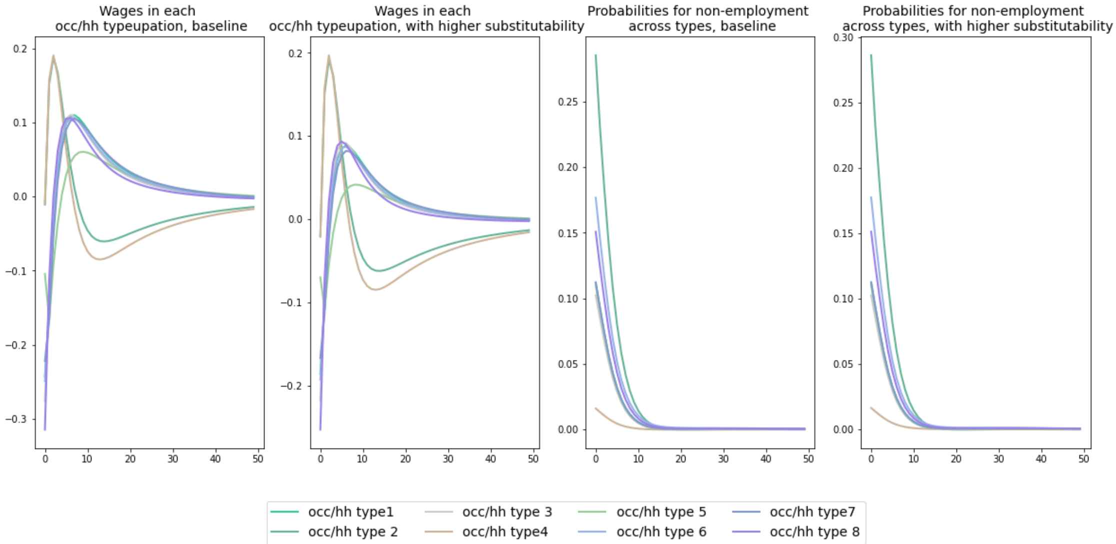

Figure 6: Impulse Responses to the Monetary Policy Shock under Different Substitutability of Labor. The figure shows the responses of the selected variables in response to a monetary policy shock. The shock is a 25 b.p. tightening and the shock follows an AR(1) process with the persistence parameter 0.7, such that the shock almost dissipates by the 13^{th} quarter. The figure shows the impulse responses from the model with the baseline substitutability of labor (with $\sigma_s = 0.2$) and from the model with a higher substitutability of labor (with $\sigma_s = 0.5$). Wages are shown for each occupation and in percentage deviations from the steady state. Probabilities of non-employment are shown for different types of households and are presented in deviations from the steady state. X-axis shows the time in quarters.

provides more or less equalizing skills and access to opportunity. This too may have important consequences for the extent to which workers can move across jobs. The model's primitive capturing the degree of workers' skill transferability across occupations is the skewness of the skill distribution matrix, Γ.

Figure 7 compares impulse responses of selected variables to the usual monetary policy shock under the model with the baseline skill distribution matrix and with one characterized by the lower variance. The dynamic of wages is now more even across occupations and wages of low-earners increase by more. Firms have now more incentives to retain them in the labour force. This induces less of the low-wage earners to opt out of the labour force (see last mobility patterns in the last panel of figure 7 and the forth panel in the figure 8). Their attachment to the labour force and the low mobility results in a rise in wage dispersion. As job flows react by less, wages react by more to the shock. Hence in system with more equal skill distribution a tightening in monetary policy may result in a rise in wage inequality.

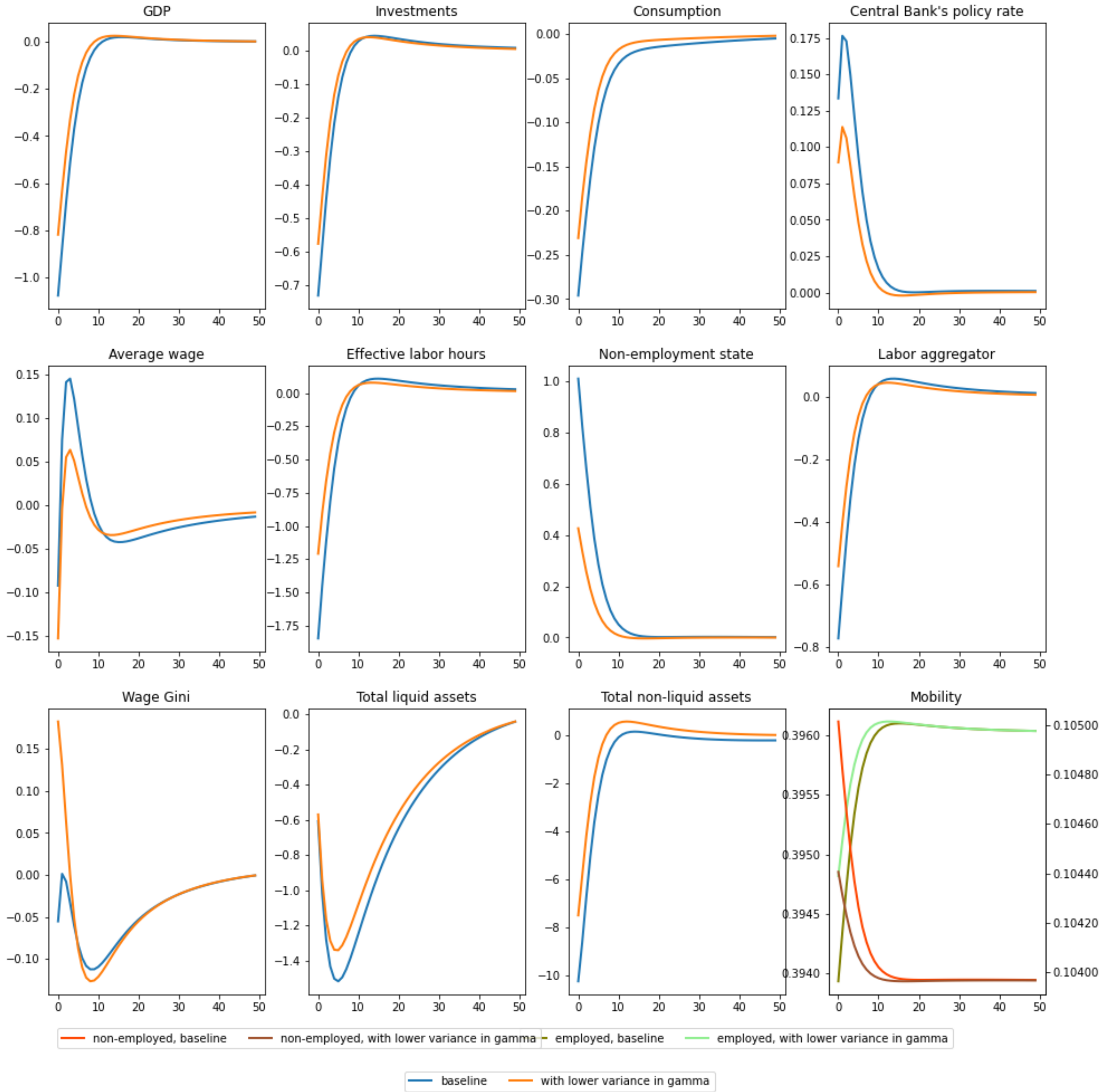

Figure 7: Impulse Responses to the Monetary Policy Shock under the Baseline and a Lower Variance of the Skill Distribution. The figure shows the percentage deviations of the selected variables from the steady state in response to a monetary policy shock. The shock is a 25 b.p. tightening and the shock follows an AR(1) process with the persistence parameter 0.7, such that the shock almost dissipates by the 13^{th} quarter. The figure shows the impulse responses from the model with the baseline degree of skill transferability across occupations (in blue), and from the model with a higher degree of skill transferability (in orange). γ in the baseline model is obtained by using $exp(\gamma)/(1+\gamma)$ transformation, γ in the version with the lower variation in gamma is obtained as $exp(\gamma)/(1+\gamma+\frac{\gamma^2}{2})$. The latter γ specification delivers four times smaller standard deviation (the skewness and kurtosis in the two specification are close to each other, with a bit larger values for the second specification, 1.7 vs. 2 for skewness and 4.5 vs. 5.3 for kurtosis). The average wage is the weighted average wage across occupations where the weights are occupational employment shares in terms of efficiency units. The effective labor hours is the sum of the labor hours across occupations. The non-employment share shows the share of households that choose non-employment. Labor aggregator is L_s and shows the aggregated labor used in the production. Mobility panel shows probability (aggregated across households and occupations) of being employed and probability of being in the non-employed state (aggregated across households). Probability of being in the non-employed state is shown on the right axis. X-axis shows the time in quarters.

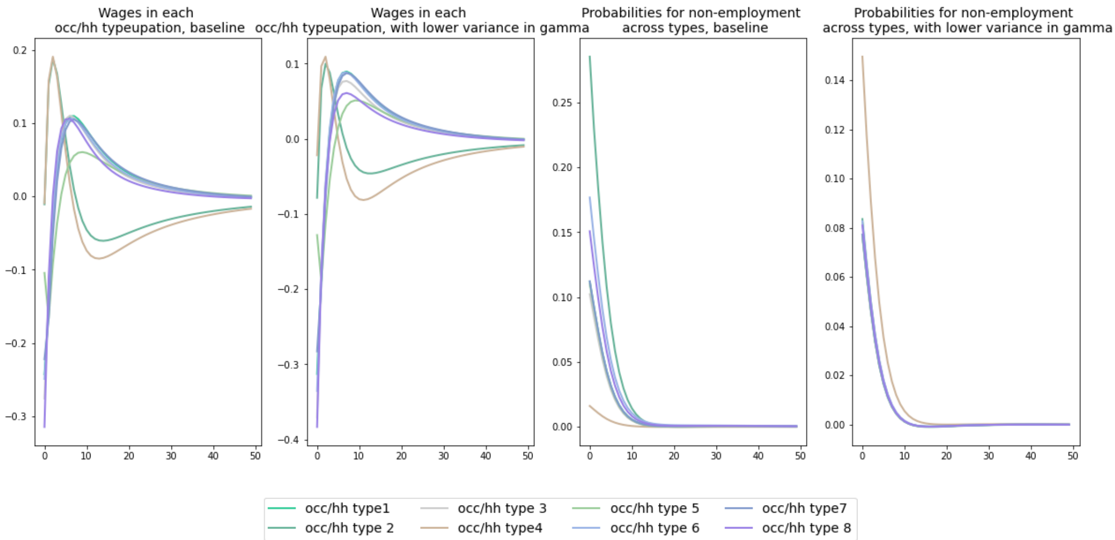

Figure 8: Impulse Responses to the Monetary Policy Shock under the Baseline and a Lower Variance of the Skill Distribution. The figure shows the responses of the selected variables in response to a monetary policy shock. The shock is a 25 b.p. tightening and the shock follows an AR(1) process with the persistence parameter 0.7, such that the shock almost dissipates by the 13^{th} quarter. The figure shows the impulse responses from the model with the baseline degree of skill transferability across occupation, and from the model with a higher degree of skill transferability. γ in the baseline model is obtained by using $exp(\gamma)/(1+\gamma)$ transformation, γ in the version with the lower variation in gamma is obtained as $exp(\gamma)/(1+\gamma+\frac{\gamma^2}{2})$. The latter γ specification delivers four times smaller standard deviation (the skewness and kurtosis in the two specification are close to each other, with a bit larger values for the second specification, 1.7 vs. 2 for skewness and 4.5 vs. 5.3 for kurtosis). Wages are shown for each occupation and in percentage deviations from the steady state. Probabilities of non-employment are shown for different types of households and are presented in deviations from the steady state. X-axis shows the time in quarters.

7. Conclusions

In times of increasing inequality, large structural reallocation and an upsurge in resignation there is increasing need to understand the distributional consequences of policies, particularly monetary policy whose stance is typically used first to combat recessions. There is indeed an expanding literature that studies the distributional consequences of monetary policies in models with uninsurable risk and nominal rigidities, hence much progress has been done on that front. This literature has focused in particular on the elasticity saving channel, which is of first order importance for the study of wealth inequalities. Much less has been done in terms of understanding a key monetary policy transmission channel, namely the one passing through the labour market. This requires indeed to extend models with uninsurable risk to include an income process whose cross-sectional variation endogenously changes through the labour market equilibrium across occupational islands. Most importantly it requires to extend such framework to include agents heterogenous both in terms of their idiosyncratic income risk (luck component) and talents/skills.

Our analysis starts by documenting a decline in wage inequality following a monetary tightening. Exploiting the heterogenous impact of monetary shocks along the income distribution we find that the decline in wage dispersion is induced primarily by the exit of low-wage earners from the labour force. The cut in the lower tails results in a wage compression. Furthermore, the stayers among the bottom earners tend to reallocate toward higher paying job, a selection through reallocation channel that favors a shrink in the wage dispersion.

Next, we build a multi-occupation model in which agents are heterogeneous in skills and are exposed to idiosyncratic income risk, along the Bewley (1980)-Aiyagari (1994) tradition. We introduce Roy-type occupational choice driven by skill comparative advantage and use the model to study economies response to monetary policy shocks, focusing on the role of the participation and the reallocation margin for growth and inequality. A key novelty is a two way link between the heterogenous responses of wealth and income and that of the transition probabilities, which depend on the first. In the model too a monetary tightening reduces inequality mainly through a larger exit of bottom earners and a move of the stayers toward higher paying wages. Model-based regressions that map the empirical equivalent confirm the empirical fit of the model.

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A. Analytical Derivations

Labour Supply Elasticities. Formula for the elasticity of labor income that is solved for its impact on reallocation reads as follows.

$$
\begin{aligned}
\varepsilon_{\xi_t^o, r_t} = & \left(1 + (1-\sigma)\sum_{j=1}^{J} m_j \gamma_j^o n_t \int e_t \theta_{j,t}^o u_c \xi_{j,t}^o dD_j\right)^{-1} \cdot \\
& \cdot \left[-\frac{\tau_t}{1-\tau_t}\varepsilon_{\tau_t, r_t} + \sigma \varepsilon_{n_t, r_t} + (1-\sigma)\frac{r_t}{r_t+\delta} + \sigma \left(\sum_{o''=1}^{O} \alpha_{o''}^{\frac{1}{1-\sigma}} w_{o'',t}^{\frac{-\sigma}{1-\sigma}} \cdot \varepsilon_{w_{o'',t}, r_t}\right)\left(\sum_{o''=1}^{O} \alpha_{o''}^{\frac{1}{1-\sigma}} w_{o'',t}^{\frac{-\sigma}{1-\sigma}}\right)^{-1} - \right. \\
& -(1-\sigma)\frac{n_t}{l_{o,t}}\sum_{j=1}^{J} m_j \gamma_j^o \int e_t \theta_j^o(e_t, a_{t-1}, b_{t-1}) \left(r_t u_c \left(\frac{\partial r_t^a}{\partial r_t} a_{t-1} + \frac{\partial r_t^b}{\partial r_t} b_{t-1} - (\Phi_1' + 1)\frac{\partial a_t^{o,*}}{\partial r_t} - \frac{\partial b_t^{o,*}}{\partial r_t}\right) + \right. \\
& \left.\left. + \ r_t u_n \frac{\partial n_t}{\partial r_t} + \beta r_t E_\phi E_e \frac{\partial V_j^o(e_{t+1}, a_t, b_t)}{\partial r_t} - r_t \frac{\sum_{o''=1}^{O+1} exp(V_j^{o''}(e_t, a_{t-1}, b_{t-1}))\frac{\partial V_j^{o''}}{\partial r_t}}{\sum_{o''=1}^{O+1} exp(V_j^{o''}(e_t, a_{t-1}, b_{t-1}))}\right) dDj \right]
\end{aligned}
\tag{A.1}
$$

Proof of Proposition 1. Starting from the definition of labor income

$$
\xi_t^o = (1-\tau_t) e_t \gamma_j^o w_t^o n_t \tag{A.2}
$$

the derivative of the labor income with respect to the interest rate could be rewritten as:

$$
\varepsilon_{\xi_t^o, r_t} = \frac{\partial \xi_t^o}{\partial r_t}\frac{r_t}{\xi_t^o} = -\frac{\partial \tau}{\partial r_t}\frac{\xi_t^o}{1-\tau_t}\frac{r_t}{\xi_t^o} + \frac{\partial n_t}{r_t}\frac{\xi_t^o}{n_t}\frac{r_t}{\xi_t^o} + \frac{\partial w_t^o}{r_t}\frac{\xi_t^o}{w_t^o}\frac{r_t}{\xi_t^o} = -\frac{\tau_t}{1-\tau_t}\varepsilon_{\tau_t, r_t} + \varepsilon_{n_t, r_t} + \varepsilon_{w_t^o, r_t} \tag{A.3}
$$

From the labor demand equation (eq. (A.16)) and the labor aggregator eq. (17):

$$
l_{o,t} = \left(\frac{mc_t(1-\nu)\alpha_o}{w_t^o}\right)^{\frac{1}{1-\sigma}} y_t^{\frac{1}{1-\sigma}} L_t^{\frac{-\sigma}{1-\sigma}} \tag{A.4}
$$

$$
L_t = \left(\sum_{o=1}^{O} \alpha_o l_{o,t}^{\sigma}\right)^{\frac{1}{\sigma}} = mc_t(1-\nu) y_t \left(\sum_{o=1}^{O} \alpha_o^{\frac{1}{1-\sigma}} (w_t^o)^{\frac{-\sigma}{1-\sigma}}\right)^{\frac{1-\sigma}{\sigma}} \tag{A.5}
$$

where in the second equation $l_{o,t}$ was substituted from the first equation and then the equation was solved for L_t. Taking derivatives with respect to r_t of the $l_{o,t}$, using the expression for L_t and solving for the $\varepsilon_{w_t^o,r_t}$ delivers:

$$\varepsilon_{w_t^o,r_t} = (1-\sigma)\frac{\partial mc_t}{\partial r_t}\frac{r_t}{mc_t} + (1-\sigma)\frac{\partial y_t}{\partial r_t}\frac{r_t}{y_t} - (1-\sigma)\varepsilon_{l_t^o,r_t} + \sigma\left(\sum_{o''=1}^{O}\alpha_{o''}^{\frac{1}{1-\sigma}}(w_t^{o''})^{\frac{-1}{1-\sigma}}\frac{\partial w_t^{o''}}{\partial r_t}\right)\left(\sum_{o''=1}^{O}\alpha_{o''}^{\frac{1}{1-\sigma}}(w_t^{o''})^{\frac{-\sigma}{1-\sigma}}\right)^{-1} \quad (A.6)$$

The derivative of l_t^o with respect to r_t could be obtained from the labor market clearing condition:

$$\varepsilon_{l_t^o,r_t} = \varepsilon_{n_t,r_t} + \frac{n_t r_t}{l_{o,t}}\sum_{j=1}^{J} m_j \gamma_j^o \int e_t \frac{\partial \theta_j^o(e_t,a_{t-1},b_{t-1})}{\partial r_t} dD_j(e_t,a_{t-1},b_{t-1}) \quad (A.7)$$

and the derivatives of mc_t and y_t could be obtained from the demand for capital (assuming no capital adjustment costs):

$$r_t + \delta = \nu mc_t \frac{y_t}{k_{t-1}} \quad (A.8)$$

$$1 = \frac{\partial mc_t}{\partial r_t}\frac{r_t+\delta}{mc_t} + \frac{\partial y_t}{\partial r_t}\frac{r_t+\delta}{y_t} \quad (A.9)$$

The final result of Proposition 1 is obtained upon substitution of all derived elasticities into eq. (A.3).

Proof of Corollary 1. The decomposition in corollary 1 is obtained by substituting elasticity of labor income to interest rates from Proposition 1 into eq. (37).

Proof of Corollary 2. Corollary 2 is derived using the formula obtained in Proposition 1 and using the definition of covariance:

$$\sum_{j=1}^{J}\int e_t\gamma_j^o \frac{\partial \theta_j^o(e_t,a_{t-1},b_{t-1})}{\partial r_t} m_j dD_j(e_t,a_{t-1},b_{t-1})$$
$$= Cov_I\left(e_t\gamma_j^o, \frac{\partial \theta_j^o(e_t,a_{t-1},b_{t-1})}{\partial r_t}\right) + \left(\sum_{j=1}^{J} m_j\gamma_j^o\right) E_I \frac{\partial \theta_j^o(e_t,a_{t-1},b_{t-1})}{\partial r_t} \quad (A.10)$$

Proof of Proposition 2. Starting from the expression for the probability of choosing occupation o by household of type j:

$$\theta_j^o(e_t, a_{t-1}, b_{t-1}) = \frac{exp(V_j^o(e_t, a_{t-1}, b_{t-1}))}{\sum_{o''=1}^{O+1} exp(V_j^{o''}(e_t, a_{t-1}, b_{t-1}))} \tag{A.11}$$

the elasticity of probability with respect to the interest rate could be written as:

$$\varepsilon_{\theta_j^o, r_t} = r_t \frac{\partial V_j^o(e_t, a_{t-1}, b_{t-1})}{\partial r_t} - r_t \frac{\sum_{o''=1}^{O+1} exp(V_j^{o''}(e_t, a_{t-1}, b_{t-1})) \frac{\partial V_j^{o''}}{\partial r_t}}{\sum_{o''=1}^{O+1} exp(V_j^{o''}(e_t, a_{t-1}, b_{t-1}))} \tag{A.12}$$

Using definition of the value function at the optimum:

$$V_j^o(e_t, a_{t-1}, b_{t-1}) = u(c_t^{o,*}, n_t^*) + \beta E_e E_\phi V_j^o(e_{t+1}, a_t^{o,*}, b_t^{o,*}) \tag{A.13}$$

The derivative of the value function with respect to the interest rate could be expressed as:

$$\frac{\partial V_j^o(e_t, a_{t-1}, b_{t-1})}{\partial r_t} = u_c \frac{\partial c_t^{o,*}}{\partial r_t} + u_n \frac{\partial n_t}{\partial r_t} + \beta E_e E_\phi \frac{\partial V_j^o(e_{t+1}, a_t^{o,*}, b_t^{o,*})}{\partial r_t} \tag{A.14}$$

Finally, derivative of the optimal consumption is equal to the following expression:

$$\frac{\partial c_t^{o,*}}{\partial r_t} = \frac{\partial r_t^a}{\partial r_t} a_{t-1} + \frac{\partial r_t^b}{\partial r_t} b_{t-1} - \Phi_1(a^{o,*}, a_{t-1}) - \frac{\partial a^{o,*}}{\partial r_t} - \frac{\partial b^{o,*}}{\partial r_t} + \frac{\partial \xi_j^o}{\partial r_t} \tag{A.15}$$

The result in Proposition 2 is obtained by substituting derived derivatives into eq. (A.12).

Proposition 3. From the intermediate firm's problem, the first-order condition for the demand for labor in each occupation reads as follows:

$$l_{o,t} = \left(\frac{p_t(1-\nu)\alpha_o}{\mu_p p_t w_{o,t}} \right)^{\frac{1}{1-\sigma}} \left(z K_{t-1}^\nu \right)^{\frac{\sigma}{(1-\nu)(1-\sigma)}} y_t^{\frac{\sigma-1+\nu}{(1-\nu)(\sigma-1)}} \tag{A.16}$$

Dividing the labor demand in one occupation by the labor demand in another, but for the same sector, delivers: $\frac{l_o}{l_{o'}} = \left(\frac{\alpha_{o'} w_o}{\alpha_o w_{o'}} \right)^{\frac{-1}{1-\sigma}}$, which implies that:

$$w_{o'}^{\frac{\sigma}{\sigma-1}} l_o^\sigma \alpha_{o'}^{\frac{-1}{\sigma_s - 1}} = \left(\frac{w_o}{\alpha_o} \right)^{\frac{\sigma}{\sigma-1}} \alpha_{o'} l_{o'}^\sigma \tag{A.17}$$

which upon expressing l_o in terms of the other variables and upon some re-shuffling, it can be summed over both sides by o' to obtain: L:

$$l_o^{\sigma}\sum_{o'=1}^{O} w_{o'}^{\frac{\sigma}{\sigma-1}}\alpha_{o'}^{\frac{-1}{\sigma-1}} = \left(\frac{w_o}{\alpha_o}\right)^{\frac{\sigma}{\sigma-1}} L^{\sigma} \tag{A.18}$$

Isolating l_o from the A.18 delivers $l_o = \left(\frac{w_o}{\alpha_o}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\sum_{o'=1}^{O} w_{o'}^{\frac{\sigma}{\sigma-1}}\alpha_{o'}^{\frac{-1}{\sigma-1}}}\right)^{\frac{1}{\sigma}} L$, which, after using Equation (A.16), can be written as:

$$l_o = \left(\frac{w_o}{\alpha_o}\right)^{\frac{1}{\sigma-1}}\left(\frac{F}{\sum_{o'=1}^{O} w_{o'}l_{o'}}\right)^{\frac{1}{\sigma}} L \tag{A.19}$$

where $F = \left((1-\nu)L^{-\sigma}\frac{Y}{\mu_p}y\right)^{\frac{1}{1-\sigma}}$. Using Equation (A.19), the elasticity of labor demand with respect to wages, with a continuum of occupations, is:

$$\varepsilon_{l,w} = \frac{\partial l_o}{\partial w_o}\frac{w_o}{l_o} = \frac{1}{\sigma-1} \tag{A.20}$$

while it reads as follows: $\varepsilon_{l,w} = \frac{1}{\sigma-1}\left(1-\frac{l_o w_o}{\sum_{o'=1}^{O} l_{o'}w_o'}\right)$ under a finite number of occupations. The markup is then the inverse of the following expression (under an infinite number of occupations):

$$\mu_w = \frac{\varepsilon_{l_{l,w}}}{\varepsilon_{l,w}+1} = \frac{1}{\sigma} \tag{A.21}$$

and the inverse of the following expression with finite number of occupations: $\mu_w = \frac{1-\frac{l_o w_o}{\sum_{o'=1}^{O} l_{o'}w_o'}}{\sigma-\frac{l_o w_o}{\sum_{o'=1}^{O} l_{o'}w_o'}}$. This proves part (1). When $\sigma = 1$ the mark-up is also one and the labor aggregator is $L = \sum_{o=1}^{O}\alpha_o l_{o,t}$, hence occupations are perfectly substitutable. This proves part (2). When $\alpha_o = 1$, then the total labor demand is equal to

$$\sum_{o=1}^{O} l_o = \frac{\sum_{o=1}^{O} w_o^{\frac{1}{\sigma-1}}}{\left(\sum_{o'=1}^{O} w_{o'}^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{\sigma}}} L \tag{A.22}$$

When $\sigma = 1$ it follows that: $l_o = I(w_o = \min_o(w_o))L = \frac{1}{w_o}\frac{p}{p}\frac{\eta-1}{\eta}(1-\nu)yI(w_o = \min_o(w_o))$

with $\sum_{o=1}^{O} l_o = L$ and where $I(w_o = \min_o(w_o))$ is an indicator function that equals 1 for the occupation with the lowest wage.

Derivations of Job Flows in the Model. We calculate gross flows across occupations in the model as follows:

1. Occupational choice in period t is described by $\theta_j(o|e_t, a_{t-1}, b_{t-1})$ and in period $t+1$ by $\theta_j(o'|e_{t+1}, a_t, b_t)$
2. There is a change in occupation (flow) between t and $t+1$ if $o' \neq o$
3. Recall that $\tilde{V}_j^o$ is the value function $V_j^o(e_t, a_{t-1}, b_{t-1})$ evaluated at the optimal consumption-saving policy $c_j^o(e_t, a_{t-1}, b_{t-1})$, $a_j^o(e_t, a_{t-1}, b_{t-1})$ and $b_j^o(e_t, a_{t-1}, b_{t-1})$. Write $V_j^o(e_t, a_{t-1}, b_{t-1})$ for the value function in t and $V_j^o(e_{t+1}, a_t, b_t)$ for the value function at $t+1$.
4. There is a change in occupation o between period t and $t+1$ if

   a) in period t

   $$V_j^o(e_t, a_{t-1}, b_{t-1}) + \phi_t^o \geq V_j^{o'}(e_t, a_{t-1}, b_{t-1}) + \phi_t^{o'} \quad \forall o' \neq o \in O+1 \tag{A.23}$$

   or, alternatively, that

   $$o = \text{argmax}_{o' \in [1,.....,O,O+1]}[V_j^{o'}(e_t, a_{t-1}, b_{t-1}) + \phi_t^{o'}] \tag{A.24}$$

   b) and in period $t+1$ there exist at least one $o' \in O+1$ such that

   $$V_j^o(e_{t+1}, a_t, b_t) + \phi_{t+1}^o \leq V_j^{o'}(e_{t+1}, a_t, b_t) + \phi_{t+1}^{o'} \tag{A.25}$$

   or, alternatively, that

   $$o \neq \text{argmax}_{o' \in [1,.....,O,O+1]}[V_j^{o'}(e_{t+1}, a_t, b_t) + \phi_{t+1}^{o'}] \tag{A.26}$$

5. The probability of the change of occupation o between t and $t+1$ is the product of the probabilities of the events in 4.1 and 4.2 above, which is

$$\theta_j(o|e_t, a_{t-1}, b_{t-1}) * (1 - \theta_j(o|e_{t+1}, a_t, b_t)) \tag{A.27}$$

6. Gross flow between t and $t+1$ for workers of type j, conditional on state (e_t, a_{t-1}, b_{t-1}), is

$$\sum_o \theta_j(o|e_t, a_{t-1}, b_{t-1}) * E_{e,t}(1 - \theta_j(o|e_{t+1}, a_t, b_t)), \tag{A.28}$$

where $E_{e,t}$ is the expectations operator with respect to idiosyncratic income shocks conditional on information at time t.

7. Integrating gross flows between t and $t+1$ over state (e_t, a_{t-1}, b_{t-1}) and summing over worker types j, yields gross flow between t and $t+1$

$$\sum_{j=1}^{J} m_j \sum_o \int_{e_t, a_{t-1}, b_{t-1}} \theta_j(o|e_t, a_{t-1}, b_{t-1}) * \sum_{e_{t+1}} (1 - \theta_j(o|e_{t+1}, a_t, b_t)) P(e_{t+1}|e_t) dD_j(e_t, a_{t-1}, b_{t-1}) \tag{A.29}$$

where $a_t = a(e_t, a_{t-1}, b_{t-1})$, $b_t = b(e_t, a_{t-1}, b_{t-1})$ and $P(e_{t+1}|e_t)$ is an element of the transition matrix (the matrix has a property $\sum_{e_{t+1}} P(e_{t+1}|e_t) = 1$).

Net flows across occupations. The net flows across occupations are as follows:

$$\frac{1}{2} \sum_o \left| \sum_{j=1}^{J} m_j \int_{e_{t+1}, a_t, b_t} \theta_j(o|e_{t+1}, a_t, b_t) dD_j(e_{t+1}, a_t, b_t) - \sum_{j=1}^{J} m_j \int_{e_t, a_{t-1}, b_{t-1}} \theta_j(o|e_t, a_{t-1}, b_{t-1}) dD_j(e_t, a_{t-1}, b_{t-1}) \right| \tag{A.30}$$

where $|\cdot|$ is the absolute value.[32]

32 One half in the equation is coming from the fact that inflow in one occupation is an outflow from another occupation and this flow should be calculated only once.

B. Appendix on Computation Method and Calibration

Adapted Solution Algorithm with Fixed Point Routine. The model is simulated by adapting the algorithm developed in Auclert et al. (2019). The underlying rationale of the method lies in finding a solution in sequence space, in which all operations like summation and multiplication are defined for sequences, see, for example, Dunford and Schwartz (1958)). Any variable in the model is therefore an infinite sequence. Multivariate Newton method is used to detect, for any variable, sequences such that the initial conditions are satisfied, namely the model delivers a deterministic steady state, and all variables' dynamic paths are generated by the model equations given specified shock sequences. The solution relies on an approximation of the infinite sequences with finite ones. Auclert et al. (2019) show that such an approximation is good enough when the finite sequences considered are long enough.

The algorithm offers a log-linear approximated solution, whereby the sequences return to the same steady state, and a fully non-linear solution, which can accommodate the possibility of transitioning across steady states. Our mode features a SIR block, which implies a transition to a new steady state. For this reason the non-linear solution is adopted and adjusted to accommodate transition between the two steady states. Technically, initial and final steady states are inserted into the Newton method. The Jacobian and errors are then computed accordingly. The steady states are obtained from the deterministic version of the model equations and by normalizing aggregate output to 1. The code is written in Python 3.7.

The household optimization problem in the main text 3.1 is solved in two-stages. Numerically this has been implemented through a guess and verify procedure with the following steps:

1. Guess future value function, $W_a(\mathbf{z}_{j,t}, a_t, b_t, \phi_{o,t+1})$, $W_b((\mathbf{z}_{j,t}, a_t, b_t, \phi_{o,t+1})$ where $W(\mathbf{z}_{j,t}, a_t, b_t, \phi_{o,t+1}) = \sum_{o=1}^{O} V(\mathbf{z}_{j,t}, a_t, b_t, \phi_{o,t+1}) p(o_{t+1}|a_t, b_t, \mathbf{z}_{j,t})$
2. For each occupation compute policy functions, i.e. calculate $a(\mathbf{z}_{j,t}, b_{t-1}, a_{t-1})$ and $b(\mathbf{z}_{j,t}, b_{t-1}, a_{t-1})$ $\forall o$ (using F.O.C. s from the optimization problem in Equation (4))
3. Feed policy functions into the value and update: $V^o(\mathbf{z}_{j,t}, a_{t-1}, b_{t-1})$

4. Compute occupation choice probabilities:

   $\delta(o_t|\mathbf{z}_{j,t}, b_{t-1}, a_{t-1}) = \frac{exp(V^o(\mathbf{z}_{j,t}, b_{t-1}, a_{t-1})}{\sum_{o'_t} exp(V^o(\mathbf{z}_{j,t}, b_{t-1}, a_{t-1})}$

5. Update guesses of $W_a(\mathbf{z}_{j,t}, a_t, b_t, \phi_{o,t+1})$, $W_b(\mathbf{z}_{j,t}, a_t, b_t, \phi_{o,t+1})$ using envelope conditions and probabilities from step 4.

Aggregation is done using steady state distribution of the idiosyncratic income shocks, liquid and illiquid assets and the probabilities of being in each occupation which we assume we can handle as masses of our population. Mathematically the guess is used for the value function that reads as:

$$\overline{V}_j(e_t, a_{t-1}, b_{t-1}) = log[\sum_{o_t} exp(\max_{a_t^o, b_t^o} u(\xi_{j,t}^o + (1 + r_t^a)a_{t-1} + (1 + r_t^b)b_{t-1} -$$

$$\Phi(a_t^o, a_{t-1}) - a_t^o - b_t^o) + \lambda_t(b_t^o - \underline{b}) + \mu_t a_t^o + \beta E_{e_{t+1}} \overline{V}_j(e_t, a_t^o, b_t^o))] \quad \text{(B.1)}$$

where $\overline{V}_j(e_t, a_{t-1}, b_{t-1}) = E_\phi V_j(e_t, a_{t-1}, b_{t-1}, \boldsymbol{\phi}_t)$. Using the above we compute the F.O.C.s with respect to liquid and illiquid assets as following:

$$u_c(c_t|o) = \lambda_t + \beta E_e \partial_b \overline{V}_j(e_{t+1}, a_t^o, b_t^o) \quad \text{(B.2)}$$

$$u_c(c_t|o)[1 + \Phi_1(a_t, a_{t-1})] = \mu_t + \beta E_e \partial_a \overline{V}_j(e_{t+1}, a_t^o, b_t^o) \quad \text{(B.3)}$$

Envelope conditions:

$$\partial_b \overline{V}_t(e_t, a_{t-1}, b_{t-1}) = \frac{\sum_{o_t} exp(\tilde{V}_t^o(e_t, b_{t-1}, a_{t-1}))u'(c_t^*|o_t)(1 + r_t^b)}{exp(\overline{V}_t(e_t, a_{t-1}, b_{t-1}))} \quad \text{(B.4)}$$

$$\partial_a \overline{V}_t(w_t, a_{t-1}, b_{t-1}) = \frac{\sum_{o_t} exp(\tilde{V}_t^o(e_t, b_{t-1}, a_{t-1}))u'(c_t^*|o_t)(1 + r_t^a - \Phi_2(a_t^*, a_{t-1}))}{exp(\overline{V}_t(e_t, a_{t-1}, b_{t-1}))} \quad \text{(B.5)}$$

where $u_c(c_t|o)$ is the marginal utility of consumption in occupation o. The above operators are inserted in the computational method for heterogeneous agents by Auclert et al. (2019).

Constructing Occupational Clusters Using k-means Algorithm. The 8 clusters obtained through k-means algorithm can be summarized as follows:

1. Cluster1, Manual trade occupations, predominantly includes occupations from the *Construction and extraction occupations*, *Installation, maintenance and repair occupations* and *Production occupations.*
2. Cluster 2, Management and supervisory occupations, includes *Management occupations*, *Business and financial operations occupations*, *Education, training and library occupations* and *Sales and related occupations.*, occupations of sales agents or supervisory workers from the *Sales and related occupations* group. Finally this cluster includes occupations from the social science field.
3. Cluster 3, Machine operators, includes *residential construction* occupations such as like carpet or drywall installers or masons, some occupations from *Production occupations* and others, such as machine operator from the *Transportation and material moving occupations* category.
4. Cluster 4, Engineering occupations, includes *Management occupations*, *Computer and mathematical science occupations*, *Life, physical and social science occupations* and *Architecture and engineering occupations.* The cluster also includes occupations, such as cashier and counter clerks from the *Sales and related occupations* group, but also scientific occupations in life and physical science (occupations contain the key word scientist)
5. Cluster 5, Healthcare and community occupations, is predominantly populated by *Healthcare practitioner and technical occupations* and almost all *Community and social service occupations.*
6. Cluster 6, Personal service occupations, mainly technical-support occupations from the larger *Architecture and engineering occupations*, *Life, physical and social science occupations* and *Computer and mathematical science occupations* groups.
7. Cluster 7, Technical-Support occupations, is mainly populated with occupations *Food*

Table 9: Occupational Clusters and Major Occupation Groups

	Occupation Clusters								
Detailed Occupation Groups	1	2	3	4	5	6	7	8	Total
Management occupations	0	19	0	4	1	2	0	0	26
Business and financial operations occupations	0	17	0	2	1	0	0	5	25
Computer and mathematical science occupations	0	4	0	6	0	1	0	0	11
Architecture and engineering occupations	0	0	0	18	0	3	0	0	21
Life, physical, and social science occupations	0	7	0	9	0	7	0	0	23
Community and social service occupation	0	3	0	0	2	0	0	0	5
Legal occupations	0	2	0	0	0	0	0	2	4
Education, training, and library occupations	0	7	0	0	0	0	3	1	11
Arts, design, entertainment, sports, and media occupations	0	5	0	0	0	6	3	3	17
Healthcare practitioner and technical occupations	0	1	0	0	23	2	0	1	27
Healthcare support occupations	0	0	0	0	2	1	3	0	6
Protective service occupations	2	1	0	0	7	2	4	0	16
Food preparation and serving related occupations	0	0	1	0	0	2	9	0	12
Building and grounds cleaning and maintenance occupations	0	0	2	0	0	2	2	0	6
Personal care and service occupations	0	2	1	0	1	1	12	1	18
Sales and related occupations	0	7	0	1	0	0	5	4	17
Office and administrative support occupations	0	3	3	0	0	2	10	30	48
Farming, fishing, and forestry occupations	3	0	2	0	0	2	1	0	8
Construction and extraction occupations	20	0	19	0	0	1	0	0	40
Installation, maintenance, and repair occupations	30	0	6	0	0	1	0	0	37
Production occupations	10	0	56	0	0	5	4	0	75
Transportation and material moving occupations	6	0	20	0	0	5	2	0	33
Total	71	78	110	40	37	45	58	47	486

preparation and serving related occupations, *Sales and related occupations*, *Healthcare support occupations* and *Personal care and service occupations*.

8. Cluster 8, Office and administrative support occupations, consists mainly of *Office and administrative support occupations*.

Table 9 cross-tabulates the major occupation group and our assignment to one of the 8 clusters for the 3-digit occupations.

Calibrating the Γ Matrix. Each household j holds an absolute advantage for the occupation along the diagonal. A measure of skill-transferability is then constructed as the Euclidean distance between skill that household j possesses for occupation $j = o$ and that required in occupation o'. In the data the skill distance between occupational clusters o and o' is the sum of the weighted (by the employment shares in 2019) absolute difference between the elements of H_o and $H_{o'}$:

$$\gamma_j^o = \sum_m^M \text{abs} \left| h_m^j - h_m^o \right| , \tag{B.6}$$

This implies that the diagonal entries are the smallest.[33] To translate the Euclidean distance into a comparative advantage, we re-normalize every row so that the diagonal entries become all ones. This implies that in the final matrix higher values correspond to higher comparative advantage in the specific occupation. The characterization described so far captures the concept of horizontal or core-task specialization. In the data however occupations are also hierarchical. To capture also vertical specialization, each entry is multiplied by the share of occupations in cluster o with the same or higher educational attainment. The latter is computed as the mode of educational attainment for all occupations in the cluster using the 2018 BLS data and considering three levels: (1) less than college, (2) college, and (3) more than college. The resulting parameters for the matrix Γ of our baseline calibration are reported in table table 10 in Appendix appendix B. At last, in the baseline parametrization, the shares of households with skill j is distributed uniformly, i.e., $\mu_j = 1/J = 1/8$ for all $j \in J$.

The calibration of the Γ matrix resulting from the k-means clustering is reported in table table 10.

33 Note that the entries are never zeros: since those are grouped occupations the set of skills is an average across all the occupations in the cluster.

Table 10: Skill-distribution Γ matrix. Each row represents one type of households. Each column represents skills that different types of households have. The higher the entry in the matrix, the more skilled is the type for that occupation. Note that the diagonal has been normalized to one so as to make all other entries comparable in relative terms.

Type of households	**Occ. 1**	**Occ. 2**	**Occ. 3**	**Occ. 4**	**Occ. 5**	**Occ. 6**	**Occ. 7**	**Occ. 8**
Type 1	1	0.34	0.26	0.33	0.38	0.54	0.19	0.21
Type 2	0.022	1	0.019	0.27	0.57	0.12	0.029	0.097
Type 3	0.38	0.32	1	0.28	0.34	0.48	0.35	0.24
Type 4	0.018	0.59	0.014	1	0.45	0.13	0.017	0.07
Type 5	0.037	0.58	0.03	0.27	1	0.25	0.038	0.12
Type 6	0.14	0.57	0.11	0.49	0.58	1	0.12	0.22
Type 7	0.18	0.46	0.23	0.32	0.42	0.45	1	0.35
Type 8	0.083	0.68	0.085	0.42	0.42	0.37	0.14	1

C. Other Results

Long Run Wealth Distributions. Below we report the long run wealth distribution in the baseline calibration of our model. The distribution features the classical leptokurtotic shape.

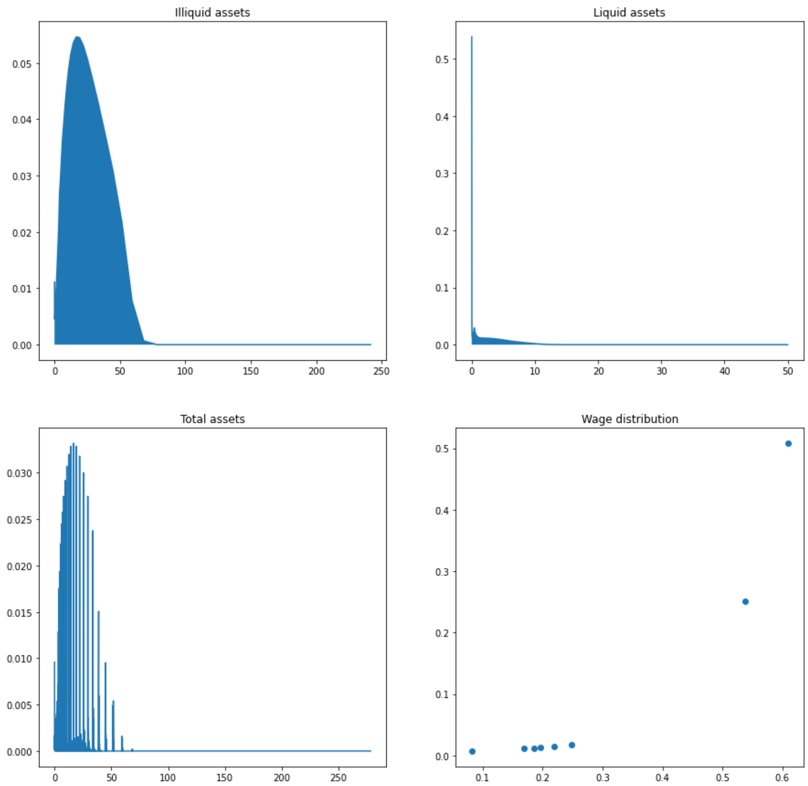

Figure 9: Distributions in the Steady State. The figure shows the distribution of total wealth, liquid/non-liquid assets or wages. X-axis represents wealth/wage values and Y-axis represents probability densities. The wage distribution plots wages for each of the eight occupations (x-axis) against the share of total population working in each occupation. The non-liquid and the wealth distributions are cut at the upper tail (the total probability of the cut parts is 10^{-13}).

Monetary Policy Shock More Variables. Below we report the impulse response of other variables to a 25 basis points monetary policy shock in the baseline calibration of the model.

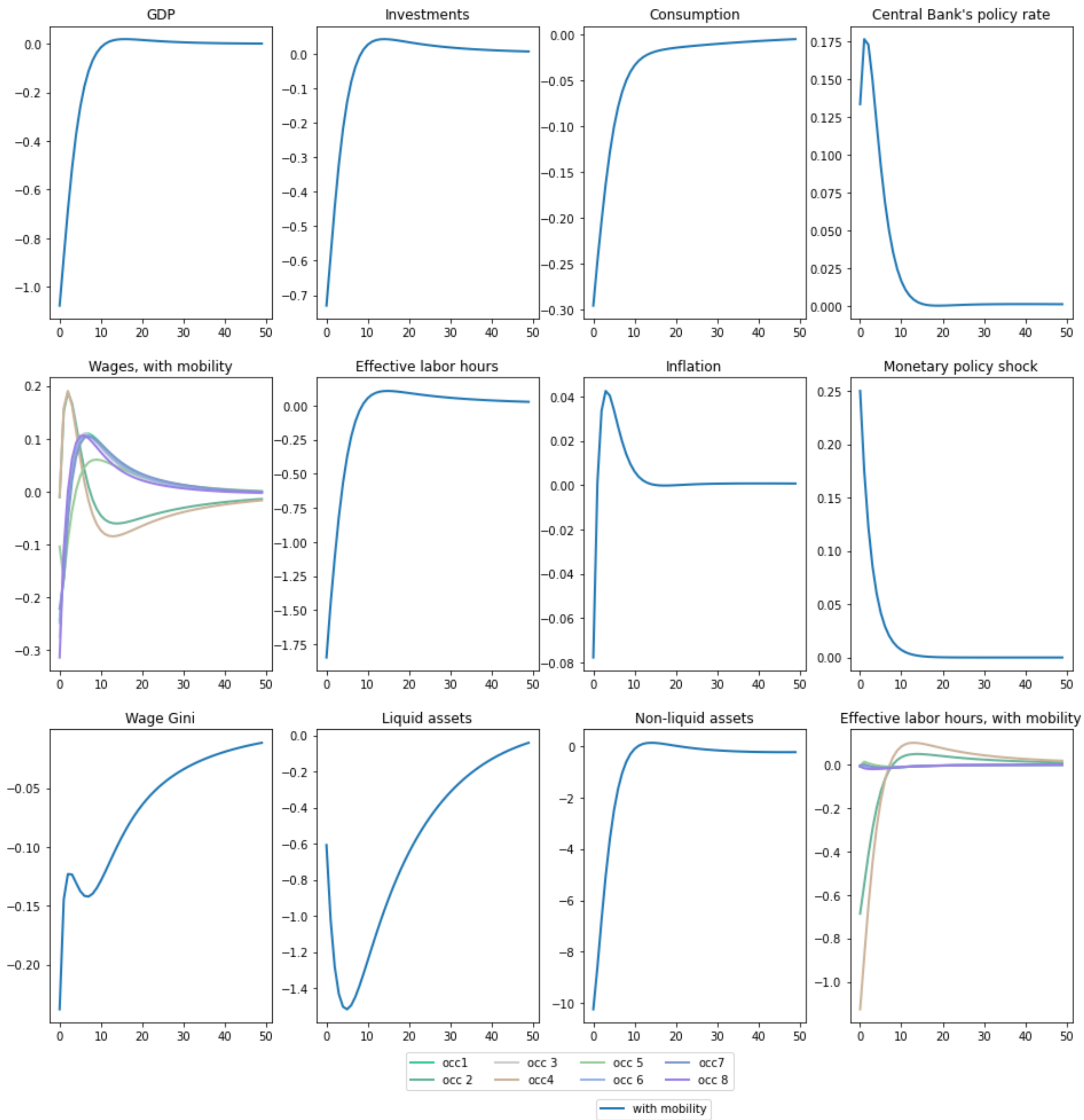

Figure 10: Impulse Responses to a Monetary Policy Shock.